

Polynomials

Question Paper, Set 04

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $-2x^3y^3$ a monomial? Give reason. [1 mark]
2. Write the degree of $-8x^2y + 2x^2 - 5y^5 - 4x^4y^5 + 1$, showing the degree of each term. [2 marks]
3. In $p(x) = 4x^2 + 5 - 9x^4$, what is the coefficient of x^2 ? [1 mark]
4. Is $p(x) = -2x + 4 - 5x^4$ a linear, quadratic, cubic or bi-quadratic polynomial (or a constant)? [1 mark]
5. If $(p + 4)y^2 + (q - 3p)y + (r + 3q)$ is a zero polynomial, find the values of p , q and r . [2 marks]
6. Which of the following are quadratic equations? (i) $x^2 + \frac{1}{x^2} = 9$ (ii) $7x^2 - 1 = 0$ (iii) $5\sqrt{5}x^2 - 7x - \sqrt{5} = 0$ [2 marks]
7. Find the value of $p(x) = -2x^2 + 8x - 2$ at $x = -4$. [1 mark]
8. Find a , if $-\frac{1}{2}$ is a zero of $p(x) = 8x^3 - 6x^2 + ax - 2$. [2 marks]
9. Solve $9x^2 - 15x - 14 = 0$ by splitting the middle term. [3 marks]
10. Find $\Delta = b^2 - 4ac$ for $2x^2 - 7x - 8 = 0$. [1 mark]
11. Divide $-6x^3 + 27x^2 - 35x + 58$ by $7 - 2x$ and write the quotient and remainder. [5 marks]
12. If the polynomials $ax^3 + 2x^2 - x + 7$ and $x^3 - x^2 - a$ leave the same remainder when divided by $x - 2$, find the value of a . [3 marks]
13. Using the factor theorem, show that $x - 5$ is a factor of $f(x) = 2x^3 - 9x^2 + x - 30$. [2 marks]
14. Factorise $15x^2 + 29x + 8$ by splitting the middle term. [2 marks]
15. Find the values of p and q so that $x^4 + px^3 - 10x^2 + 9x + q$ is divisible by $x^2 - 9$. [5 marks]
16. If $x + 2$ is a factor of $x^3 + 4x^2 - 20x - 48$, factorise it completely. [3 marks]
17. Factorise $x^4 - 5x^3 + 5x^2 + 5x - 6$ completely. [5 marks]
18. Divide $x^3 + 2x^2 - 2x - 11$ by $x - 3$ by synthetic division. [3 marks]

- 19.** Find the remainder when $24^{42} - 17^{42}$ is divided by 7. [1 mark]
-
- 20.** Find the quadratic polynomial $f(x)$ which leaves the remainders 1, -4 and 4 when divided by $x + 3$, $x + 2$ and $x + 1$ respectively. [5 marks]
-

Key sheet

Q	Answer
1	Yes, the powers of x and y are 3 and 3, non-negative integers
2	9
3	4
4	Bi-quadratic polynomial
5	$p = -4, q = -12, r = 36$
6	(ii) and (iii)
7	-66
8	$a = -9$
9	$x = -\frac{2}{3}, \frac{7}{3}$
10	$\Delta = 113$
11	Quotient = $3x^2 - 3x + 7$, Remainder = 9
12	$a = -1$
13	$f(5) = 0$, so $x - 5$ is a factor
14	$(3x + 1)(5x + 8)$
15	$p = -1, q = 9$
16	$(x + 2)(x - 4)(x + 6)$
17	$(x - 1)(x + 1)(x - 2)(x - 3)$
18	Quotient = $x^2 + 5x + 13$, Remainder = 28
19	0
20	$f(x) = \frac{13}{2}x^2 + \frac{55}{2}x + 25$

Solutions

1. Is $-2x^3y^3$ a monomial? Give reason. [1]

In $-2x^3y^3$, the powers of x and y are 3 and 3, non-negative integers.

So $-2x^3y^3$ is a monomial.

Ans: Yes, the powers of x and y are 3 and 3, non-negative integers

2. Write the degree of $-8x^2y + 2x^2 - 5y^5 - 4x^4y^5 + 1$, showing the degree of each term. [2]

Degrees of the terms: 3, 2, 5, 9, 0

Greatest among them = 9

So the degree of the polynomial is 9.

Ans: 9

3. In $p(x) = 4x^2 + 5 - 9x^4$, what is the coefficient of x^2 ? [1]

The term with x^2 in $p(x)$ is $4x^2$.

So the coefficient of x^2 is 4.

Ans: 4

4. Is $p(x) = -2x + 4 - 5x^4$ a linear, quadratic, cubic or bi-quadratic polynomial (or a constant)? [1]

The highest power of x is 4, so the degree is 4.

A polynomial of degree 4 is a bi-quadratic polynomial.

Ans: Bi-quadratic polynomial

5. If $(p + 4)y^2 + (q - 3p)y + (r + 3q)$ is a zero polynomial, find the values of p , q and r . [2]

In a zero polynomial all the coefficients are zero.

$$p + 4 = 0 \Rightarrow p = -4$$

$$q - 3p = 0 \Rightarrow q + 12 = 0 \Rightarrow q = -12$$

$$r + 3q = 0 \Rightarrow r - 36 = 0 \Rightarrow r = 36$$

Ans: $p = -4, q = -12, r = 36$

6. Which of the following are quadratic equations? (i) $x^2 + \frac{1}{x^2} = 9$ (ii) $7x^2 - 1 = 0$ (iii) $5\sqrt{5}x^2 - 7x - \sqrt{5} = 0$ [2]

(i) $\frac{1}{x^2} = 1x^{-2}$ has a negative power, so it is not a quadratic equation.

(ii) It has the form $ax^2 + bx + c = 0$ with $a \neq 0$.

(iii) It has the form $ax^2 + bx + c = 0$ with $a \neq 0$.

Ans: (ii) and (iii)

7. Find the value of $p(x) = -2x^2 + 8x - 2$ at $x = -4$. [1]

$$p(-4) = -2(-4)^2 + 8(-4) - 2$$

$$= -32 - 32 - 2$$

$$= -66$$

Ans: -66

8. Find a , if $-\frac{1}{2}$ is a zero of $p(x) = 8x^3 - 6x^2 + ax - 2$. [2]

$$\begin{aligned} p\left(-\frac{1}{2}\right) &= 8\left(-\frac{1}{2}\right)^3 - 6\left(-\frac{1}{2}\right)^2 + a\left(-\frac{1}{2}\right) - 2 \\ &= -1 - \frac{3}{2} - \frac{1}{2}a - 2 \\ &= -\frac{1}{2}a - \frac{9}{2} \end{aligned}$$

Given $p\left(-\frac{1}{2}\right) = 0$:

$$\begin{aligned} -\frac{1}{2}a - \frac{9}{2} &= 0 \\ \Rightarrow -\frac{1}{2}a &= \frac{9}{2} \\ \Rightarrow a &= -9 \end{aligned}$$

Ans: $a = -9$

9. Solve $9x^2 - 15x - 14 = 0$ by splitting the middle term. [3]

$$\begin{aligned} 9 \times (-14) &= -126 = 6 \times (-21) \text{ and } 6 + (-21) = -15 \\ 9x^2 - 15x - 14 &= 9x^2 + 6x - 21x - 14 \\ &= 3x(3x + 2) - 7(3x + 2) \\ &= (3x + 2)(3x - 7) \\ \Rightarrow 3x + 2 &= 0 \text{ (or) } 3x - 7 = 0 \\ \Rightarrow x &= -\frac{2}{3}, \frac{7}{3} \end{aligned}$$

Ans: $x = -\frac{2}{3}, \frac{7}{3}$

10. Find $\Delta = b^2 - 4ac$ for $2x^2 - 7x - 8 = 0$. [1]

$$\begin{aligned} a &= 2, b = -7, c = -8 \\ \Delta &= b^2 - 4ac = (-7)^2 - 4(2)(-8) \\ &= 49 + 64 = 113 \end{aligned}$$

Ans: $\Delta = 113$

11. Divide $-6x^3 + 27x^2 - 35x + 58$ by $7 - 2x$ and write the quotient and remainder. [5]

Write the divisor in descending powers: $7 - 2x = -2x + 7$

$$-6x^3 \div (-2x) = 3x^2; 3x^2 \times (-2x + 7) = -6x^3 + 21x^2; \text{ subtract: } 6x^2 - 35x + 58$$

$$6x^2 \div (-2x) = -3x; (-3x) \times (-2x + 7) = 6x^2 - 21x; \text{ subtract: } -14x + 58$$

$$-14x \div (-2x) = 7; 7 \times (-2x + 7) = -14x + 49; \text{ subtract: } 9$$

Quotient = $3x^2 - 3x + 7$, Remainder = 9

Ans: Quotient = $3x^2 - 3x + 7$, Remainder = 9

12. If the polynomials $ax^3 + 2x^2 - x + 7$ and $x^3 - x^2 - a$ leave the same remainder when divided by $x - 2$, find the value of a . [3]

Let $f(x) = ax^3 + 2x^2 - x + 7$ and $g(x) = x^3 - x^2 - a$.

Given $f(2) = g(2)$

$$f(2) = a(2)^3 + 2(2)^2 - (2) + 7$$

$$= 8a + 8 - 2 + 7$$

$$= 8a + 13$$

$$g(2) = (2)^3 - (2)^2 - a$$

$$= 8 - 4 - a$$

$$= -a + 4$$

$$8a + 13 = -a + 4$$

$$\Rightarrow 9a = -9$$

$$\Rightarrow a = -1$$

Ans: $a = -1$

13. Using the factor theorem, show that $x - 5$ is a factor of $f(x) = 2x^3 - 9x^2 + x - 30$. [2]

Let $f(x) = 2x^3 - 9x^2 + x - 30$; $x - 5 = 0 \Rightarrow x = 5$

$$f(5) = 2(5)^3 - 9(5)^2 + (5) - 30$$

$$= 250 - 225 + 5 - 30$$

$$= 0$$

$\Rightarrow x - 5$ is a factor of $f(x)$.

Ans: $f(5) = 0$, so $x - 5$ is a factor

14. Factorise $15x^2 + 29x + 8$ by splitting the middle term. [2]

$$15 \times 8 = 120 = 5 \times 24 \text{ and } 5 + 24 = 29$$

$$15x^2 + 29x + 8 = 15x^2 + 5x + 24x + 8$$

$$= 5x(3x + 1) + 8(3x + 1)$$

$$= (3x + 1)(5x + 8)$$

Ans: $(3x + 1)(5x + 8)$

15. Find the values of p and q so that $x^4 + px^3 - 10x^2 + 9x + q$ is divisible by $x^2 - 9$. [5]

$$x^2 - 9 = (x + 3)(x - 3)$$

Let $f(x) = x^4 + px^3 - 10x^2 + 9x + q$; $x + 3$ and $x - 3$ are factors.

$$f(-3) = 0:$$

$$f(-3) = (-3)^4 + p(-3)^3 - 10(-3)^2 + 9(-3) + q$$

$$= 81 - 27p - 90 - 27 + q$$

$$= -27p + q - 36$$

$$\Rightarrow 27p - q = -36 \dots (1)$$

$$f(3) = 0:$$

$$f(3) = (3)^4 + p(3)^3 - 10(3)^2 + 9(3) + q$$

$$= 81 + 27p - 90 + 27 + q$$

$$= 27p + q + 18$$

$$\Rightarrow 27p + q = -18 \dots (2)$$

$$(1) + (2): 54p = -54$$

$$\Rightarrow p = -1$$

$$\text{Put } p = -1 \text{ in (1): } -27 - q = -36 \Rightarrow q = 9$$

$$\text{Ans: } p = -1, q = 9$$

16. If $x + 2$ is a factor of $x^3 + 4x^2 - 20x - 48$, factorise it completely. [3]

Divide $x^3 + 4x^2 - 20x - 48$ by $x + 2$:

$$x^3 \div x = x^2; x^2 \times (x + 2) = x^3 + 2x^2; \text{ subtract: } 2x^2 - 20x - 48$$

$$2x^2 \div x = 2x; 2x \times (x + 2) = 2x^2 + 4x; \text{ subtract: } -24x - 48$$

$$-24x \div x = -24; (-24) \times (x + 2) = -24x - 48; \text{ subtract: } 0$$

$$\therefore x^3 + 4x^2 - 20x - 48 = (x + 2)(x^2 + 2x - 24)$$

$$x^2 + 2x - 24 = x^2 - 4x + 6x - 24$$

$$= x(x - 4) + 6(x - 4)$$

$$= (x - 4)(x + 6)$$

$$\therefore x^3 + 4x^2 - 20x - 48 = (x + 2)(x - 4)(x + 6)$$

$$\text{Ans: } (x + 2)(x - 4)(x + 6)$$

17. Factorise $x^4 - 5x^3 + 5x^2 + 5x - 6$ completely. [5]

Let $f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$. By trial and error:

$$f(1) = 1 - 5 + 5 + 5 - 6 = 0$$

$$f(-1) = 1 + 5 + 5 - 5 - 6 = 0$$

$\Rightarrow x - 1$ and $x + 1$ are factors, so $(x - 1)(x + 1) = x^2 - 1$ is a factor

$$x^4 \div x^2 = x^2; x^2 \times (x^2 - 1) = x^4 - x^2; \text{ subtract: } -5x^3 + 6x^2 + 5x - 6$$

$$-5x^3 \div x^2 = -5x; (-5x) \times (x^2 - 1) = -5x^3 + 5x; \text{ subtract: } 6x^2 - 6$$

$$6x^2 \div x^2 = 6; 6 \times (x^2 - 1) = 6x^2 - 6; \text{ subtract: } 0$$

$$\therefore f(x) = (x^2 - 1)(x^2 - 5x + 6)$$

$$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$$

$$= x(x - 2) - 3(x - 2)$$

$$= (x - 2)(x - 3)$$

$$\therefore f(x) = (x - 1)(x + 1)(x - 2)(x - 3)$$

$$\text{Ans: } (x - 1)(x + 1)(x - 2)(x - 3)$$

18. Divide $x^3 + 2x^2 - 2x - 11$ by $x - 3$ by synthetic division. [3]

$$x - 3 = 0 \Rightarrow x = 3$$

Write the coefficients and 3 on the left: $3 \mid 1 \quad 2 \quad -2 \quad -11$

Bring down the first number; multiply by 3 and add, each time.

Second row: 0 3 15 39

Bottom row: 1 5 13 28

$$\text{Quotient} = x^2 + 5x + 13, \text{ Remainder} = 28$$

$$\text{Ans: Quotient} = x^2 + 5x + 13, \text{ Remainder} = 28$$

19. Find the remainder when $24^{42} - 17^{42}$ is divided by 7. [1]

$x - a$ is a factor of $x^n - a^n$ for every positive integer n .

So $24 - 17 = 7$ divides $24^{42} - 17^{42}$.

$$\text{Remainder} = 0$$

$$\text{Ans: } 0$$

20. Find the quadratic polynomial $f(x)$ which leaves the remainders 1, -4 and 4 when divided by $x + 3$, $x + 2$ and $x + 1$ respectively. [5]

Let $f(x) = ax^2 + bx + c$. The remainder on dividing by $x - k$ is $f(k)$.

$$f(-3) = 1 \Rightarrow 9a - 3b + c = 1 \dots (1)$$

$$f(-2) = -4 \Rightarrow 4a - 2b + c = -4 \dots (2)$$

$$f(-1) = 4 \Rightarrow a - b + c = 4 \dots (3)$$

$$(2) - (1): -5a + b = -5 \dots (4)$$

$$(3) - (2): -3a + b = 8 \dots (5)$$

$$(5) - (4): 2a = 13 \Rightarrow a = \frac{13}{2}$$

$$\text{From (4): } b = \frac{55}{2}; \text{ from (1): } c = 25$$

$$\therefore f(x) = \frac{13}{2}x^2 + \frac{55}{2}x + 25$$

$$\text{Ans: } f(x) = \frac{13}{2}x^2 + \frac{55}{2}x + 25$$