

Polynomials

Question Paper, Set 03

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Which of the following are monomials? Give reasons. (i) $-8\sqrt{xy}^2$ (ii) $\frac{7}{3}x^4$ (iii) $2x^{\frac{1}{3}}y$ (iv) $-4x^2y^4$ [2 marks]
2. Write the degree of the polynomial $-8x - 2x^3 + 5x^5 + 9$. [1 mark]
3. Write $p(x) = -9x^2 + 5x^3 + 7 + 2x$ in standard form. [1 mark]
4. Is $4\sqrt{2}x - 6x^{\frac{2}{3}} + 2$ a multinomial? Is it a polynomial? [1 mark]
5. If $ax^{n-3} + bx^{n-4} + cx^{n-5}$ is a bi-quadratic polynomial, where $n \in \mathbb{N}$, find the value of n . [2 marks]
6. What type of equation is $-x^3 - 5x^2 - 6x = 4$? [1 mark]
7. If $p(x) = 5x^3 + 2x^2 + x + 8$, find $p(-3)$. [2 marks]
8. Solve $x^2 - 10x + 24 = 0$ by splitting the middle term. [2 marks]
9. Using the formula, find the roots of $2x^2 - 3x + 1 = 0$. [2 marks]
10. Divide $x^3 + x^2 - 6x - 11$ by $x + 2$ and write the quotient and remainder. [3 marks]
11. Find the remainder when $f(x) = 4x^2 - 4x + 4$ is divided by $x - 3$. [1 mark]
12. Find the value of a for which $x^4 - 6x^3 + 6x^2 - ax + 3$ is divisible by $1 - x$. [3 marks]
13. What must be subtracted from $-4x^4 + 6x^3 + 25x^2 + 21x + 5$ so that the result is exactly divisible by $x^2 - 3x - 2$? [5 marks]
14. Find the values of p and q so that $x^4 + px^3 - 8x^2 + 12x + q$ is divisible by $x^2 - 4$. [5 marks]
15. Find the zeros of $15x^2 + 49x + 40$ using the formula and hence factorise it. [3 marks]
16. Factorise $2x^3 + 7x^2 - 3x - 18$ by trial and error. [5 marks]
17. Is $x - 4$ a factor of $x^3 - 64$? Give reason. [1 mark]
18. Show that $27^{120} - 11^{120}$ is divisible by both 16 and 38. [2 marks]
19. Divide $\frac{8}{27}a^3 + \frac{8}{3}a^2x + 8ax^2 + 8x^3$ by $\frac{2}{3}a + 2x$. [3 marks]

- 20.** Find the quadratic polynomial $f(x)$ which leaves the remainders -2 , 10 and 2 when divided by $x + 1$, $x - 2$ and $x - 3$ respectively. [5 marks]
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Key sheet

Q	Answer
1	(ii) and (iv) are monomials
2	5
3	$p(x) = 5x^3 - 9x^2 + 2x + 7$
4	It is a multinomial but not a polynomial
5	$n = 7$
6	Cubic equation
7	-112
8	$x = 4, 6$
9	$x = 1, \frac{1}{2}$
10	Quotient = $x^2 - x - 4$, Remainder = -3
11	28
12	$a = 4$
13	$6x + 3$
14	$p = -3, q = 16$
15	$(3x + 5)(5x + 8)$
16	$(x + 2)(2x - 3)(x + 3)$
17	Yes, $f(4) = 0$
18	Divisible by 16 and by 38
19	Quotient = $\frac{4}{9}a^2 + \frac{8}{3}ax + 4x^2$
20	$f(x) = -3x^2 + 7x + 8$

Solutions

1. Which of the following are monomials? Give reasons. (i) $-8\sqrt{x}y^2$ (ii) $\frac{7}{3}x^4$ (iii) $2x^{\frac{1}{3}}y$ (iv) $-4x^2y^4$ [2]

(i) $-8\sqrt{x}y^2$: $\sqrt{x} = x^{\frac{1}{2}}$, the power $\frac{1}{2}$ is not a whole number, so it is not a monomial.

(ii) $\frac{7}{3}x^4$: the powers of x are 4, non-negative integers, so it is a monomial.

(iii) $2x^{\frac{1}{3}}y$: the power $\frac{1}{3}$ of x is not a whole number, so it is not a monomial.

(iv) $-4x^2y^4$: the powers of x and y are 2 and 4, non-negative integers, so it is a monomial.

Ans: (ii) and (iv) are monomials

2. Write the degree of the polynomial $-8x - 2x^3 + 5x^5 + 9$. [1]

The greatest power of x in the polynomial is 5.

So its degree is 5.

Ans: 5

3. Write $p(x) = -9x^2 + 5x^3 + 7 + 2x$ in standard form. [1]

Standard form: terms in descending powers of x .

$$p(x) = 5x^3 - 9x^2 + 2x + 7$$

Ans: $p(x) = 5x^3 - 9x^2 + 2x + 7$

4. Is $4\sqrt{2}x - 6x^{\frac{2}{3}} + 2$ a multinomial? Is it a polynomial? [1]

$4\sqrt{2}x - 6x^{\frac{2}{3}} + 2$ is an algebraic expression with three terms, so it is a multinomial.

The power $\frac{2}{3}$ of x is not a whole number, so it is not a polynomial.

Ans: It is a multinomial but not a polynomial

5. If $ax^{n-3} + bx^{n-4} + cx^{n-5}$ is a bi-quadratic polynomial, where $n \in \mathbb{N}$, find the value of n . [2]

The degree of a bi-quadratic polynomial is 4.

So the highest power: $n - 3 = 4$

$$\Rightarrow n = 7$$

Ans: $n = 7$

6. What type of equation is $-x^3 - 5x^2 - 6x = 4$? [1]

The highest power of x in the equation is 3.

So it is a cubic equation.

Ans: Cubic equation

7. If $p(x) = 5x^3 + 2x^2 + x + 8$, find $p(-3)$. [2]

$$p(-3) = 5(-3)^3 + 2(-3)^2 + (-3) + 8$$

$$= -135 + 18 - 3 + 8$$

$$= -112$$

Ans: -112

8. Solve $x^2 - 10x + 24 = 0$ by splitting the middle term. [2]

$$1 \times 24 = 24 = (-4) \times (-6) \text{ and } (-4) + (-6) = -10$$

$$x^2 - 10x + 24 = x^2 - 4x - 6x + 24$$

$$= x(x - 4) - 6(x - 4)$$

$$= (x - 4)(x - 6)$$

$$\Rightarrow x - 4 = 0 \text{ (or) } x - 6 = 0$$

$$\Rightarrow x = 4, 6$$

Ans: $x = 4, 6$

9. Using the formula, find the roots of $2x^2 - 3x + 1 = 0$. [2]

$$a = 2, b = -3, c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(2)(1)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{1}}{4}$$

$$= \frac{3+1}{4}, \frac{3-1}{4} = 1, \frac{1}{2}$$

Ans: $x = 1, \frac{1}{2}$

10. Divide $x^3 + x^2 - 6x - 11$ by $x + 2$ and write the quotient and remainder. [3]

$$x^3 \div x = x^2; x^2 \times (x + 2) = x^3 + 2x^2; \text{ subtract: } -x^2 - 6x - 11$$

$$-x^2 \div x = -x; (-x) \times (x + 2) = -x^2 - 2x; \text{ subtract: } -4x - 11$$

$$-4x \div x = -4; (-4) \times (x + 2) = -4x - 8; \text{ subtract: } -3$$

$$\text{Quotient} = x^2 - x - 4, \text{ Remainder} = -3$$

Ans: **Quotient** $= x^2 - x - 4$, **Remainder** $= -3$

11. Find the remainder when $f(x) = 4x^2 - 4x + 4$ is divided by $x - 3$. [1]

$$x - 3 = 0 \Rightarrow x = 3$$

By the remainder theorem, remainder $= f(3)$

$$f(3) = 4(3)^2 - 4(3) + 4$$

$$= 36 - 12 + 4$$

$$= 28$$

Ans: 28

12. Find the value of a for which $x^4 - 6x^3 + 6x^2 - ax + 3$ is divisible by $1 - x$. [3]

$$1 - x = 0 \Rightarrow x = 1$$

Divisible means remainder $= f(1) = 0$

$$f(1) = (1)^4 - 6(1)^3 + 6(1)^2 - a(1) + 3$$

$$= 1 - 6 + 6 - a + 3$$

$$= -a + 4$$

$$-a + 4 = 0$$

$$\Rightarrow -a = -4$$

$$\Rightarrow a = 4$$

Ans: $a = 4$

13. What must be subtracted from $-4x^4 + 6x^3 + 25x^2 + 21x + 5$ so that the result is exactly divisible by $x^2 - 3x - 2$? [5]

$$-4x^4 \div x^2 = -4x^2; (-4x^2) \times (x^2 - 3x - 2) = -4x^4 + 12x^3 + 8x^2; \text{ subtract: } -6x^3 + 17x^2 + 21x + 5$$

$$-6x^3 \div x^2 = -6x; (-6x) \times (x^2 - 3x - 2) = -6x^3 + 18x^2 + 12x; \text{ subtract: } -x^2 + 9x + 5$$

$$-x^2 \div x^2 = -1; (-1) \times (x^2 - 3x - 2) = -x^2 + 3x + 2; \text{ subtract: } 6x + 3$$

$$\text{Remainder} = 6x + 3$$

Subtracting the remainder leaves an exact multiple: $6x + 3$ must be subtracted.

Ans: $6x + 3$

14. Find the values of p and q so that $x^4 + px^3 - 8x^2 + 12x + q$ is divisible by $x^2 - 4$. [5]

$$x^2 - 4 = (x + 2)(x - 2)$$

Let $f(x) = x^4 + px^3 - 8x^2 + 12x + q$; $x + 2$ and $x - 2$ are factors.

$$f(-2) = 0:$$

$$f(-2) = (-2)^4 + p(-2)^3 - 8(-2)^2 + 12(-2) + q$$

$$= 16 - 8p - 32 - 24 + q$$

$$= -8p + q - 40$$

$$\Rightarrow 8p - q = -40 \dots (1)$$

$$f(2) = 0:$$

$$f(2) = (2)^4 + p(2)^3 - 8(2)^2 + 12(2) + q$$

$$= 16 + 8p - 32 + 24 + q$$

$$= 8p + q + 8$$

$$\Rightarrow 8p + q = -8 \dots (2)$$

$$(1) + (2): 16p = -48$$

$$\Rightarrow p = -3$$

$$\text{Put } p = -3 \text{ in (1): } -24 - q = -40 \Rightarrow q = 16$$

Ans: $p = -3, q = 16$

15. Find the zeros of $15x^2 + 49x + 40$ using the formula and hence factorise it. [3]

Zeros of $ax^2 + bx + c$ are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$a = 15, b = 49, c = 40$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-49 \pm \sqrt{2401 - 4(15)(40)}}{2(15)}$$

$$= \frac{-49 \pm \sqrt{1}}{30}$$

$$= \frac{-49+1}{30}, \frac{-49-1}{30} = -\frac{8}{5}, -\frac{5}{3}$$

$$\therefore 15x^2 + 49x + 40 = 15 \left(x + \frac{8}{5}\right) \left(x + \frac{5}{3}\right)$$

$$= (3x + 5)(5x + 8)$$

Ans: $(3x + 5)(5x + 8)$

16. Factorise $2x^3 + 7x^2 - 3x - 18$ by trial and error. [5]

Let $f(x) = 2x^3 + 7x^2 - 3x - 18$. By trial and error:

$$f(1) = 2 + 7 - 3 - 18 \neq 0$$

$$f(-1) = -2 + 7 + 3 - 18 \neq 0$$

$$f(2) = 16 + 28 - 6 - 18 \neq 0$$

$$f(-2) = -16 + 28 + 6 - 18 = 0$$

$\Rightarrow x + 2$ is a factor of $f(x)$

$$2x^3 \div x = 2x^2; 2x^2 \times (x + 2) = 2x^3 + 4x^2; \text{ subtract: } 3x^2 - 3x - 18$$

$$3x^2 \div x = 3x; 3x \times (x + 2) = 3x^2 + 6x; \text{ subtract: } -9x - 18$$

$$-9x \div x = -9; (-9) \times (x + 2) = -9x - 18; \text{ subtract: } 0$$

$$\therefore f(x) = (x + 2)(2x^2 + 3x - 9)$$

$$2x^2 + 3x - 9 = 2x^2 - 3x + 6x - 9$$

$$= x(2x - 3) + 3(2x - 3)$$

$$= (2x - 3)(x + 3)$$

$$\therefore f(x) = (x + 2)(2x - 3)(x + 3)$$

$$\text{Ans: } (x + 2)(2x - 3)(x + 3)$$

17. Is $x - 4$ a factor of $x^3 - 64$? Give reason. [1]

$$x^3 - 64 = x^3 - 4^3$$

$$f(4) = (4)^3 - 64 = 0$$

So $x - 4$ is a factor.

$$\text{Ans: Yes, } f(4) = 0$$

18. Show that $27^{120} - 11^{120}$ is divisible by both 16 and 38. [2]

$x - a$ is a factor of $x^n - a^n$ for every positive integer n : so $27 - 11 = 16$ divides it.

$x + a$ is also a factor of $x^n - a^n$ when n is even.

Here $n = 120$ is even, so $27 + 11 = 38$ also divides it.

Ans: Divisible by 16 and by 38

19. Divide $\frac{8}{27}a^3 + \frac{8}{3}a^2x + 8ax^2 + 8x^3$ by $\frac{2}{3}a + 2x$. [3]

$$\frac{8}{27}a^3 \div \frac{2}{3}a = \frac{4}{9}a^2; \frac{4}{9}a^2 \times (\frac{2}{3}a + 2x) = \frac{8}{27}a^3 + \frac{8}{9}a^2x; \text{ subtract: } \frac{16}{9}a^2x + 8ax^2 + 8x^3$$

$$\frac{16}{9}a^2x \div \frac{2}{3}a = \frac{8}{3}ax; \frac{8}{3}ax \times (\frac{2}{3}a + 2x) = \frac{16}{9}a^2x + \frac{16}{3}ax^2; \text{ subtract: } \frac{8}{3}ax^2 + 8x^3$$

$$\frac{8}{3}ax^2 \div \frac{2}{3}a = 4x^2; 4x^2 \times (\frac{2}{3}a + 2x) = \frac{8}{3}ax^2 + 8x^3; \text{ subtract: } 0$$

$$\text{Remainder} = 0; \text{Quotient} = \frac{4}{9}a^2 + \frac{8}{3}ax + 4x^2$$

$$\text{Ans: Quotient} = \frac{4}{9}a^2 + \frac{8}{3}ax + 4x^2$$

20. Find the quadratic polynomial $f(x)$ which leaves the remainders -2 , 10 and 2 when divided by $x + 1$, $x - 2$ and $x - 3$ respectively. [5]

Let $f(x) = ax^2 + bx + c$. The remainder on dividing by $x - k$ is $f(k)$.

$$f(-1) = -2 \Rightarrow a - b + c = -2 \dots (1)$$

$$f(2) = 10 \Rightarrow 4a + 2b + c = 10 \dots (2)$$

$$f(3) = 2 \Rightarrow 9a + 3b + c = 2 \dots (3)$$

$$(2) - (1), \text{ divide by 3: } a + b = 4 \dots (4)$$

$$(3) - (2): 5a + b = -8 \dots (5)$$

$$(5) - (4): 4a = -12 \Rightarrow a = -3$$

$$\text{From (4): } b = 7; \text{ from (1): } c = 8$$

$$\therefore f(x) = -3x^2 + 7x + 8$$

$$\text{Ans: } f(x) = -3x^2 + 7x + 8$$
