

Polynomials

Question Paper, Set 02

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $-6x^3y^3 + 8x^2y^3$ a monomial, a binomial or a trinomial? [1 mark]
2. Find the degree of the monomial $5x^4y^4z^3$. [1 mark]
3. Write $p(x) = 8 + 8x^2 - 5x^3 + 4x$ in the standard form $a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$. Also write n , a_n and a_0 . [2 marks]
4. Is $-8x^2 + 8x^{-3}$ a polynomial? Give reason. [1 mark]
5. If $(p + 2)x^2 + (q - p)x + (r + q)$ is a zero polynomial, find the values of p , q and r . [2 marks]
6. Name the type of the equation $8x^2 + 3x - 4 = 0$ (linear, quadratic, cubic or bi-quadratic). [1 mark]
7. Find the value of the polynomial $p(x) = x^3 + 4x^2 + 9x + 1$ at $x = \frac{1}{2}$. [2 marks]
8. Find the zero of the polynomial $p(x) = 7x + 3$. [1 mark]
9. Check whether -1 and -8 are roots of $x^2 + 9x + 8 = 0$. [2 marks]
10. Find the roots of $x^2 + 4x - 1 = 0$ using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. [3 marks]
11. Find the quotient and remainder when $f(x) = 2x^3 + 7x^2 + x - 10$ is divided by $x + 1$. [3 marks]
12. Using the remainder theorem, find the remainder when $54x^3 + 27x^2 - 9x - 9$ is divided by $3x - 2$. [2 marks]
13. If $ax^3 + bx^2 + 3x - 9$ has $x - 1$ as a factor and leaves the remainder -8 when divided by $x + 1$, find the values of a and b . [5 marks]
14. Factorise $3\sqrt{3}x^2 + 3x - 2\sqrt{3}$. [2 marks]
15. What must be subtracted from $-x^3 + 4x^2 - 8$ so that the result is exactly divisible by $x^2 - 2x - 1$? [3 marks]
16. Find the zeros of $3x^2 - 10x + 5$ and hence write it as a product of linear factors. [5 marks]
17. Factorise $18t^3 - 3t^2 - 178t - 112$, if it is given that $3t + 8$ is a factor of it. [5 marks]
18. Factorise $2x^3 + 9x^2 - 6x - 5$ using synthetic division. [5 marks]

19. Find the remainder when $26^{141} + 5^{141}$ is divided by 31. [1 mark]

20. Find the quotient when $\frac{2}{3}a^3 - \frac{5}{6}a^2x - \frac{7}{3}ax^2 - x^3$ is divided by $\frac{1}{3}a + \frac{1}{4}x$. [3 marks]

Key sheet

Q	Answer
1	Binomial
2	11
3	$p(x) = -5x^3 + 8x^2 + 4x + 8; n = 3, a_3 = -5, a_0 = 8$
4	No, the power -3 of x is negative
5	$p = -2, q = -2, r = 2$
6	Quadratic equation
7	$\frac{53}{8}$
8	$x = -\frac{3}{7}$
9	Both -1 and -8 are roots
10	$x = -2 + \sqrt{5}, -2 - \sqrt{5}$
11	Quotient $= 2x^2 + 5x - 4$, Remainder $= -6$
12	13
13	$a = 1, b = 5$
14	$(\sqrt{3}x + 2)(3x - \sqrt{3})$
15	$3x - 6$
16	$3 \left[x - \left(\frac{5+\sqrt{10}}{3} \right) \right] \left[x - \left(\frac{5-\sqrt{10}}{3} \right) \right]$
17	$(3t + 8)(3t + 2)(2t - 7)$
18	$(x - 1)(2x + 1)(x + 5)$
19	0
20	Quotient $= 2a^2 - 4ax - 4x^2$

Solutions

1. Is $-6x^3y^3 + 8x^2y^3$ a monomial, a binomial or a trinomial? [1]

$-6x^3y^3 + 8x^2y^3$ has two terms: $-6x^3y^3, 8x^2y^3$.

An expression with two terms is a binomial.

Ans: Binomial

2. Find the degree of the monomial $5x^4y^4z^3$. [1]

Degree of a monomial = sum of the exponents of its variables

$$= 4 + 4 + 3 = 11$$

Ans: 11

3. Write $p(x) = 8 + 8x^2 - 5x^3 + 4x$ in the standard form $a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$. Also write n , a_n and a_0 . [2]

In standard form the powers of x go down: $p(x) = -5x^3 + 8x^2 + 4x + 8$

Highest power = 3, so $n = 3$; its coefficient $a_3 = -5$

Constant term $a_0 = 8$

Ans: $p(x) = -5x^3 + 8x^2 + 4x + 8$; $n = 3$, $a_3 = -5$, $a_0 = 8$

4. Is $-8x^2 + 8x^{-3}$ a polynomial? Give reason. [1]

In $-8x^2 + 8x^{-3}$, the power -3 of x is negative.

So it is not a polynomial.

Ans: No, the power -3 of x is negative

5. If $(p + 2)x^2 + (q - p)x + (r + q)$ is a zero polynomial, find the values of p , q and r . [2]

In a zero polynomial all the coefficients are zero.

$$p + 2 = 0 \Rightarrow p = -2$$

$$q - p = 0 \Rightarrow q + 2 = 0 \Rightarrow q = -2$$

$$r + q = 0 \Rightarrow r - 2 = 0 \Rightarrow r = 2$$

Ans: $p = -2, q = -2, r = 2$

6. Name the type of the equation $8x^2 + 3x - 4 = 0$ (linear, quadratic, cubic or bi-quadratic). [1]

The highest power of x in the equation is 2.

So it is a quadratic equation.

Ans: Quadratic equation

7. Find the value of the polynomial $p(x) = x^3 + 4x^2 + 9x + 1$ at $x = \frac{1}{2}$. [2]

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^2 + 9\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{8} + 1 + \frac{9}{2} + 1$$

$$= \frac{53}{8}$$

Ans: $\frac{53}{8}$

8. Find the zero of the polynomial $p(x) = 7x + 3$. [1]

$$7x + 3 = 0$$

$$\Rightarrow 7x = -3$$

$$\Rightarrow x = -\frac{3}{7}$$

$$\text{Ans: } x = -\frac{3}{7}$$

9. Check whether -1 and -8 are roots of $x^2 + 9x + 8 = 0$. [2]

$$\text{Let } p(x) = x^2 + 9x + 8.$$

$$p(-1) = (-1)^2 + 9(-1) + 8$$

$$= 1 - 9 + 8$$

$$= 0$$

$$p(-8) = (-8)^2 + 9(-8) + 8$$

$$= 64 - 72 + 8$$

$$= 0$$

Both -1 and -8 are roots.

Ans: Both -1 and -8 are roots

10. Find the roots of $x^2 + 4x - 1 = 0$ using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. [3]

$$a = 1, b = 4, c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{20}}{2}$$

$$= \frac{-4 \pm 2\sqrt{5}}{2}$$

$$= -2 \pm \sqrt{5}$$

$$= -2 + \sqrt{5}, -2 - \sqrt{5}$$

$$\text{Ans: } x = -2 + \sqrt{5}, -2 - \sqrt{5}$$

11. Find the quotient and remainder when $f(x) = 2x^3 + 7x^2 + x - 10$ is divided by $x + 1$. [3]

$$2x^3 \div x = 2x^2; 2x^2 \times (x + 1) = 2x^3 + 2x^2; \text{ subtract: } 5x^2 + x - 10$$

$$5x^2 \div x = 5x; 5x \times (x + 1) = 5x^2 + 5x; \text{ subtract: } -4x - 10$$

$$-4x \div x = -4; (-4) \times (x + 1) = -4x - 4; \text{ subtract: } -6$$

$$\text{Quotient} = 2x^2 + 5x - 4, \text{ Remainder} = -6$$

$$\text{Ans: Quotient} = 2x^2 + 5x - 4, \text{ Remainder} = -6$$

12. Using the remainder theorem, find the remainder when $54x^3 + 27x^2 - 9x - 9$ is divided by $3x - 2$. [2]

$$3x - 2 = 0 \Rightarrow x = \frac{2}{3}$$

$$\text{Remainder} = f\left(\frac{2}{3}\right)$$

$$f\left(\frac{2}{3}\right) = 54\left(\frac{2}{3}\right)^3 + 27\left(\frac{2}{3}\right)^2 - 9\left(\frac{2}{3}\right) - 9$$

$$= 16 + 12 - 6 - 9$$

$$= 13$$

$$\text{Ans: } 13$$

13. If $ax^3 + bx^2 + 3x - 9$ has $x - 1$ as a factor and leaves the remainder -8 when divided by $x + 1$, find the values of a and b . [5]

$$\text{Let } f(x) = ax^3 + bx^2 + 3x - 9.$$

$$x - 1 \text{ is a factor} \Rightarrow f(1) = 0$$

$$f(1) = a(1)^3 + b(1)^2 + 3(1) - 9$$

$$= a + b + 3 - 9$$

$$= a + b - 6$$

$$\Rightarrow a + b = 6 \dots (1)$$

$$\text{Remainder on dividing by } x + 1 \text{ is } -8 \Rightarrow f(-1) = -8$$

$$f(-1) = a(-1)^3 + b(-1)^2 + 3(-1) - 9$$

$$= -a + b - 3 - 9$$

$$= -a + b - 12$$

$$\Rightarrow a - b = -4 \dots (2)$$

$$(1) + (2): 2a = 2$$

$$\Rightarrow a = 1$$

$$\text{Put } a = 1 \text{ in (1): } 1 + b = 6 \Rightarrow b = 5$$

$$\text{Ans: } a = 1, b = 5$$

14. Factorise $3\sqrt{3}x^2 + 3x - 2\sqrt{3}$. [2]

$$3\sqrt{3} \times (-2\sqrt{3}) = -18; \text{ split } 3x \text{ as } -3x \text{ and } 6x$$

$$3\sqrt{3}x^2 + 3x - 2\sqrt{3} = 3\sqrt{3}x^2 - 3x + 6x - 2\sqrt{3}$$

$$= \sqrt{3}x(3x - \sqrt{3}) + 2(3x - \sqrt{3})$$

$$= (\sqrt{3}x + 2)(3x - \sqrt{3})$$

$$\text{Ans: } (\sqrt{3}x + 2)(3x - \sqrt{3})$$

15. What must be subtracted from $-x^3 + 4x^2 - 8$ so that the result is exactly divisible by $x^2 - 2x - 1$? [3]

$$-x^3 \div x^2 = -x; (-x) \times (x^2 - 2x - 1) = -x^3 + 2x^2 + x; \text{ subtract: } 2x^2 - x - 8$$

$$2x^2 \div x^2 = 2; 2 \times (x^2 - 2x - 1) = 2x^2 - 4x - 2; \text{ subtract: } 3x - 6$$

$$\text{Remainder} = 3x - 6$$

Subtracting the remainder leaves an exact multiple: $3x - 6$ must be subtracted.

$$\text{Ans: } 3x - 6$$

16. Find the zeros of $3x^2 - 10x + 5$ and hence write it as a product of linear factors. [5]

$$a = 3, b = -10, c = 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{10 \pm \sqrt{100 - 4(3)(5)}}{2(3)}$$

$$= \frac{10 \pm \sqrt{40}}{6}$$

$$= \frac{10 \pm 2\sqrt{10}}{6}$$

$$= \frac{5 \pm \sqrt{10}}{3}$$

$$= \frac{5 + \sqrt{10}}{3}, \frac{5 - \sqrt{10}}{3}$$

$$\therefore 3x^2 - 10x + 5 = a(x - \alpha)(x - \beta)$$

$$= 3 \left[x - \left(\frac{5 + \sqrt{10}}{3} \right) \right] \left[x - \left(\frac{5 - \sqrt{10}}{3} \right) \right]$$

$$\text{Ans: } 3 \left[x - \left(\frac{5 + \sqrt{10}}{3} \right) \right] \left[x - \left(\frac{5 - \sqrt{10}}{3} \right) \right]$$

17. Factorise $18t^3 - 3t^2 - 178t - 112$, if it is given that $3t + 8$ is a factor of it. [5]

$$18t^3 \div 3t = 6t^2; 6t^2 \times (3t + 8) = 18t^3 + 48t^2; \text{ subtract: } -51t^2 - 178t - 112$$

$$-51t^2 \div 3t = -17t; (-17t) \times (3t + 8) = -51t^2 - 136t; \text{ subtract: } -42t - 112$$

$$-42t \div 3t = -14; (-14) \times (3t + 8) = -42t - 112; \text{ subtract: } 0$$

$$\therefore 18t^3 - 3t^2 - 178t - 112 = (3t + 8)(6t^2 - 17t - 14)$$

$$6 \times (-14) = -84 = 4 \times (-21) \text{ and } 4 + (-21) = -17$$

$$6t^2 - 17t - 14 = 6t^2 + 4t - 21t - 14$$

$$= 2t(3t + 2) - 7(3t + 2)$$

$$= (3t + 2)(2t - 7)$$

$$\therefore 18t^3 - 3t^2 - 178t - 112 = (3t + 8)(3t + 2)(2t - 7)$$

$$\text{Ans: } (3t + 8)(3t + 2)(2t - 7)$$

18. Factorise $2x^3 + 9x^2 - 6x - 5$ using synthetic division. [5]

$$\text{Let } f(x) = 2x^3 + 9x^2 - 6x - 5.$$

$$f(1) = 2 + 9 - 6 - 5 = 0$$

$$\therefore x - 1 \text{ is a factor}$$

Write the coefficients and 1 on the left: $1 \mid 2 \quad 9 \quad -6 \quad -5$

Bring down the first number; multiply by 1 and add, each time.

$$\text{Second row: } 0 \quad 2 \quad 11 \quad 5$$

$$\text{Bottom row: } 2 \quad 11 \quad 5 \mid 0$$

$$\therefore f(x) = (x - 1)(2x^2 + 11x + 5)$$

$$2x^2 + 11x + 5 = 2x^2 + x + 10x + 5$$

$$= x(2x + 1) + 5(2x + 1)$$

$$= (2x + 1)(x + 5)$$

$$\therefore f(x) = (x - 1)(2x + 1)(x + 5)$$

$$\text{Ans: } (x - 1)(2x + 1)(x + 5)$$

19. Find the remainder when $26^{141} + 5^{141}$ is divided by 31. [1]

$x + a$ is a factor of $x^n + a^n$ when n is odd.

Here $n = 141$ is odd, so $26 + 5 = 31$ divides $26^{141} + 5^{141}$.

Remainder = 0

Ans: 0

20. Find the quotient when $\frac{2}{3}a^3 - \frac{5}{6}a^2x - \frac{7}{3}ax^2 - x^3$ is divided by $\frac{1}{3}a + \frac{1}{4}x$. [3]

$\frac{2}{3}a^3 \div \frac{1}{3}a = 2a^2$; $2a^2 \times (\frac{1}{3}a + \frac{1}{4}x) = \frac{2}{3}a^3 + \frac{1}{2}a^2x$; subtract: $-\frac{4}{3}a^2x - \frac{7}{3}ax^2 - x^3$
 $-\frac{4}{3}a^2x \div \frac{1}{3}a = -4ax$; $(-4ax) \times (\frac{1}{3}a + \frac{1}{4}x) = -\frac{4}{3}a^2x - ax^2$; subtract: $-\frac{4}{3}ax^2 - x^3$
 $-\frac{4}{3}ax^2 \div \frac{1}{3}a = -4x^2$; $(-4x^2) \times (\frac{1}{3}a + \frac{1}{4}x) = -\frac{4}{3}ax^2 - x^3$; subtract: 0

Remainder = 0; Quotient = $2a^2 - 4ax - 4x^2$

Ans: Quotient = $2a^2 - 4ax - 4x^2$
