

Polynomials

Question Paper, Set 10

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. State whether $9xy^3$ is a monomial. Give reason. [1 mark]
2. Write the degree of $3y^6 - 2x^4y^3 - 8x^6y^3 + 7x^2y^4 - 6$, showing the degree of each term. [2 marks]
3. In $p(x) = 6 - 4x^3 - 2x^4 - 3x$, what is the coefficient of x^3 ? [1 mark]
4. Write the type of the polynomial $p(x) = -1 + 8x + 4x^2$ according to its degree. [1 mark]
5. If $(p - 4)x^2 + (q - 2p)x + (r - q)$ is a zero polynomial, find the values of p , q and r . [2 marks]
6. Which of the following are quadratic equations? (i) $x^2 - 6\sqrt{x} + 1 = 0$ (ii) $x^2 + \frac{2}{x^2} = 8$ (iii) $2x^2 + 2x - 5 = 0$ [2 marks]
7. Find the value of $p(x) = 2x^2 + 6x - 12$ at $x = -1$. [1 mark]
8. Find a , if -2 is a zero of $p(x) = 2x^3 - 4x^2 + ax + 2$. [2 marks]
9. Solve $12x^2 - 37x + 21 = 0$ by splitting the middle term. [3 marks]
10. Find $\Delta = b^2 - 4ac$ for $x^2 - 2x - 2 = 0$. [1 mark]
11. Divide $-9x^3 + 18x^2 + 19x - 23$ by $7 - 3x$ and write the quotient and remainder. [5 marks]
12. If the polynomials $ax^3 + 4x^2 - x - 6$ and $2x^3 + x + a$ leave the same remainder when divided by $x + 1$, find the value of a . [3 marks]
13. Using the factor theorem, show that $x - 3$ is a factor of $f(x) = x^3 - x^2 - 13x + 21$. [2 marks]
14. Factorise $5x^2 + 23x + 12$. [2 marks]
15. Find the values of p and q so that $x^4 + px^3 + 26x^2 - 43x + q$ is exactly divisible by $x^2 - 5x + 6$. [5 marks]
16. If $x - 5$ is a factor of $x^3 - 18x^2 + 107x - 210$, factorise it completely. [3 marks]
17. Factorise $x^3 + 5x^2 + 2x - 8$. [5 marks]
18. Divide $x^3 + 4x^2 - 2x - 2$ by $x + 1$ by synthetic division. [3 marks]

- 19.** Find the remainder when $23^{99} + 18^{99}$ is divided by 41. [1 mark]
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- 20.** Find the quadratic polynomial $f(x)$ which leaves the remainders 5, -3 and -7 when divided by x , $x - 1$ and $x - 2$ respectively. [5 marks]
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Key sheet

Q	Answer
1	Yes, the powers of x and y are 1 and 3, non-negative integers
2	9
3	-4
4	Quadratic polynomial
5	$p = 4, q = 8, r = 8$
6	(iii) only
7	-16
8	$a = -15$
9	$x = \frac{3}{4}, \frac{7}{3}$
10	$\Delta = 12$
11	Quotient $= 3x^2 + x - 4$, Remainder $= 5$
12	$a = 1$
13	$f(3) = 0$, so $x - 3$ is a factor
14	$(5x + 3)(x + 4)$
15	$p = -8, q = 30$
16	$(x - 5)(x - 6)(x - 7)$
17	$(x - 1)(x + 2)(x + 4)$
18	Quotient $= x^2 + 3x - 5$, Remainder $= 3$
19	0
20	$f(x) = 2x^2 - 10x + 5$

Solutions

1. State whether $9xy^3$ is a monomial. Give reason. [1]

In $9xy^3$, the powers of x and y are 1 and 3, non-negative integers.

So $9xy^3$ is a monomial.

Ans: Yes, the powers of x and y are 1 and 3, non-negative integers

2. Write the degree of $3y^6 - 2x^4y^3 - 8x^6y^3 + 7x^2y^4 - 6$, showing the degree of each term. [2]

Degrees of the terms: 6, 7, 9, 6, 0

Greatest among them = 9

So the degree of the polynomial is 9.

Ans: 9

3. In $p(x) = 6 - 4x^3 - 2x^4 - 3x$, what is the coefficient of x^3 ? [1]

The term with x^3 in $p(x)$ is $-4x^3$.

So the coefficient of x^3 is -4 .

Ans: -4

4. Write the type of the polynomial $p(x) = -1 + 8x + 4x^2$ according to its degree. [1]

The highest power of x is 2, so the degree is 2.

A polynomial of degree 2 is a quadratic polynomial.

Ans: Quadratic polynomial

5. If $(p - 4)x^2 + (q - 2p)x + (r - q)$ is a zero polynomial, find the values of p , q and r . [2]

In a zero polynomial all the coefficients are zero.

$$p - 4 = 0 \Rightarrow p = 4$$

$$q - 2p = 0 \Rightarrow q - 8 = 0 \Rightarrow q = 8$$

$$r - q = 0 \Rightarrow r - 8 = 0 \Rightarrow r = 8$$

Ans: $p = 4, q = 8, r = 8$

6. Which of the following are quadratic equations? (i) $x^2 - 6\sqrt{x} + 1 = 0$ (ii) $x^2 + \frac{2}{x^2} = 8$ (iii) $2x^2 + 2x - 5 = 0$ [2]

(i) $\sqrt{x} = x^{\frac{1}{2}}$ is not a whole-number power, so it is not a quadratic equation.

(ii) $\frac{2}{x^2} = 2x^{-2}$ has a negative power, so it is not a quadratic equation.

(iii) It has the form $ax^2 + bx + c = 0$ with $a \neq 0$.

Ans: (iii) only

7. Find the value of $p(x) = 2x^2 + 6x - 12$ at $x = -1$. [1]

$$p(-1) = 2(-1)^2 + 6(-1) - 12$$

$$= 2 - 6 - 12$$

$$= -16$$

Ans: -16

8. Find a , if -2 is a zero of $p(x) = 2x^3 - 4x^2 + ax + 2$. [2]

$$\begin{aligned} p(-2) &= 2(-2)^3 - 4(-2)^2 + a(-2) + 2 \\ &= -16 - 16 - 2a + 2 \\ &= -2a - 30 \end{aligned}$$

Given $p(-2) = 0$:

$$-2a - 30 = 0$$

$$\Rightarrow -2a = 30$$

$$\Rightarrow a = -15$$

Ans: $a = -15$

9. Solve $12x^2 - 37x + 21 = 0$ by splitting the middle term. [3]

$$12 \times 21 = 252 = (-9) \times (-28) \text{ and } (-9) + (-28) = -37$$

$$12x^2 - 37x + 21 = 12x^2 - 9x - 28x + 21$$

$$= 3x(4x - 3) - 7(4x - 3)$$

$$= (4x - 3)(3x - 7)$$

$$\Rightarrow 4x - 3 = 0 \text{ (or) } 3x - 7 = 0$$

$$\Rightarrow x = \frac{3}{4}, \frac{7}{3}$$

Ans: $x = \frac{3}{4}, \frac{7}{3}$

10. Find $\Delta = b^2 - 4ac$ for $x^2 - 2x - 2 = 0$. [1]

$$a = 1, b = -2, c = -2$$

$$\Delta = b^2 - 4ac = (-2)^2 - 4(1)(-2)$$

$$= 4 + 8 = 12$$

Ans: $\Delta = 12$

11. Divide $-9x^3 + 18x^2 + 19x - 23$ by $7 - 3x$ and write the quotient and remainder. [5]

Write the divisor in descending powers: $7 - 3x = -3x + 7$

$$-9x^3 \div (-3x) = 3x^2; 3x^2 \times (-3x + 7) = -9x^3 + 21x^2; \text{ subtract: } -3x^2 + 19x - 23$$

$$-3x^2 \div (-3x) = x; x \times (-3x + 7) = -3x^2 + 7x; \text{ subtract: } 12x - 23$$

$$12x \div (-3x) = -4; (-4) \times (-3x + 7) = 12x - 28; \text{ subtract: } 5$$

$$\text{Quotient} = 3x^2 + x - 4, \text{ Remainder} = 5$$

Ans: **Quotient** $= 3x^2 + x - 4$, **Remainder** $= 5$

12. If the polynomials $ax^3 + 4x^2 - x - 6$ and $2x^3 + x + a$ leave the same remainder when divided by $x + 1$, find the value of a . [3]

Let $f(x) = ax^3 + 4x^2 - x - 6$ and $g(x) = 2x^3 + x + a$.

Given $f(-1) = g(-1)$

$$f(-1) = a(-1)^3 + 4(-1)^2 - (-1) - 6$$

$$= -a + 4 + 1 - 6$$

$$= -a - 1$$

$$g(-1) = 2(-1)^3 + (-1) + a$$

$$= -2 - 1 + a$$

$$= a - 3$$

$$-a - 1 = a - 3$$

$$\Rightarrow -2a = -2$$

$$\Rightarrow a = 1$$

Ans: $a = 1$

13. Using the factor theorem, show that $x - 3$ is a factor of $f(x) = x^3 - x^2 - 13x + 21$. [2]

Let $f(x) = x^3 - x^2 - 13x + 21$; $x - 3 = 0 \Rightarrow x = 3$

$$f(3) = (3)^3 - (3)^2 - 13(3) + 21$$

$$= 27 - 9 - 39 + 21$$

$$= 0$$

$\Rightarrow x - 3$ is a factor of $f(x)$.

Ans: $f(3) = 0$, so $x - 3$ is a factor

14. Factorise $5x^2 + 23x + 12$. [2]

$$5 \times 12 = 60 = 3 \times 20 \text{ and } 3 + 20 = 23$$

$$5x^2 + 23x + 12 = 5x^2 + 3x + 20x + 12$$

$$= x(5x + 3) + 4(5x + 3)$$

$$= (5x + 3)(x + 4)$$

Ans: $(5x + 3)(x + 4)$

15. Find the values of p and q so that $x^4 + px^3 + 26x^2 - 43x + q$ is exactly divisible by $x^2 - 5x + 6$. [5]

$$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$$

$$= x(x - 2) - 3(x - 2)$$

$$= (x - 2)(x - 3)$$

Let $f(x) = x^4 + px^3 + 26x^2 - 43x + q$; $x - 3$ and $x - 2$ are factors.

$$f(3) = 0:$$

$$f(3) = (3)^4 + p(3)^3 + 26(3)^2 - 43(3) + q$$

$$= 81 + 27p + 234 - 129 + q$$

$$= 27p + q + 186$$

$$\Rightarrow 27p + q = -186 \dots (1)$$

$$f(2) = 0:$$

$$f(2) = (2)^4 + p(2)^3 + 26(2)^2 - 43(2) + q$$

$$= 16 + 8p + 104 - 86 + q$$

$$= 8p + q + 34$$

$$\Rightarrow 8p + q = -34 \dots (2)$$

$$(1) - (2): 19p = -152$$

$$\Rightarrow p = -8$$

$$\text{Put } p = -8 \text{ in (1): } -216 + q = -186 \Rightarrow q = 30$$

$$\text{Ans: } p = -8, q = 30$$

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16. If $x - 5$ is a factor of $x^3 - 18x^2 + 107x - 210$, factorise it completely. [3]

Divide $x^3 - 18x^2 + 107x - 210$ by $x - 5$:

$$x^3 \div x = x^2; x^2 \times (x - 5) = x^3 - 5x^2; \text{ subtract: } -13x^2 + 107x - 210$$

$$-13x^2 \div x = -13x; (-13x) \times (x - 5) = -13x^2 + 65x; \text{ subtract: } 42x - 210$$

$$42x \div x = 42; 42 \times (x - 5) = 42x - 210; \text{ subtract: } 0$$

$$\therefore x^3 - 18x^2 + 107x - 210 = (x - 5)(x^2 - 13x + 42)$$

$$x^2 - 13x + 42 = x^2 - 6x - 7x + 42$$

$$= x(x - 6) - 7(x - 6)$$

$$= (x - 6)(x - 7)$$

$$\therefore x^3 - 18x^2 + 107x - 210 = (x - 5)(x - 6)(x - 7)$$

$$\text{Ans: } (x - 5)(x - 6)(x - 7)$$

17. Factorise $x^3 + 5x^2 + 2x - 8$. [5]

Let $f(x) = x^3 + 5x^2 + 2x - 8$. By trial and error:

$$f(1) = 1 + 5 + 2 - 8 = 0$$

$\Rightarrow x - 1$ is a factor of $f(x)$

$$x^3 \div x = x^2; x^2 \times (x - 1) = x^3 - x^2; \text{ subtract: } 6x^2 + 2x - 8$$

$$6x^2 \div x = 6x; 6x \times (x - 1) = 6x^2 - 6x; \text{ subtract: } 8x - 8$$

$$8x \div x = 8; 8 \times (x - 1) = 8x - 8; \text{ subtract: } 0$$

$$\therefore f(x) = (x - 1)(x^2 + 6x + 8)$$

$$x^2 + 6x + 8 = x^2 + 2x + 4x + 8$$

$$= x(x + 2) + 4(x + 2)$$

$$= (x + 2)(x + 4)$$

$$\therefore f(x) = (x - 1)(x + 2)(x + 4)$$

$$\text{Ans: } (x - 1)(x + 2)(x + 4)$$

18. Divide $x^3 + 4x^2 - 2x - 2$ by $x + 1$ by synthetic division. [3]

$$x + 1 = 0 \Rightarrow x = -1$$

Write the coefficients and -1 on the left: $-1 \mid 1 \quad 4 \quad -2 \quad -2$

Bring down the first number; multiply by -1 and add, each time.

$$\text{Second row: } 0 \quad -1 \quad -3 \quad 5$$

$$\text{Bottom row: } 1 \quad 3 \quad -5 \mid 3$$

$$\text{Quotient} = x^2 + 3x - 5, \text{ Remainder} = 3$$

$$\text{Ans: Quotient} = x^2 + 3x - 5, \text{ Remainder} = 3$$

19. Find the remainder when $23^{99} + 18^{99}$ is divided by 41. [1]

$x + a$ is a factor of $x^n + a^n$ when n is odd.

Here $n = 99$ is odd, so $23 + 18 = 41$ divides $23^{99} + 18^{99}$.

$$\text{Remainder} = 0$$

$$\text{Ans: } 0$$

20. Find the quadratic polynomial $f(x)$ which leaves the remainders 5, -3 and -7 when divided by x , $x - 1$ and $x - 2$ respectively. [5]

Let $f(x) = ax^2 + bx + c$. The remainder on dividing by $x - k$ is $f(k)$.

$$f(0) = 5 \Rightarrow c = 5 \dots (1)$$

$$f(1) = -3 \Rightarrow a + b + c = -3 \dots (2)$$

$$f(2) = -7 \Rightarrow 4a + 2b + c = -7 \dots (3)$$

$$(2) - (1): a + b = -8 \dots (4)$$

$$(3) - (2): 3a + b = -4 \dots (5)$$

$$(5) - (4): 2a = 4 \Rightarrow a = 2$$

$$\text{From (4): } b = -10; \text{ from (1): } c = 5$$

$$\therefore f(x) = 2x^2 - 10x + 5$$

$$\text{Ans: } f(x) = 2x^2 - 10x + 5$$
