

Polynomials

Question Paper, Set 01

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $-2x^{-1}y$ a monomial? Give reason. [1 mark]
2. Find the degree of the polynomial $-8xy^3 - 9x^6y^2 + 4x^3 - 8x^5y^4 - 3$. [2 marks]
3. Write the coefficient of x^3 in $p(x) = 4x^3 - 2x^4 - 3 - 9x^2$. [1 mark]
4. Write the type of the polynomial $p(x) = -3 - x^3$ according to its degree. [1 mark]
5. If $ax^{n-4} - bx^{n-5} - cx^{n-6}$ is a quadratic polynomial, where $n \in N$, find the value of n . [2 marks]
6. Which of the following are quadratic equations? (i) $x^2 - \frac{5}{x^2} = 2$ (ii) $-8x^3 - 9 = 0$ (iii) $9x^2 + x - 2 = 0$ [2 marks]
7. If $p(x) = 7x^2 + 6x - 11$, find $p(-2)$. [1 mark]
8. If $x = -\frac{1}{2}$ is a zero of the polynomial $16x^3 + ax^2 - 2x + 6$, find a . [2 marks]
9. Solve $5x^2 - x - 4 = 0$. [3 marks]
10. Find the discriminant of $5x^2 + 3x - 10 = 0$. [1 mark]
11. Divide $4x^4 + 3x^3 - 21x^2 - 6$ by $x^2 + x - 5$. Write the quotient and the remainder. [5 marks]
12. If the polynomials $ax^3 - 4x^2 - x + 9$ and $2x^3 - 4x + a$ leave the same remainder when divided by $x + 1$, find the value of a . [3 marks]
13. Show that $x - 5$ is a factor of $x^3 - 5x^2 + 4x - 20$. [2 marks]
14. Factorise $15x^2 - 31x - 24$. [2 marks]
15. Find the values of p and q so that $x^4 + px^3 - x^2 - 24x + q$ is exactly divisible by $x^2 + 2x - 3$. [5 marks]
16. Factorise $x^3 - 9x^2 + 8x + 60$, given that $x - 6$ is a factor of it. [3 marks]
17. Factorise $x^3 + 2x^2 - 23x - 60$. [5 marks]
18. Using synthetic division, find the quotient and remainder when $5x^3 - x + 5$ is divided by $x - 3$. [3 marks]

- 19.** Is $x + 3$ a factor of $x^3 + 27$? Give reason. [1 mark]
-
- 20.** Find the quadratic polynomial $f(x)$ which leaves the remainders -3 , -7 and 9 when divided by $x + 3$, $x + 2$ and $x + 1$ respectively. [5 marks]
-

Key sheet

Q	Answer
1	No, the power of x is -1 , a negative number
2	9
3	4
4	Cubic polynomial
5	$n = 6$
6	(iii) only
7	5
8	$a = -20$
9	$x = -\frac{4}{5}, 1$
10	$\Delta = 209$
11	Quotient $= 4x^2 - x$, Remainder $= -5x - 6$
12	$a = 2$
13	$f(5) = 0$, so $x - 5$ is a factor
14	$(5x + 3)(3x - 8)$
15	$p = 6, q = 18$
16	$(x - 6)(x + 2)(x - 5)$
17	$(x + 3)(x + 4)(x - 5)$
18	Quotient $= 5x^2 + 15x + 44$, Remainder $= 137$
19	Yes, $f(-3) = 0$
20	$f(x) = 10x^2 + 46x + 45$

Solutions

1. Is $-2x^{-1}y$ a monomial? Give reason. [1]

The power of x is -1 , a negative number.

So $-2x^{-1}y$ is not a monomial.

Ans: No, the power of x is -1 , a negative number

2. Find the degree of the polynomial $-8xy^3 - 9x^6y^2 + 4x^3 - 8x^5y^4 - 3$. [2]

Degrees of the terms: 4, 8, 3, 9, 0

Greatest among them = 9

So the degree of the polynomial is 9.

Ans: 9

3. Write the coefficient of x^3 in $p(x) = 4x^3 - 2x^4 - 3 - 9x^2$. [1]

The term with x^3 in $p(x)$ is $4x^3$.

So the coefficient of x^3 is 4.

Ans: 4

4. Write the type of the polynomial $p(x) = -3 - x^3$ according to its degree. [1]

The highest power of x is 3, so the degree is 3.

A polynomial of degree 3 is a cubic polynomial.

Ans: Cubic polynomial

5. If $ax^{n-4} - bx^{n-5} - cx^{n-6}$ is a quadratic polynomial, where $n \in N$, find the value of n . [2]

The degree of a quadratic polynomial is 2.

So the highest power: $n - 4 = 2$

$\Rightarrow n = 6$

Ans: $n = 6$

6. Which of the following are quadratic equations? (i) $x^2 - \frac{5}{x^2} = 2$ (ii) $-8x^3 - 9 = 0$ (iii) $9x^2 + x - 2 = 0$ [2]

(i) $\frac{5}{x^2} = 5x^{-2}$ has a negative power, so it is not a quadratic equation.

(ii) Its degree is 3, so it is a cubic equation.

(iii) It has the form $ax^2 + bx + c = 0$ with $a \neq 0$.

Ans: (iii) only

7. If $p(x) = 7x^2 + 6x - 11$, find $p(-2)$. [1]

$$p(-2) = 7(-2)^2 + 6(-2) - 11$$

$$= 28 - 12 - 11$$

$$= 5$$

Ans: 5

8. If $x = -\frac{1}{2}$ is a zero of the polynomial $16x^3 + ax^2 - 2x + 6$, find a . [2]

$$\begin{aligned} p\left(-\frac{1}{2}\right) &= 16\left(-\frac{1}{2}\right)^3 + a\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 6 \\ &= -2 + \frac{1}{4}a + 1 + 6 \\ &= \frac{1}{4}a + 5 \end{aligned}$$

Given $p\left(-\frac{1}{2}\right) = 0$:

$$\frac{1}{4}a + 5 = 0$$

$$\Rightarrow \frac{1}{4}a = -5$$

$$\Rightarrow a = -20$$

Ans: $a = -20$

9. Solve $5x^2 - x - 4 = 0$. [3]

$$5 \times (-4) = -20 = 4 \times (-5) \text{ and } 4 + (-5) = -1$$

$$5x^2 - x - 4 = 5x^2 + 4x - 5x - 4$$

$$= x(5x + 4) - 1(5x + 4)$$

$$= (5x + 4)(x - 1)$$

$$\Rightarrow 5x + 4 = 0 \text{ (or) } x - 1 = 0$$

$$\Rightarrow x = -\frac{4}{5}, 1$$

Ans: $x = -\frac{4}{5}, 1$

10. Find the discriminant of $5x^2 + 3x - 10 = 0$. [1]

$$a = 5, b = 3, c = -10$$

$$\Delta = b^2 - 4ac = 3^2 - 4(5)(-10)$$

$$= 9 + 200 = 209$$

Ans: $\Delta = 209$

11. Divide $4x^4 + 3x^3 - 21x^2 - 6$ by $x^2 + x - 5$. Write the quotient and the remainder. [5]

$$4x^4 \div x^2 = 4x^2; 4x^2 \times (x^2 + x - 5) = 4x^4 + 4x^3 - 20x^2; \text{ subtract: } -x^3 - x^2 - 6$$

$$-x^3 \div x^2 = -x; (-x) \times (x^2 + x - 5) = -x^3 - x^2 + 5x; \text{ subtract: } -5x - 6$$

$$\text{Quotient} = 4x^2 - x, \text{ Remainder} = -5x - 6$$

Ans: Quotient = $4x^2 - x$, Remainder = $-5x - 6$

12. If the polynomials $ax^3 - 4x^2 - x + 9$ and $2x^3 - 4x + a$ leave the same remainder when divided by $x + 1$, find the value of a . [3]

Let $f(x) = ax^3 - 4x^2 - x + 9$ and $g(x) = 2x^3 - 4x + a$.

Given $f(-1) = g(-1)$

$$f(-1) = a(-1)^3 - 4(-1)^2 - (-1) + 9$$

$$= -a - 4 + 1 + 9$$

$$= -a + 6$$

$$g(-1) = 2(-1)^3 - 4(-1) + a$$

$$= -2 + 4 + a$$

$$= a + 2$$

$$-a + 6 = a + 2$$

$$\Rightarrow -2a = -4$$

$$\Rightarrow a = 2$$

Ans: $a = 2$

13. Show that $x - 5$ is a factor of $x^3 - 5x^2 + 4x - 20$. [2]

Let $f(x) = x^3 - 5x^2 + 4x - 20$; $x - 5 = 0 \Rightarrow x = 5$

$$f(5) = (5)^3 - 5(5)^2 + 4(5) - 20$$

$$= 125 - 125 + 20 - 20$$

$$= 0$$

$\Rightarrow x - 5$ is a factor of $f(x)$.

Ans: $f(5) = 0$, so $x - 5$ is a factor

14. Factorise $15x^2 - 31x - 24$. [2]

$$15 \times (-24) = -360 = 9 \times (-40) \text{ and } 9 + (-40) = -31$$

$$15x^2 - 31x - 24 = 15x^2 + 9x - 40x - 24$$

$$= 3x(5x + 3) - 8(5x + 3)$$

$$= (5x + 3)(3x - 8)$$

Ans: $(5x + 3)(3x - 8)$

15. Find the values of p and q so that $x^4 + px^3 - x^2 - 24x + q$ is exactly divisible by $x^2 + 2x - 3$. [5]

$$x^2 + 2x - 3 = x^2 - x + 3x - 3$$

$$= x(x - 1) + 3(x - 1)$$

$$= (x - 1)(x + 3)$$

Let $f(x) = x^4 + px^3 - x^2 - 24x + q$; $x - 1$ and $x + 3$ are factors.

$$f(1) = 0:$$

$$f(1) = (1)^4 + p(1)^3 - (1)^2 - 24(1) + q$$

$$= 1 + p - 1 - 24 + q$$

$$= p + q - 24$$

$$\Rightarrow p + q = 24 \dots (1)$$

$$f(-3) = 0:$$

$$f(-3) = (-3)^4 + p(-3)^3 - (-3)^2 - 24(-3) + q$$

$$= 81 - 27p - 9 + 72 + q$$

$$= -27p + q + 144$$

$$\Rightarrow 27p - q = 144 \dots (2)$$

$$(1) + (2): 28p = 168$$

$$\Rightarrow p = 6$$

$$\text{Put } p = 6 \text{ in (1): } 6 + q = 24 \Rightarrow q = 18$$

$$\text{Ans: } p = 6, q = 18$$

-
16. Factorise $x^3 - 9x^2 + 8x + 60$, given that $x - 6$ is a factor of it. [3]

Divide $x^3 - 9x^2 + 8x + 60$ by $x - 6$:

$$x^3 \div x = x^2; x^2 \times (x - 6) = x^3 - 6x^2; \text{ subtract: } -3x^2 + 8x + 60$$

$$-3x^2 \div x = -3x; (-3x) \times (x - 6) = -3x^2 + 18x; \text{ subtract: } -10x + 60$$

$$-10x \div x = -10; (-10) \times (x - 6) = -10x + 60; \text{ subtract: } 0$$

$$\therefore x^3 - 9x^2 + 8x + 60 = (x - 6)(x^2 - 3x - 10)$$

$$x^2 - 3x - 10 = x^2 + 2x - 5x - 10$$

$$= x(x + 2) - 5(x + 2)$$

$$= (x + 2)(x - 5)$$

$$\therefore x^3 - 9x^2 + 8x + 60 = (x - 6)(x + 2)(x - 5)$$

$$\text{Ans: } (x - 6)(x + 2)(x - 5)$$

17. Factorise $x^3 + 2x^2 - 23x - 60$. [5]

Let $f(x) = x^3 + 2x^2 - 23x - 60$. By trial and error:

$$f(1) = 1 + 2 - 23 - 60 \neq 0$$

$$f(-1) = -1 + 2 + 23 - 60 \neq 0$$

$$f(2) = 8 + 8 - 46 - 60 \neq 0$$

$$f(-2) = -8 + 8 + 46 - 60 \neq 0$$

$$f(3) = 27 + 18 - 69 - 60 \neq 0$$

$$f(-3) = -27 + 18 + 69 - 60 = 0$$

$\Rightarrow x + 3$ is a factor of $f(x)$

$$x^3 \div x = x^2; x^2 \times (x + 3) = x^3 + 3x^2; \text{ subtract: } -x^2 - 23x - 60$$

$$-x^2 \div x = -x; (-x) \times (x + 3) = -x^2 - 3x; \text{ subtract: } -20x - 60$$

$$-20x \div x = -20; (-20) \times (x + 3) = -20x - 60; \text{ subtract: } 0$$

$$\therefore f(x) = (x + 3)(x^2 - x - 20)$$

$$x^2 - x - 20 = x^2 + 4x - 5x - 20$$

$$= x(x + 4) - 5(x + 4)$$

$$= (x + 4)(x - 5)$$

$$\therefore f(x) = (x + 3)(x + 4)(x - 5)$$

Ans: $(x + 3)(x + 4)(x - 5)$

18. Using synthetic division, find the quotient and remainder when $5x^3 - x + 5$ is divided by $x - 3$. [3]

$$x - 3 = 0 \Rightarrow x = 3$$

The missing power of x gets the coefficient 0.

Write the coefficients and 3 on the left: $3 \mid 5 \quad 0 \quad -1 \quad 5$

Bring down the first number; multiply by 3 and add, each time.

Second row: 0 15 45 132

Bottom row: 5 15 44 | 137

Quotient = $5x^2 + 15x + 44$, Remainder = 137

Ans: Quotient = $5x^2 + 15x + 44$, Remainder = 137

19. Is $x + 3$ a factor of $x^3 + 27$? Give reason. [1]

$$x^3 + 27 = x^3 + 3^3$$

$$f(-3) = (-3)^3 + 27 = 0$$

So $x + 3$ is a factor.

($x + a$ is a factor of $x^n + a^n$ when n is odd.)

Ans: Yes, $f(-3) = 0$

20. Find the quadratic polynomial $f(x)$ which leaves the remainders -3 , -7 and 9 when divided by $x + 3$, $x + 2$ and $x + 1$ respectively. [5]

Let $f(x) = ax^2 + bx + c$. The remainder on dividing by $x - k$ is $f(k)$.

$$f(-3) = -3 \Rightarrow 9a - 3b + c = -3 \dots (1)$$

$$f(-2) = -7 \Rightarrow 4a - 2b + c = -7 \dots (2)$$

$$f(-1) = 9 \Rightarrow a - b + c = 9 \dots (3)$$

$$(2) - (1): -5a + b = -4 \dots (4)$$

$$(3) - (2): -3a + b = 16 \dots (5)$$

$$(5) - (4): 2a = 20 \Rightarrow a = 10$$

From (4): $b = 46$; from (1): $c = 45$

$$\therefore f(x) = 10x^2 + 46x + 45$$

$$\text{Ans: } f(x) = 10x^2 + 46x + 45$$
