EXPRESSWALA ACADEMY

Maths 1A: Mathematical Induction

MCQ Test, Set 09

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If $P(n)$: $1 + 8 + 15 + \cdots + (7n - 6) = \frac{n(7n-5)}{2}$, then the value of each side of $P(2)$ is

A. 1 **B.** 24 **C.** 8 **D.** 9

2. In the inductive step for a sum formula, we

A. multiply both sides of $P(k)$ by $k + 1$

B. subtract $P(1)$ from $P(k)$

C. add the $(k + 1)$th term to both sides of $P(k)$

D. add the kth term to both sides of $P(k)$

3. To prove $2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \cdots + (n + 1)(n + 2) = \frac{n(n^2+6n+11)}{3}$ by induction, the term added to both sides of $P(k)$ is

A. $(k + 1)(k + 2)$ **B.** $(k + 3)(k + 4)$ **C.** $(k + 1)(k + 2) + 1$ **D.** $(k + 2)(k + 3)$

4. By induction it can be shown that $7 + 17 + 27 + \cdots + (10n - 3) =$

A. $(n - 1)(5n - 3)$ **B.** $(n + 1)(5n + 7)$ **C.** $n(5n + 2)$ **D.** $2n(5n + 2)$

5. $1 + 2 + 3 + \cdots + 66 =$

A. 2211 **B.** 2278 **C.** 4422 **D.** 2145

6. If $1^3 + 2^3 + \cdots + n^3 = 14400$, then $1 + 2 + \cdots + n =$

A. 135 **B.** 15 **C.** 120 **D.** 7200

7. The sum of the first 41 odd natural numbers is

A. 1680 **B.** 1681 **C.** 1764 **D.** 1722

8. The sum of the first 27 even natural numbers is

A. 702 **B.** 1458 **C.** 729 **D.** 756

9. $1^2 - 2^2 + 3^2 - \cdots + 21^2 =$

A. 231 **B.** -231 **C.** 462 **D.** -210

10. $\sum_{k=1}^{n} k(2k-3) =$

A. $\frac{n(n-1)(4n-11)}{6}$ **B.** $\frac{n(n+1)(4n-7)}{6}$

C. $\frac{n(n+1)(4n-7)}{3}$ **D.** $\frac{(n+1)(n+2)(4n-3)}{6}$

11. $\sum_{k=1}^{10} k^2 - k + 1 =$

A. 910 **B.** 451 **C.** 249 **D.** 340

12. The 15th term of the A.P. $1, 11, 21, \ldots$ is

A. 150 **B.** 151 **C.** 131 **D.** 141

13. The 5th term of the G.P. $4, 16, 64, \ldots$ is

A. 4096 **B.** 256 **C.** 1024 **D.** 65536

14. The sum of the first 6 terms of $3 + 9 + 27 + \ldots$ is

A. 3279 **B.** 1092 **C.** 2187 **D.** 2184

15. The sum to infinity of $3 + 7 \cdot \frac{1}{3} + 11 \cdot \frac{1}{9} + 15 \cdot \frac{1}{27} + \ldots$ is

A. $\frac{15}{2}$ **B.** $\frac{13}{2}$ **C.** $\frac{9}{2}$ **D.** $\frac{27}{2}$

16. The nth term of $4, 9, 18, 31, \ldots$ is

A. $2n^2 - n + 3$ **B.** $2n^2 - n$ **C.** $2n^2 + 3n + 4$ **D.** $3n^2 - 2n + 2$

17. The nth term of $10, 46, 190, 766, \ldots$ is

A. $3 \cdot 4^{n+1} - 2$ **B.** $3 \cdot 4^n$ **C.** $3 \cdot 4^n - 2$ **D.** $3 \cdot 4^n + 2$

18. Using $n = 1$ and $n = 2$, the nth term of $3 \cdot 4 + 4 \cdot 5 + 5 \cdot 6 + \ldots$ is

A. $n^2 + 6n + 5$ **B.** $(n+1)(n+2)$ **C.** $(n+3)(n+4)$ **D.** $(n+2)(n+3)$

19. Testing with $n = 1, 2, 3$, the sum of n terms of $5 + 12 + 19 + \ldots$ is

A. $\frac{n(7n+3)}{2}$ **B.** $\frac{7n^2 - 11n + 14}{2}$

C. $n(7n-2)$ **D.** $\frac{(n+1)(7n-2)}{2}$

20. The largest number that divides $19^n - 7^n$ for every $n \in \mathbb{N}$ is

A. 156 **B.** 13 **C.** 12 **D.** 24

21. The largest number that divides $8^{2n} - 1$ for every $n \in \mathbb{N}$ is

A. 126 **B.** 63 **C.** 64 **D.** 189

22. To prove $14^n - 5^n$ is divisible by 9, assume $14^k - 5^k = 9\lambda$. Then $14^{k+1} - 5^{k+1} =$

A. $9(\lambda + 5^k)$ **B.** $9(14\lambda + 5^k)$ **C.** $9(5\lambda + 14^k)$ **D.** $9(14\lambda - 5^k)$

23. If $61 \equiv r \pmod{14}$ with $0 \le r < 14$, then $r =$

A. 4 **B.** 6 **C.** 9 **D.** 5

24. If $113 \equiv r \pmod{6}$ with $0 \le r < 6$, then $r =$

A. 18 **B.** 1 **C.** 0 **D.** 5

25. If $145 \equiv r \pmod{12}$ with $0 \leq r < 12$, then $r =$

A. 1 **B.** 11 **C.** 12 **D.** 2

Key sheet

1. D	**2.** C	**3.** D	**4.** C	**5.** A
6. C	**7.** B	**8.** D	**9.** A	**10.** B
11. D	**12.** D	**13.** C	**14.** B	**15.** A
16. A	**17.** C	**18.** D	**19.** A	**20.** C
21. B	**22.** B	**23.** D	**24.** D	**25.** A

Solutions

1. If $P(n)$: $1+8+15+\cdots+(7n-6)=\frac{n(7n-5)}{2}$, then the value of each side of $P(2)$ is

$1+8=9$

and the right side at $n=2$ is also 9.

Ans: (D) 9

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2. In the inductive step for a sum formula, we

Assume $P(k)$, add the next term, the $(k+1)$th, to both sides and simplify to get $P(k+1)$.

Ans: (C) add the $(k+1)$th term to both sides of $P(k)$

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3. To prove $2\cdot3+3\cdot4+4\cdot5+\cdots+(n+1)(n+2)=\frac{n(n^2+6n+11)}{3}$ by induction, the term added to both sides of $P(k)$ is

The $(k+1)$th term is got by putting $n=k+1$ in $(n+1)(n+2)$.

It is $(k+2)(k+3)$.

Ans: (D) $(k+2)(k+3)$

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4. By induction it can be shown that $7+17+27+\cdots+(10n-3)=$

Check with $n=1$: the sum is 7 and the formula gives 7.

The formula $n(5n+2)$ is the one proved by induction.

Ans: (C) $n(5n+2)$

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5. $1+2+3+\cdots+66=$

$\frac{n(n+1)}{2}=\frac{66\cdot67}{2}=2211$

Ans: (A) 2211

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6. If $1^3+2^3+\cdots+n^3=14400$, then $1+2+\cdots+n=$

$\sum n^3=(\sum n)^2$

$\therefore \sum n=\sqrt{14400}=120$

Ans: (C) 120

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7. The sum of the first 41 odd natural numbers is

$1+3+\cdots+(2n-1)=n^2=1681$

Ans: (B) 1681

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8. The sum of the first 27 even natural numbers is

$2+4+\cdots+2n=n(n+1)=27\cdot28=756$

Ans: (D) 756

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9. $1^2-2^2+3^2-\cdots+21^2=$

$n=21$ is odd.

$\frac{21\cdot22}{2}=231$

Ans: (A) 231

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10. $\sum_{k=1}^{n} k(2k-3) =$

Expand $k(2k-3) = 2k^2 - 3k$ and use $\sum k^2$, $\sum k$.

The total factorises as $\frac{n(n+1)(4n-7)}{6}$.

Ans: (B) $\frac{n(n+1)(4n-7)}{6}$

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11. $\sum_{k=1}^{10} k^2 - k + 1 =$

Expand $k^2 - k + 1 = k^2 - k + 1$ and use the standard sums with $n = 10$.

The sum is 340.

Ans: (D) 340

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12. The 15th term of the A.P. $1, 11, 21, \ldots$ is

$t_n = a + (n-1)d = 1 + 14(10) = 141$

Ans: (D) 141

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13. The 5th term of the G.P. $4, 16, 64, \ldots$ is

$t_n = ar^{n-1} = 4 \cdot 4^4 = 1024$

Ans: (C) 1024

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14. The sum of the first 6 terms of $3 + 9 + 27 + \ldots$ is

$S_n = a\frac{r^n - 1}{r - 1} = 3 \cdot \frac{729 - 1}{2} = 1092$

Ans: (B) 1092

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15. The sum to infinity of $3 + 7 \cdot \frac{1}{3} + 11 \cdot \frac{1}{9} + 15 \cdot \frac{1}{27} + \ldots$ is

$a = 3$, $d = 4$, $r = \frac{1}{3}$.

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = \frac{9}{2} + 3 = \frac{15}{2}$

Ans: (A) $\frac{15}{2}$

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16. The nth term of $4, 9, 18, 31, \ldots$ is

Differences: $5, 9, 13$.

Fitting $n = 1, 2, 3$ gives $T_n = 2n^2 - n + 3$.

Ans: (A) $2n^2 - n + 3$

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17. The nth term of $10, 46, 190, 766, \ldots$ is

Differences: $36, 144, 576$.

Fitting $n = 1, 2, 3$ gives $T_n = 3 \cdot 4^n - 2$.

Ans: (C) $3 \cdot 4^n - 2$

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18. Using $n = 1$ and $n = 2$, the nth term of $3 \cdot 4 + 4 \cdot 5 + 5 \cdot 6 + \ldots$ is

$T_1 = 12$, $T_2 = 20$.

Only $(n+2)(n+3)$ gives both.

Ans: (D) $(n+2)(n+3)$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $5 + 12 + 19 + \ldots$ is

$S_1 = 5$, $S_2 = 17$, $S_3 = 36$.

Only $\frac{n(7n+3)}{2}$ gives all three.

Ans: (A) $\frac{n(7n+3)}{2}$

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20. The largest number that divides $19^n - 7^n$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 12, \ n = 2 : \ 312$

$\text{G.C.D.}(12, 312) = 12$

Ans: (C) 12

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21. The largest number that divides $8^{2n} - 1$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 63, \ n = 2 : \ 4095$

$\text{G.C.D.}(63, 4095) = 63$

Ans: (B) 63

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22. To prove $14^n - 5^n$ is divisible by 9, assume $14^k - 5^k = 9\lambda$. Then $14^{k+1} - 5^{k+1} =$

$14^{k+1} - 5^{k+1} = 14(9\lambda + 5^k) - 5 \cdot 5^k$

$= 126\lambda + 9 \cdot 5^k = 9(14\lambda + 5^k)$

Ans: (B) $9(14\lambda + 5^k)$

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23. If $61 \equiv r \pmod{14}$ with $0 \leq r < 14$, then $r =$

$61 = 14 \times 4 + 5$

$\therefore 61 - 5 = 56$ is divisible by 14, so $r = 5$.

Ans: (D) 5

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24. If $113 \equiv r \pmod{6}$ with $0 \leq r < 6$, then $r =$

$113 = 6 \times 18 + 5$

$\therefore 113 - 5 = 108$ is divisible by 6, so $r = 5$.

Ans: (D) 5

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25. If $145 \equiv r \pmod{12}$ with $0 \leq r < 12$, then $r =$

$145 = 12 \times 12 + 1$

$\therefore 145 - 1 = 144$ is divisible by 12, so $r = 1$.

Ans: (A) 1
