EXPRESSWALA ACADEMY

Maths 1A: Mathematical Induction

MCQ Test, Set 08

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Let $P(n)$ be $3 + 11 + 19 + \cdots + (8n - 5) = n(4n - 1)$. In $P(4)$, the left side $3 + 11 + 19 + 27$ equals

A. 95 **B.** 27 **C.** 33 **D.** 60

2. A statement $P(n)$ is true for $n = 1$, but the inductive step cannot be proved. Then

A. $P(n)$ is true for all n

B. $P(n)$ is false for all n

C. we cannot conclude that $P(n)$ is true for all n

D. $P(n)$ is true for all $n \geq 2$

3. To prove $1 + 9 + 81 + \cdots + 9^{n-1} = \frac{9^n - 1}{8}$ by induction, the term added to both sides of $P(k)$ is

A. 9^{k-1} **B.** 9^k **C.** $9^{k-1} + 1$ **D.** 9^{k+1}

4. By induction it can be shown that $\frac{1}{1 \cdot 7} + \frac{1}{7 \cdot 13} + \frac{1}{13 \cdot 19} + \cdots + \frac{1}{(6n-5)(6n+1)} =$

A. $\frac{2n}{6n+1}$ **B.** $\frac{n-1}{6n-5}$ **C.** $\frac{n+1}{6n+7}$ **D.** $\frac{n}{6n+1}$

5. If $1 + 2 + 3 + \cdots + n = 276$, then $n =$

A. 22 **B.** 46 **C.** 24 **D.** 23

6. $1^2 + 2^2 + \cdots + 17^2 =$

A. 23409 **B.** 3570 **C.** 1785 **D.** 1683

7. $2^2 + 4^2 + \cdots + 22^2 =$

A. 2024 **B.** 3795 **C.** 1012 **D.** 506

8. $1^3 + 3^3 + 5^3 + \ldots$ to 9 terms $=$

A. 1377 **B.** 13203 **C.** 2025 **D.** 13041

9. $1^2 - 2^2 + 3^2 - \cdots - 30^2 =$

A. -930 **B.** 465 **C.** 435 **D.** -465

10. $\sum_{k=1}^{n}(k+1)(3k+1) =$

A. $\frac{n(2n^2+7n+7)}{2}$ **B.** $n(2n^2+7n+7)$

C. $\frac{(n-1)(2n^2+3n+2)}{2}$ **D.** $\frac{(n+1)(2n^2+11n+16)}{2}$

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11. $\sum_{k=1}^{5} k(k+1)(k+2) =$

A. 1050 **B.** 420 **C.** 756 **D.** 210

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12. The sum of the first 26 terms of the A.P. $9, 11, 13, \ldots$ is

A. 910 **B.** 884 **C.** 767 **D.** 1534

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13. The sum of the first 8 terms of $5 + 10 + 20 + \ldots$ is

A. 2555 **B.** 635 **C.** 1275 **D.** 1280

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14. The sum to infinity of $15 - 10 + \frac{20}{3} - \frac{40}{9} + \ldots$ is

A. 25 **B.** 45 **C.** 9 **D.** $-\frac{45}{2}$

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15. The sum to infinity of $5 + 6 \cdot \frac{1}{4} + 7 \cdot \frac{1}{16} + 8 \cdot \frac{1}{64} + \ldots$ is

A. $\frac{64}{9}$ **B.** 7 **C.** $\frac{20}{3}$ **D.** $\frac{76}{9}$

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16. The nth term of $8, 11, 16, 23, \ldots$ is

A. $2n^2 - n + 6$ **B.** $n^2 + 7$ **C.** n^2 **D.** $n^2 + 2n + 8$

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17. The nth term of $11, 51, 251, 1251, \ldots$ is

A. $2 \cdot 5^{n+1} + 1$ **B.** $2 \cdot 5^n$ **C.** $2 \cdot 5^n + 1$ **D.** $2 \cdot 5^n - 1$

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18. Using $n = 1$ and $n = 2$, the nth term of $5 + 17 + 29 + \ldots$ is

A. $12n - 7$ **B.** $12n + 5$ **C.** $13n - 8$ **D.** $12n - 19$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $4 + 7 + 10 + \ldots$ is

A. $\frac{n(3n+5)}{2}$ **B.** $\frac{3n^2+7n-2}{2}$

C. $n(3n+1)$ **D.** $\frac{3n^2-n+6}{2}$

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20. The largest number that divides $4^{2n} - 1$ for every $n \in \mathbb{N}$ is

A. 30 **B.** 15 **C.** 45 **D.** 16

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21. The largest number that divides $6^n - 1^n$ for every $n \in \mathbb{N}$ is

A. 5 **B.** 6 **C.** 10 **D.** 15

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22. To prove $15^n - 4^n$ is divisible by 11, assume $15^k - 4^k = 11\lambda$. Then $15^{k+1} - 4^{k+1} =$

A. $11(15\lambda - 4^k)$ **B.** $11(15\lambda + 4^k)$ **C.** $11(4\lambda + 15^k)$ **D.** $11(\lambda + 4^k)$

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23. With $P = 5$, $P^{n+1} + (P+1)^{2n-1}$ is divisible by

A. 30 **B.** 31 **C.** 11 **D.** 36

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24. If $137 \equiv r \pmod{15}$ with $0 \le r < 15$, then $r =$

A. 13 **B.** 9 **C.** 3 **D.** 2

25. If $82 \equiv r \pmod 9$ with $0 \leq r < 9$, then $r =$

A. 2 **B.** 8 **C.** 9 **D.** 1

Key sheet

1. D	**2.** C	**3.** B	**4.** D	**5.** D
6. C	**7.** A	**8.** D	**9.** D	**10.** A
11. B	**12.** B	**13.** C	**14.** C	**15.** A
16. B	**17.** C	**18.** A	**19.** A	**20.** B
21. A	**22.** B	**23.** B	**24.** D	**25.** D

Solutions

1. Let $P(n)$ be $3 + 11 + 19 + \cdots + (8n - 5) = n(4n - 1)$. In $P(4)$, the left side $3 + 11 + 19 + 27$ equals

$3 + 11 + 19 + 27 = 60$

and the right side at $n = 4$ is also 60.

Ans: (D) 60

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2. A statement $P(n)$ is true for $n = 1$, but the inductive step cannot be proved. Then

Both steps are needed. Without the inductive step, the principle gives no conclusion.

Ans: (C) we cannot conclude that $P(n)$ is true for all n

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3. To prove $1 + 9 + 81 + \cdots + 9^{n-1} = \frac{9^n - 1}{8}$ by induction, the term added to both sides of $P(k)$ is

The $(k+1)$th term is got by putting $n = k + 1$ in 9^{n-1}.

It is 9^k.

Ans: (B) 9^k

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4. By induction it can be shown that $\frac{1}{1 \cdot 7} + \frac{1}{7 \cdot 13} + \frac{1}{13 \cdot 19} + \cdots + \frac{1}{(6n-5)(6n+1)} =$

Check with $n = 1$: the sum is $\frac{1}{7}$ and the formula gives $\frac{1}{7}$.

The formula $\frac{n}{6n+1}$ is the one proved by induction.

Ans: (D) $\frac{n}{6n+1}$

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5. If $1 + 2 + 3 + \cdots + n = 276$, then $n =$

$\frac{n(n+1)}{2} = 276 \Rightarrow n(n+1) = 552 = 23 \cdot 24$

$\therefore n = 23$

Ans: (D) 23

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6. $1^2 + 2^2 + \cdots + 17^2 =$

$\frac{n(n+1)(2n+1)}{6} = \frac{17 \cdot 18 \cdot 35}{6} = 1785$

Ans: (C) 1785

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7. $2^2 + 4^2 + \cdots + 22^2 =$

$4 \sum_{k=1}^{11} k^2 = 4 \cdot 506 = 2024$

Ans: (A) 2024

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8. $1^3 + 3^3 + 5^3 + \ldots$ to 9 terms $=$

$n^2(2n^2 - 1) = 81(161) = 13041$

Ans: (D) 13041

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9. $1^2 - 2^2 + 3^2 - \cdots - 30^2 =$

$n = 30$ is even.

$-\frac{30 \cdot 31}{2} = -465$

Ans: (D) -465

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10. $\sum_{k=1}^{n}(k+1)(3k+1) =$

Expand $(k+1)(3k+1) = 3k^2+4k+1$ and use $\sum k^2$, $\sum k$.

The total factorises as $\frac{n(2n^2+7n+7)}{2}$.

Ans: (A) $\frac{n(2n^2+7n+7)}{2}$

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11. $\sum_{k=1}^{5} k(k+1)(k+2) =$

Expand $k(k+1)(k+2) = k^3+3k^2+2k$ and use the standard sums with $n=5$.

The sum is 420.

Ans: (B) 420

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12. The sum of the first 26 terms of the A.P. $9, 11, 13, \ldots$ is

$S_n = \frac{n}{2}[2a+(n-1)d] = \frac{26}{2}[18+50] = 884$

Ans: (B) 884

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13. The sum of the first 8 terms of $5+10+20+\ldots$ is

$S_n = a\frac{r^n-1}{r-1} = 5\cdot\frac{256-1}{1} = 1275$

Ans: (C) 1275

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14. The sum to infinity of $15-10+\frac{20}{3}-\frac{40}{9}+\ldots$ is

$a=15$, $r=-\frac{2}{3}$.

$S_\infty = \frac{a}{1-r} = 9$

Ans: (C) 9

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15. The sum to infinity of $5+6\cdot\frac{1}{4}+7\cdot\frac{1}{16}+8\cdot\frac{1}{64}+\ldots$ is

$a=5$, $d=1$, $r=\frac{1}{4}$.

$S_\infty = \frac{a}{1-r}+\frac{dr}{(1-r)^2} = \frac{20}{3}+\frac{4}{9} = \frac{64}{9}$

Ans: (A) $\frac{64}{9}$

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16. The nth term of $8, 11, 16, 23, \ldots$ is

Differences: $3, 5, 7$.

Fitting $n=1,2,3$ gives $T_n = n^2+7$.

Ans: (B) n^2+7

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17. The nth term of $11, 51, 251, 1251, \ldots$ is

Differences: $40, 200, 1000$.

Fitting $n=1,2,3$ gives $T_n = 2\cdot 5^n+1$.

Ans: (C) $2\cdot 5^n+1$

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18. Using $n=1$ and $n=2$, the nth term of $5+17+29+\ldots$ is

$T_1=5$, $T_2=17$.

Only $12n-7$ gives both.

Ans: (A) $12n-7$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $4 + 7 + 10 + \ldots$ is

$S_1 = 4$, $S_2 = 11$, $S_3 = 21$.

Only $\frac{n(3n+5)}{2}$ gives all three.

Ans: (A) $\frac{n(3n+5)}{2}$

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20. The largest number that divides $4^{2n} - 1$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 15, \ n = 2 : \ 255$

$\text{G.C.D.}(15, 255) = 15$

Ans: (B) 15

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21. The largest number that divides $6^n - 1^n$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 5, \ n = 2 : \ 35$

$\text{G.C.D.}(5, 35) = 5$

Ans: (A) 5

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22. To prove $15^n - 4^n$ is divisible by 11, assume $15^k - 4^k = 11\lambda$. Then $15^{k+1} - 4^{k+1} =$

$15^{k+1} - 4^{k+1} = 15(11\lambda + 4^k) - 4 \cdot 4^k$

$= 165\lambda + 11 \cdot 4^k = 11(15\lambda + 4^k)$

Ans: (B) $11(15\lambda + 4^k)$

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23. With $P = 5$, $P^{n+1} + (P+1)^{2n-1}$ is divisible by

$P^2 + P + 1 = 25 + 5 + 1 = 31$.

Ans: (B) 31

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24. If $137 \equiv r \pmod{15}$ with $0 \leq r < 15$, then $r =$

$137 = 15 \times 9 + 2$

$\therefore 137 - 2 = 135$ is divisible by 15, so $r = 2$.

Ans: (D) 2

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25. If $82 \equiv r \pmod{9}$ with $0 \leq r < 9$, then $r =$

$82 = 9 \times 9 + 1$

$\therefore 82 - 1 = 81$ is divisible by 9, so $r = 1$.

Ans: (D) 1
