EXPRESSWALA ACADEMY

Maths 1A: Mathematical Induction

MCQ Test, Set 07

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If $P(n)$: $7 + 13 + 19 + \cdots + (6n + 1) = n(3n + 4)$, then the value of each side of $P(3)$ is

A. 19 **B.** 20 **C.** 64 **D.** 39

2. In this chapter n takes values in $\mathbb{N}$, which is

A. $\{1, 2, 3, \dots\}$ **B.** $\{2, 3, 4, \dots\}$

C. $\{\dots, -1, 0, 1, \dots\}$ **D.** $\{0, 1, 2, \dots\}$

3. To prove $1^2 + 4^2 + 7^2 + \cdots + (3n - 2)^2 = \frac{n(6n^2 - 3n - 1)}{2}$ by induction, the term added to both sides of $P(k)$ is

A. $(3k + 4)^2$ **B.** $(3k - 2)^2 + 1$ **C.** $(3k - 2)^2$ **D.** $(3k + 1)^2$

4. By induction it can be shown that $1^2 + 5^2 + 9^2 + \cdots + (4n - 3)^2 =$

A. $\frac{(n-1)(16n^2 - 44n + 27)}{3}$ **B.** $\frac{(n+1)(16n^2 + 20n + 3)}{3}$

C. $\frac{n(16n^2 - 12n - 1)}{3}$ **D.** $\frac{2n(16n^2 - 12n - 1)}{3}$

5. $1 + 2 + 3 + \cdots + 85 =$

A. 3570 **B.** 3655 **C.** 7310 **D.** 3741

6. $\sum_{k=1}^{21} k^3 =$

A. 3311 **B.** 44100 **C.** 106722 **D.** 53361

7. The sum of the first 38 even natural numbers is

A. 1406 **B.** 2888 **C.** 1482 **D.** 1444

8. $1^2 + 3^2 + 5^2 + \dots$ to 6 terms $=$

A. 506 **B.** 286 **C.** 364 **D.** 290

9. $1^2 - 2^2 + 3^2 - \dots$ to $(2n + 1)$ terms $=$

A. $n(2n + 1)$ **B.** $-(n + 1)(2n + 1)$ **C.** $\frac{(n+1)(2n+1)}{2}$ **D.** $(n + 1)(2n + 1)$

10. $\sum_{k=1}^{n} k(k + 6) =$

A. $\frac{n(n+1)(2n+19)}{3}$ **B.** $\frac{n(n+1)(2n+19)}{6}$

C. $\frac{(n+1)(n+2)(2n+21)}{6}$ **D.** $\frac{n(n-1)(2n+17)}{6}$

11. $\sum_{k=1}^{6} k^3 + k =$

A. 1332 **B.** 240 **C.** 812 **D.** 462

12. The 9th term of the A.P. $4, 15, 26, \ldots$ is

A. 99 **B.** 103 **C.** 81 **D.** 92

13. The sum to infinity of $36 + 6 + 1 + \frac{1}{6} + \ldots$ is

A. 216 **B.** 30 **C.** $\frac{216}{5}$ **D.** $\frac{216}{7}$

14. The 5th term of the G.P. $6, 18, 54, \ldots$ is

A. 486 **B.** 104976 **C.** 162 **D.** 1458

15. The sum to infinity of $2 + 5 \cdot \frac{1}{2} + 8 \cdot \frac{1}{4} + 11 \cdot \frac{1}{8} + \ldots$ is

A. 10 **B.** 16 **C.** 4 **D.** 7

16. The nth term of $6, 17, 34, 57, \ldots$ is

A. $3n^2 + 8n + 6$ **B.** $3n^2 + 2n + 1$ **C.** $4n^2 + n$ **D.** $3n^2 + 2n$

17. The nth term of $5, 35, 215, 1295, \ldots$ is

A. 6^n **B.** $6^{n+1} - 1$ **C.** $6^n + 1$ **D.** $6^n - 1$

18. Using $n = 1$ and $n = 2$, the nth term of $2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + 4 \cdot 5 \cdot 6 + \ldots$ is

A. $n^3 + 6n^2 + 12n + 5$ **B.** $(n+1)(n+2)(n+3)$

C. $(n+2)(n+3)(n+4)$ **D.** $n(n+1)(n+2)$

19. Testing with $n = 1, 2, 3$, the sum of n terms of $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \ldots$ is

A. $n^2(n+2)$ **B.** $\frac{2n^3+3n^2-5n+18}{6}$

C. $\frac{n(n+1)(2n+7)}{6}$ **D.** $\frac{2n^3+9n^2+13n-6}{6}$

20. The largest number that divides $11^n - 1$ for every $n \in \mathbb{N}$ is

A. 10 **B.** 60 **C.** 11 **D.** 20

21. The largest number that divides $17^n - 5^n$ for every $n \in \mathbb{N}$ is

A. 13 **B.** 24 **C.** 12 **D.** 132

22. To prove $11^n - 3^n$ is divisible by 8, assume $11^k - 3^k = 8\lambda$. Then $11^{k+1} - 3^{k+1} =$

A. $8(\lambda + 3^k)$ **B.** $8(11\lambda + 3^k)$ **C.** $8(3\lambda + 11^k)$ **D.** $8(11\lambda - 3^k)$

23. If $99 \equiv r \pmod{12}$ with $0 \le r < 12$, then $r =$

A. 8 **B.** 4 **C.** 3 **D.** 9

24. The statement $8^n - 1$ is divisible by 7 can be written as

A. $8^n \equiv 0 \pmod{7}$ **B.** $7^n \equiv 1 \pmod{8}$ **C.** $8^n \equiv 1 \pmod{7}$ **D.** $8^n \equiv 7 \pmod{1}$

25. If $46 \equiv r \pmod{5}$ with $0 \le r < 5$, then $r =$

A. 1 **B.** 4 **C.** 9 **D.** 2

Key sheet

1. D	**2.** A	**3.** D	**4.** C	**5.** B
6. D	**7.** C	**8.** B	**9.** D	**10.** B
11. D	**12.** D	**13.** C	**14.** A	**15.** A
16. B	**17.** D	**18.** B	**19.** C	**20.** A
21. C	**22.** B	**23.** C	**24.** C	**25.** A

Solutions

1. If $P(n)$: $7 + 13 + 19 + \cdots + (6n+1) = n(3n+4)$, then the value of each side of $P(3)$ is

$7 + 13 + 19 = 39$

and the right side at $n = 3$ is also 39.

Ans: (D) 39

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2. In this chapter n takes values in $\mathbb{N}$, which is

$\mathbb{N} = \{1, 2, 3, \dots\}$, the natural numbers.

Ans: (A) $\{1, 2, 3, \dots\}$

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3. To prove $1^2 + 4^2 + 7^2 + \cdots + (3n-2)^2 = \frac{n(6n^2-3n-1)}{2}$ by induction, the term added to both sides of $P(k)$ is

The $(k+1)$th term is got by putting $n = k+1$ in $(3n-2)^2$.

It is $(3k+1)^2$.

Ans: (D) $(3k+1)^2$

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4. By induction it can be shown that $1^2 + 5^2 + 9^2 + \cdots + (4n-3)^2 =$

Check with $n = 1$: the sum is 1 and the formula gives 1.

The formula $\frac{n(16n^2-12n-1)}{3}$ is the one proved by induction.

Ans: (C) $\frac{n(16n^2-12n-1)}{3}$

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5. $1 + 2 + 3 + \cdots + 85 =$

$\frac{n(n+1)}{2} = \frac{85 \cdot 86}{2} = 3655$

Ans: (B) 3655

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6. $\sum_{k=1}^{21} k^3 =$

$\left(\frac{n(n+1)}{2}\right)^2 = 231^2 = 53361$

Ans: (D) 53361

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7. The sum of the first 38 even natural numbers is

$2 + 4 + \cdots + 2n = n(n+1) = 38 \cdot 39 = 1482$

Ans: (C) 1482

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8. $1^2 + 3^2 + 5^2 + \dots$ to 6 terms $=$

$\frac{n}{3}(4n^2 - 1) = \frac{6}{3}(143) = 286$

Ans: (B) 286

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9. $1^2 - 2^2 + 3^2 - \dots$ to $(2n+1)$ terms $=$

$2n+1$ is odd, so the sum is $\frac{(2n+1)(2n+2)}{2} = (n+1)(2n+1)$.

Ans: (D) $(n+1)(2n+1)$

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10. $\sum_{k=1}^{n} k(k+6) =$

Expand $k(k+6) = k^2 + 6k$ and use $\sum k^2$, $\sum k$.

The total factorises as $\frac{n(n+1)(2n+19)}{6}$.

Ans: (B) $\frac{n(n+1)(2n+19)}{6}$

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11. $\sum_{k=1}^{6} k^3 + k =$

Expand $k^3 + k = k^3 + k$ and use the standard sums with $n = 6$.

The sum is 462.

Ans: (D) 462

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12. The 9th term of the A.P. $4, 15, 26, \ldots$ is

$t_n = a + (n-1)d = 4 + 8(11) = 92$

Ans: (D) 92

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13. The sum to infinity of $36 + 6 + 1 + \frac{1}{6} + \ldots$ is

$a = 36$, $r = \frac{1}{6}$.

$S_\infty = \frac{a}{1-r} = \frac{216}{5}$

Ans: (C) $\frac{216}{5}$

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14. The 5th term of the G.P. $6, 18, 54, \ldots$ is

$t_n = ar^{n-1} = 6 \cdot 3^4 = 486$

Ans: (A) 486

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15. The sum to infinity of $2 + 5 \cdot \frac{1}{2} + 8 \cdot \frac{1}{4} + 11 \cdot \frac{1}{8} + \ldots$ is

$a = 2$, $d = 3$, $r = \frac{1}{2}$.

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = 4 + 6 = 10$

Ans: (A) 10

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16. The nth term of $6, 17, 34, 57, \ldots$ is

Differences: $11, 17, 23$.

Fitting $n = 1, 2, 3$ gives $T_n = 3n^2 + 2n + 1$.

Ans: (B) $3n^2 + 2n + 1$

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17. The nth term of $5, 35, 215, 1295, \ldots$ is

Differences: $30, 180, 1080$.

Fitting $n = 1, 2, 3$ gives $T_n = 6^n - 1$.

Ans: (D) $6^n - 1$

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18. Using $n = 1$ and $n = 2$, the nth term of $2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + 4 \cdot 5 \cdot 6 + \ldots$ is

$T_1 = 24$, $T_2 = 60$.

Only $(n+1)(n+2)(n+3)$ gives both.

Ans: (B) $(n+1)(n+2)(n+3)$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \ldots$ is

$S_1 = 3$, $S_2 = 11$, $S_3 = 26$.

Only $\frac{n(n+1)(2n+7)}{6}$ gives all three.

Ans: (C) $\frac{n(n+1)(2n+7)}{6}$

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20. The largest number that divides $11^n - 1$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 10, \ n = 2 : \ 120$

$\text{G.C.D.}(10, 120) = 10$

Ans: (A) 10

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21. The largest number that divides $17^n - 5^n$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 12, \ n = 2 : \ 264$

$\text{G.C.D.}(12, 264) = 12$

Ans: (C) 12

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22. To prove $11^n - 3^n$ is divisible by 8, assume $11^k - 3^k = 8\lambda$. Then $11^{k+1} - 3^{k+1} =$

$11^{k+1} - 3^{k+1} = 11(8\lambda + 3^k) - 3 \cdot 3^k$

$= 88\lambda + 8 \cdot 3^k = 8(11\lambda + 3^k)$

Ans: (B) $8(11\lambda + 3^k)$

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23. If $99 \equiv r \pmod{12}$ with $0 \leq r < 12$, then $r =$

$99 = 12 \times 8 + 3$

$\therefore 99 - 3 = 96$ is divisible by 12, so $r = 3$.

Ans: (C) 3

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24. The statement $8^n - 1$ is divisible by 7 can be written as

$8^n - 1$ divisible by 7 means $8^n \equiv 1 \pmod{7}$.

Ans: (C) $8^n \equiv 1 \pmod{7}$

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25. If $46 \equiv r \pmod{5}$ with $0 \leq r < 5$, then $r =$

$46 = 5 \times 9 + 1$

$\therefore 46 - 1 = 45$ is divisible by 5, so $r = 1$.

Ans: (A) 1
