EXPRESSWALA ACADEMY

Maths 1A: Mathematical Induction

MCQ Test, Set 06

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Let $P(n)$ be $1+6+11+\cdots+(5n-4)=\frac{n(5n-3)}{2}$. In $P(2)$, the left side $1+6$ equals

A. 6 **B.** 18 **C.** 1 **D.** 7

2. In the domino picture of induction, the inductive step says that

A. the first domino falls

B. the last domino falls first

C. each falling domino knocks over the next one

D. all the dominoes fall at once

3. To prove $8+11+14+\cdots+(3n+5)=\frac{n(3n+13)}{2}$ by induction, the term added to both sides of $P(k)$ is

A. $3k+6$ **B.** $3k+8$ **C.** $3k+5$ **D.** $3k+11$

4. By induction it can be shown that $7+16+25+\cdots+(9n-2)=$

A. $n(9n+5)$ **B.** $\frac{(n+1)(9n+14)}{2}$

C. $\frac{n(9n+5)}{2}$ **D.** $\frac{(n-1)(9n-4)}{2}$

5. $1^2+2^2+\cdots+20^2=$

A. 2730 **B.** 44100 **C.** 5740 **D.** 2870

6. $1^4+2^4+\cdots+9^4=$

A. 81225 **B.** 4050 **C.** 15333 **D.** 8772

7. $1^3+3^3+5^3+\ldots$ to 8 terms $=$

A. 1296 **B.** 8128 **C.** 960 **D.** 8256

8. The sum of the first 52 odd natural numbers is

A. 2756 **B.** 2704 **C.** 2703 **D.** 2809

9. $1^2-2^2+3^2-\cdots+17^2=$

A. 153 **B.** -153 **C.** 306 **D.** -136

10. $\sum_{k=1}^{n}(k-1)(k+3) =$

A. $\frac{n(n-1)(2n+11)}{6}$ **B.** $\frac{n(n+1)(2n+13)}{6}$

C. $\frac{(n-2)(n-1)(2n+9)}{6}$ **D.** $\frac{n(n-1)(2n+11)}{3}$

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11. $\sum_{k=1}^{8} 3k^2 - 1 =$

A. 846 **B.** 413 **C.** 1528 **D.** 604

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12. The sum of the first 20 terms of the A.P. $12, 15, 18, \ldots$ is

A. 690 **B.** 840 **C.** 810 **D.** 1380

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13. The 6th term of the G.P. $7, 14, 28, \ldots$ is

A. 112 **B.** 537824 **C.** 224 **D.** 448

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14. The sum of the first 4 terms of $2 + 10 + 50 + \ldots$ is

A. 312 **B.** 1248 **C.** 1562 **D.** 1250

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15. The sum to infinity of $1 + 2 \cdot \frac{1}{5} + 3 \cdot \frac{1}{25} + 4 \cdot \frac{1}{125} + \ldots$ is

A. $\frac{3}{2}$ **B.** $\frac{5}{4}$ **C.** $\frac{45}{16}$ **D.** $\frac{25}{16}$

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16. The nth term of $6, 14, 24, 36, \ldots$ is

A. $n^2 + 7n + 6$ **B.** $n^2 + 6n - 2$ **C.** $n^2 + 5n$ **D.** $2n^2 + 4n - 1$

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17. The nth term of $11, 21, 41, 81, \ldots$ is

A. $5 \cdot 2^n$ **B.** $5 \cdot 2^{n+1} + 1$ **C.** $5 \cdot 2^n - 1$ **D.** $5 \cdot 2^n + 1$

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18. Using $n = 1$ and $n = 2$, the nth term of $1^2 + 4^2 + 7^2 + \ldots$ is

A. $9n^2 - 11n + 3$ **B.** $(3n - 2)^2$ **C.** $(3n - 5)^2$ **D.** $(3n + 1)^2$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $5 + 11 + 17 + \ldots$ is

A. $n(3n + 2)$ **B.** $3n^2 - 4n + 6$ **C.** $3n^2 + 3n - 1$ **D.** $n(6n - 1)$

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20. The largest number that divides $3^{3n} - 1$ for every $n \in \mathbb{N}$ is

A. 364 **B.** 26 **C.** 52 **D.** 27

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21. The largest number that divides $7^{2n} - 1$ for every $n \in \mathbb{N}$ is

A. 49 **B.** 1200 **C.** 48 **D.** 96

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22. To prove $10^n - 7^n$ is divisible by 3, assume $10^k - 7^k = 3\lambda$. Then $10^{k+1} - 7^{k+1} =$

A. $3(\lambda + 7^k)$ **B.** $3(10\lambda + 7^k)$ **C.** $3(7\lambda + 10^k)$ **D.** $3(10\lambda - 7^k)$

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23. For all positive integers n, $7^n - 4^n$ is divisible by

A. 3 **B.** 11 **C.** 28 **D.** 4

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24. If $150 \equiv r \pmod{17}$ with $0 \le r < 17$, then $r =$

A. 8 **B.** 3 **C.** 15 **D.** 14

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25. $5^{2n-1}+2^{2n-1}$ is divisible by

A. 3 **B.** 10 **C.** 7 **D.** 29

Key sheet

1. D	**2.** C	**3.** B	**4.** C	**5.** D
6. C	**7.** B	**8.** B	**9.** A	**10.** A
11. D	**12.** C	**13.** C	**14.** A	**15.** D
16. C	**17.** D	**18.** B	**19.** A	**20.** B
21. C	**22.** B	**23.** A	**24.** D	**25.** C

Solutions

1. Let $P(n)$ be $1+6+11+\cdots+(5n-4)=\frac{n(5n-3)}{2}$. In $P(2)$, the left side $1+6$ equals

$1+6=7$

and the right side at $n=2$ is also 7.

Ans: (D) 7

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2. In the domino picture of induction, the inductive step says that

The base step knocks over the first domino.

The inductive step: each falling domino knocks over the next.

Ans: (C) each falling domino knocks over the next one

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3. To prove $8+11+14+\cdots+(3n+5)=\frac{n(3n+13)}{2}$ by induction, the term added to both sides of $P(k)$ is

The $(k+1)$th term is got by putting $n=k+1$ in $(3n+5)$.

It is $3k+8$.

Ans: (B) $3k+8$

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4. By induction it can be shown that $7+16+25+\cdots+(9n-2)=$

Check with $n=1$: the sum is 7 and the formula gives 7.

The formula $\frac{n(9n+5)}{2}$ is the one proved by induction.

Ans: (C) $\frac{n(9n+5)}{2}$

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5. $1^2+2^2+\cdots+20^2=$

$\frac{n(n+1)(2n+1)}{6}=\frac{20\cdot21\cdot41}{6}=2870$

Ans: (D) 2870

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6. $1^4+2^4+\cdots+9^4=$

$\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$ with $n=9$

$=15333$

Ans: (C) 15333

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7. $1^3+3^3+5^3+\ldots$ to 8 terms $=$

$n^2(2n^2-1)=64(127)=8128$

Ans: (B) 8128

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8. The sum of the first 52 odd natural numbers is

$1+3+\cdots+(2n-1)=n^2=2704$

Ans: (B) 2704

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9. $1^2-2^2+3^2-\cdots+17^2=$

$n=17$ is odd.

$\frac{17\cdot18}{2}=153$

Ans: (A) 153

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10. $\sum_{k=1}^{n}(k-1)(k+3) =$

Expand $(k-1)(k+3) = k^2+2k-3$ and use $\sum k^2$, $\sum k$.

The total factorises as $\frac{n(n-1)(2n+11)}{6}$.

Ans: (A) $\frac{n(n-1)(2n+11)}{6}$

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11. $\sum_{k=1}^{8} 3k^2 - 1 =$

Expand $3k^2 - 1 = 3k^2 - 1$ and use the standard sums with $n = 8$.

The sum is 604.

Ans: (D) 604

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12. The sum of the first 20 terms of the A.P. $12, 15, 18, \ldots$ is

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{20}{2}[24 + 57] = 810$

Ans: (C) 810

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13. The 6th term of the G.P. $7, 14, 28, \ldots$ is

$t_n = ar^{n-1} = 7 \cdot 2^5 = 224$

Ans: (C) 224

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14. The sum of the first 4 terms of $2 + 10 + 50 + \ldots$ is

$S_n = a\frac{r^n-1}{r-1} = 2 \cdot \frac{625-1}{4} = 312$

Ans: (A) 312

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15. The sum to infinity of $1 + 2 \cdot \frac{1}{5} + 3 \cdot \frac{1}{25} + 4 \cdot \frac{1}{125} + \ldots$ is

$a = 1$, $d = 1$, $r = \frac{1}{5}$.

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = \frac{5}{4} + \frac{5}{16} = \frac{25}{16}$

Ans: (D) $\frac{25}{16}$

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16. The nth term of $6, 14, 24, 36, \ldots$ is

Differences: $8, 10, 12$.

Fitting $n = 1, 2, 3$ gives $T_n = n^2 + 5n$.

Ans: (C) $n^2 + 5n$

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17. The nth term of $11, 21, 41, 81, \ldots$ is

Differences: $10, 20, 40$.

Fitting $n = 1, 2, 3$ gives $T_n = 5 \cdot 2^n + 1$.

Ans: (D) $5 \cdot 2^n + 1$

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18. Using $n = 1$ and $n = 2$, the nth term of $1^2 + 4^2 + 7^2 + \ldots$ is

$T_1 = 1$, $T_2 = 16$.

Only $(3n-2)^2$ gives both.

Ans: (B) $(3n-2)^2$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $5 + 11 + 17 + \ldots$ is

$S_1 = 5$, $S_2 = 16$, $S_3 = 33$.

Only $n(3n + 2)$ gives all three.

Ans: (A) $n(3n + 2)$

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20. The largest number that divides $3^{3n} - 1$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 26, \ n = 2 : \ 728$

$\text{G.C.D.}(26, 728) = 26$

Ans: (B) 26

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21. The largest number that divides $7^{2n} - 1$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 48, \ n = 2 : \ 2400$

$\text{G.C.D.}(48, 2400) = 48$

Ans: (C) 48

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22. To prove $10^n - 7^n$ is divisible by 3, assume $10^k - 7^k = 3\lambda$. Then $10^{k+1} - 7^{k+1} =$

$10^{k+1} - 7^{k+1} = 10(3\lambda + 7^k) - 7 \cdot 7^k$

$= 30\lambda + 3 \cdot 7^k = 3(10\lambda + 7^k)$

Ans: (B) $3(10\lambda + 7^k)$

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23. For all positive integers n, $7^n - 4^n$ is divisible by

$x^n - y^n$ is divisible by $x - y$.

$7 - 4 = 3$.

Ans: (A) 3

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24. If $150 \equiv r \pmod{17}$ with $0 \leq r < 17$, then $r =$

$150 = 17 \times 8 + 14$

$\therefore 150 - 14 = 136$ is divisible by 17, so $r = 14$.

Ans: (D) 14

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25. $5^{2n-1} + 2^{2n-1}$ is divisible by

$x^{2n-1} + y^{2n-1}$ is divisible by $x + y = 5 + 2 = 7$.

Ans: (C) 7
