EXPRESSWALA ACADEMY

Maths 1A: Mathematical Induction

MCQ Test, Set 05

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If $P(n)$: $7 + 10 + 13 + \cdots + (3n + 4) = \frac{n(3n+11)}{2}$, then the value of each side of $P(4)$ is

A. 46 **B.** 30 **C.** 16 **D.** 65

2. Strong induction is another name for

A. the principle of complete induction

B. the induction hypothesis

C. the principle of finite induction

D. the base step

3. To prove $1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 + \cdots + n(n + 3) = \frac{n(n+1)(n+5)}{3}$ by induction, the term added to both sides of $P(k)$ is

A. $k(k + 3) + 1$ **B.** $(k + 2)(k + 5)$ **C.** $(k + 1)(k + 4)$ **D.** $k(k + 3)$

4. By induction it can be shown that $\frac{1}{1 \cdot 6} + \frac{1}{6 \cdot 11} + \frac{1}{11 \cdot 16} + \cdots + \frac{1}{(5n-4)(5n+1)} =$

A. $\frac{n-1}{5n-4}$ **B.** $\frac{n}{5n+1}$ **C.** $\frac{n+1}{5n+6}$ **D.** $\frac{2n}{5n+1}$

5. If $1^3 + 2^3 + \cdots + n^3 = 6084$, then $1 + 2 + \cdots + n =$

A. 78 **B.** 12 **C.** 90 **D.** 3042

6. $1^3 + 2^3 + 3^3 + \cdots + n^3$ is equal to

A. $\frac{n(n+1)^2}{2}$ **B.** $\left(\frac{n(n+1)}{2}\right)^2$

C. $\frac{n^2(n+1)}{4}$ **D.** $\frac{n(n+1)(2n+1)}{6}$

7. $1^2 + 3^2 + 5^2 + \ldots$ to 11 terms $=$

A. 3311 **B.** 1793 **C.** 2024 **D.** 1771

8. $2^2 + 4^2 + \cdots + 24^2 =$

A. 4900 **B.** 2600 **C.** 650 **D.** 1300

9. $1^2 - 2^2 + 3^2 - \cdots - 24^2 =$

A. -300 **B.** 276 **C.** -600 **D.** 300

10. $\sum_{k=1}^{n} k(3k-1) =$

A. $n(n-1)^2$ **B.** $n^2(n+1)$ **C.** $(n+1)^2(n+2)$ **D.** $2n^2(n+1)$

11. $\sum_{k=1}^{5} k(2k+1) =$

A. 275 **B.** 70 **C.** 203 **D.** 125

12. The 17th term of the A.P. $5, 11, 17, \ldots$ is

A. 101 **B.** 102 **C.** 107 **D.** 95

13. The sum of the first 9 terms of $4 + 8 + 16 + \ldots$ is

A. 4092 **B.** 1020 **C.** 2044 **D.** 2048

14. The sum to infinity of $10 + 6 + \frac{18}{5} + \frac{54}{25} + \ldots$ is

A. 25 **B.** $\frac{50}{3}$ **C.** $\frac{25}{4}$ **D.** 4

15. The sum to infinity of $4 + 7 \cdot \frac{1}{2} + 10 \cdot \frac{1}{4} + 13 \cdot \frac{1}{8} + \ldots$ is

A. 20 **B.** 11 **C.** 14 **D.** 8

16. The nth term of $1, 8, 19, 34, \ldots$ is

A. $2n^2 + 5n + 1$ **B.** $2n^2 + n$ **C.** $3n^2 - 3$ **D.** $2n^2 + n - 2$

17. The nth term of $2, 14, 62, 254, \ldots$ is

A. $4^n + 2$ **B.** $4^{n+1} - 2$ **C.** 4^n **D.** $4^n - 2$

18. Using $n = 1$ and $n = 2$, the nth term of $2 \cdot 4 + 3 \cdot 5 + 4 \cdot 6 + \ldots$ is

A. $(n+2)(n+4)$ **B.** $n(n+2)$ **C.** $(n+1)(n+3)$ **D.** $n^2 + 5n + 2$

19. Testing with $n = 1, 2, 3$, the sum of n terms of $3 + 7 + 11 + \ldots$ is

A. $n(2n+1)$ **B.** $2n^2 + 2n - 1$ **C.** $n(4n-1)$ **D.** $2n^2 - 3n + 4$

20. The largest number that divides $9^n - 1$ for every $n \in \mathbb{N}$ is

A. 40 **B.** 16 **C.** 9 **D.** 8

21. The largest number that divides $14^n - 9^n$ for every $n \in \mathbb{N}$ is

A. 5 **B.** 10 **C.** 15 **D.** 6

22. To prove $12^n - 5^n$ is divisible by 7, assume $12^k - 5^k = 7\lambda$. Then $12^{k+1} - 5^{k+1} =$

A. $7(5\lambda + 12^k)$ **B.** $7(\lambda + 5^k)$ **C.** $7(12\lambda - 5^k)$ **D.** $7(12\lambda + 5^k)$

23. If $88 \equiv r \pmod{13}$ with $0 \le r < 13$, then $r =$

A. 3 **B.** 11 **C.** 10 **D.** 6

24. $x \equiv y \pmod{m}$ means

A. x and y are both divisible by m **B.** xy is divisible by m

C. $x - y$ is divisible by m **D.** $x + y$ is divisible by m

25. If $71 \equiv r \pmod 4$ with $0 \le r < 4$, then $r =$

A. 1 **B.** 3 **C.** 17 **D.** 0

Key sheet

1. A	**2.** A	**3.** C	**4.** B	**5.** A
6. B	**7.** D	**8.** B	**9.** A	**10.** B
11. D	**12.** A	**13.** C	**14.** A	**15.** C
16. D	**17.** D	**18.** C	**19.** A	**20.** D
21. A	**22.** D	**23.** C	**24.** C	**25.** B

Solutions

1. If $P(n)$: $7 + 10 + 13 + \cdots + (3n + 4) = \frac{n(3n+11)}{2}$, then the value of each side of $P(4)$ is

$7 + 10 + 13 + 16 = 46$

and the right side at $n = 4$ is also 46.

Ans: (A) 46

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2. Strong induction is another name for

The principle of complete induction is also called strong induction.

Ans: (A) the principle of complete induction

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3. To prove $1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 + \cdots + n(n + 3) = \frac{n(n+1)(n+5)}{3}$ by induction, the term added to both sides of $P(k)$ is

The $(k + 1)$th term is got by putting $n = k + 1$ in $n(n + 3)$.

It is $(k + 1)(k + 4)$.

Ans: (C) $(k + 1)(k + 4)$

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4. By induction it can be shown that $\frac{1}{1\cdot6} + \frac{1}{6\cdot11} + \frac{1}{11\cdot16} + \cdots + \frac{1}{(5n-4)(5n+1)} =$

Check with $n = 1$: the sum is $\frac{1}{6}$ and the formula gives $\frac{1}{6}$.

The formula $\frac{n}{5n+1}$ is the one proved by induction.

Ans: (B) $\frac{n}{5n+1}$

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5. If $1^3 + 2^3 + \cdots + n^3 = 6084$, then $1 + 2 + \cdots + n =$

$\sum n^3 = (\sum n)^2$

$\therefore \sum n = \sqrt{6084} = 78$

Ans: (A) 78

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6. $1^3 + 2^3 + 3^3 + \cdots + n^3$ is equal to

The sum of the cubes is the square of the sum: $(\sum n)^2 = \frac{n^2(n+1)^2}{4}$.

Ans: (B) $\left(\frac{n(n+1)}{2}\right)^2$

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7. $1^2 + 3^2 + 5^2 + \ldots$ to 11 terms $=$

$\frac{n}{3}(4n^2 - 1) = \frac{11}{3}(483) = 1771$

Ans: (D) 1771

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8. $2^2 + 4^2 + \cdots + 24^2 =$

$4\sum_{k=1}^{12} k^2 = 4 \cdot 650 = 2600$

Ans: (B) 2600

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9. $1^2 - 2^2 + 3^2 - \cdots - 24^2 =$

$n = 24$ is even.

$-\frac{24\cdot25}{2} = -300$

Ans: (A) -300

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10. $\sum_{k=1}^{n} k(3k-1) =$

Expand $k(3k-1) = 3k^2 - k$ and use $\sum k^2$, $\sum k$.

The total factorises as $n^2(n+1)$.

Ans: (B) $n^2(n+1)$

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11. $\sum_{k=1}^{5} k(2k+1) =$

Expand $k(2k+1) = 2k^2 + k$ and use the standard sums with $n = 5$.

The sum is 125.

Ans: (D) 125

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12. The 17th term of the A.P. $5, 11, 17, \ldots$ is

$t_n = a + (n-1)d = 5 + 16(6) = 101$

Ans: (A) 101

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13. The sum of the first 9 terms of $4 + 8 + 16 + \ldots$ is

$S_n = a\frac{r^n - 1}{r - 1} = 4 \cdot \frac{512 - 1}{1} = 2044$

Ans: (C) 2044

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14. The sum to infinity of $10 + 6 + \frac{18}{5} + \frac{54}{25} + \ldots$ is

$a = 10$, $r = \frac{3}{5}$.

$S_\infty = \frac{a}{1 - r} = 25$

Ans: (A) 25

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15. The sum to infinity of $4 + 7 \cdot \frac{1}{2} + 10 \cdot \frac{1}{4} + 13 \cdot \frac{1}{8} + \ldots$ is

$a = 4$, $d = 3$, $r = \frac{1}{2}$.

$S_\infty = \frac{a}{1 - r} + \frac{dr}{(1 - r)^2} = 8 + 6 = 14$

Ans: (C) 14

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16. The nth term of $1, 8, 19, 34, \ldots$ is

Differences: $7, 11, 15$.

Fitting $n = 1, 2, 3$ gives $T_n = 2n^2 + n - 2$.

Ans: (D) $2n^2 + n - 2$

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17. The nth term of $2, 14, 62, 254, \ldots$ is

Differences: $12, 48, 192$.

Fitting $n = 1, 2, 3$ gives $T_n = 4^n - 2$.

Ans: (D) $4^n - 2$

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18. Using $n = 1$ and $n = 2$, the nth term of $2 \cdot 4 + 3 \cdot 5 + 4 \cdot 6 + \ldots$ is

$T_1 = 8$, $T_2 = 15$.

Only $(n+1)(n+3)$ gives both.

Ans: (C) $(n+1)(n+3)$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $3 + 7 + 11 + \ldots$ is

$S_1 = 3$, $S_2 = 10$, $S_3 = 21$.

Only $n(2n + 1)$ gives all three.

Ans: (A) $n(2n + 1)$

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20. The largest number that divides $9^n - 1$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 8, \ n = 2 : \ 80$

$\text{G.C.D.}(8, 80) = 8$

Ans: (D) 8

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21. The largest number that divides $14^n - 9^n$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 5, \ n = 2 : \ 115$

$\text{G.C.D.}(5, 115) = 5$

Ans: (A) 5

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22. To prove $12^n - 5^n$ is divisible by 7, assume $12^k - 5^k = 7\lambda$. Then $12^{k+1} - 5^{k+1} =$

$12^{k+1} - 5^{k+1} = 12(7\lambda + 5^k) - 5 \cdot 5^k$

$= 84\lambda + 7 \cdot 5^k = 7(12\lambda + 5^k)$

Ans: (D) $7(12\lambda + 5^k)$

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23. If $88 \equiv r \pmod{13}$ with $0 \leq r < 13$, then $r =$

$88 = 13 \times 6 + 10$

$\therefore 88 - 10 = 78$ is divisible by 13, so $r = 10$.

Ans: (C) 10

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24. $x \equiv y \pmod{m}$ means

This is the definition of congruence modulo m.

Ans: (C) $x - y$ **is divisible by** m

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25. If $71 \equiv r \pmod{4}$ with $0 \leq r < 4$, then $r =$

$71 = 4 \times 17 + 3$

$\therefore 71 - 3 = 68$ is divisible by 4, so $r = 3$.

Ans: (B) 3
