EXPRESSWALA ACADEMY

# Maths 1A: Mathematical Induction

## MCQ Test, Set 04

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

**1.** Let $P(n)$ be $5 + 7 + 9 + \cdots + (2n + 3) = n(n + 4)$. In $P(3)$, the left side $5 + 7 + 9$ equals

**A.** $32$ **B.** $21$ **C.** $12$ **D.** $9$

**2.** If $P(1)$ is true and $P(k)$ true implies $P(k + 1)$ true for every $k \in \mathbb{N}$, then

**A.** $P(n)$ is false for some $n$ **B.** $P(n)$ is true only for $n = 1$

**C.** $P(n)$ is true only for $n = k$ **D.** $P(n)$ is true for all $n \in \mathbb{N}$

**3.** To prove $6 + 36 + 216 + \cdots + 6 \cdot 6^{n-1} = \frac{6(6^n - 1)}{5}$ by induction, the term added to both sides of $P(k)$ is

**A.** $6 \cdot 6^k$ **B.** $6 \cdot 6^{k+1}$ **C.** $6 \cdot 6^{k-1}$ **D.** $6 \cdot 6^{k-1} + 1$

**4.** By induction it can be shown that $2^2 + 3^2 + 4^2 + \cdots + (n + 1)^2 =$

**A.** $\frac{(n-1)(2n^2+5n+6)}{6}$ **B.** $\frac{n(2n^2+9n+13)}{3}$

**C.** $\frac{(n+1)(2n^2+13n+24)}{6}$ **D.** $\frac{n(2n^2+9n+13)}{6}$

**5.** $1 + 2 + 3 + \cdots + 55 =$

**A.** $1540$ **B.** $1596$ **C.** $3080$ **D.** $1485$

**6.** $\sum_{k=1}^{9} k^3 =$

**A.** $1296$ **B.** $4050$ **C.** $285$ **D.** $2025$

**7.** The sum of the first $37$ odd natural numbers is

**A.** $1369$ **B.** $1406$ **C.** $1444$ **D.** $1368$

**8.** The sum of the first $45$ even natural numbers is

**A.** $2070$ **B.** $4050$ **C.** $1980$ **D.** $2025$

**9.** $1^2 - 2^2 + 3^2 - \cdots + 13^2 =$

**A.** $-78$ **B.** $182$ **C.** $91$ **D.** $-91$

**10.** $\sum_{k=1}^{n} (k + 2)(2k + 1) =$

**A.** $\frac{n(4n^2+21n+29)}{3}$ **B.** $\frac{(n+1)(4n^2+29n+54)}{6}$

**C.** $\frac{(n-1)(4n^2+13n+12)}{6}$ **D.** $\frac{n(4n^2+21n+29)}{6}$

**11.** $\sum_{k=1}^{6} k^2 + 3k =$

**A.** $224$ **B.** $324$ **C.** $154$ **D.** $100$

**12.** The sum of the first $10$ terms of the A.P. $2, 11, 20, \ldots$ is

**A.** $470$ **B.** $415$ **C.** $830$ **D.** $425$

**13.** The sum to infinity of $18 + 6 + 2 + \frac{2}{3} + \ldots$ is

**A.** $12$ **B.** $27$ **C.** $54$ **D.** $\frac{27}{2}$

**14.** The $5$th term of the G.P. $3, 15, 75, \ldots$ is

**A.** $50625$ **B.** $9375$ **C.** $1875$ **D.** $375$

**15.** The sum to infinity of $2 + 4 \cdot \frac{1}{3} + 6 \cdot \frac{1}{9} + 8 \cdot \frac{1}{27} + \ldots$ is

**A.** $\frac{9}{2}$ **B.** $3$ **C.** $\frac{15}{2}$ **D.** $4$

**16.** The $n$th term of $2, 11, 26, 47, \ldots$ is

**A.** $3n^2 + 6n + 2$ **B.** $3n^2$ **C.** $3n^2 - 1$ **D.** $4n^2 - n - 2$

**17.** The $n$th term of $10, 22, 58, 166, \ldots$ is

**A.** $2 \cdot 3^n - 4$ **B.** $2 \cdot 3^n + 4$ **C.** $2 \cdot 3^n$ **D.** $2 \cdot 3^{n+1} + 4$

**18.** Using $n = 1$ and $n = 2$, the $n$th term of $4 + 10 + 16 + \ldots$ is

**A.** $6n - 2$ **B.** $7n - 3$ **C.** $6n + 4$ **D.** $6n - 8$

**19.** Testing with $n = 1, 2, 3$, the sum of $n$ terms of $2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \ldots$ is

**A.** $n(n+1)(n+2)$ **B.** $\frac{n^3+3n^2+2n+12}{3}$

**C.** $\frac{n(n^2+6n+11)}{3}$ **D.** $\frac{n^3+6n^2+14n-3}{3}$

**20.** The largest number that divides $15^n - 8^n$ for every $n \in \mathbb{N}$ is

**A.** $21$ **B.** $7$ **C.** $14$ **D.** $8$

**21.** The largest number that divides $5^n - 1^n$ for every $n \in \mathbb{N}$ is

**A.** $12$ **B.** $4$ **C.** $5$ **D.** $8$

**22.** To prove $8^n - 3^n$ is divisible by $5$, assume $8^k - 3^k = 5\lambda$. Then $8^{k+1} - 3^{k+1} =$

**A.** $5(8\lambda - 3^k)$ **B.** $5(3\lambda + 8^k)$ **C.** $5(\lambda + 3^k)$ **D.** $5(8\lambda + 3^k)$

**23.** With $P = 3$, $P^{n+1} + (P+1)^{2n-1}$ is divisible by

**A.** $12$ **B.** $13$ **C.** $16$ **D.** $7$

**24.** If $123 \equiv r \pmod{10}$ with $0 \le r < 10$, then $r =$

**A.** $3$ **B.** $7$ **C.** $4$ **D.** $12$

**25.** By Fermat's result, $n^{11} - n$ is divisible by

**A.** $22$ **B.** $12$ **C.** $11$ **D.** $10$

## Key sheet

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|---|---|---|---|---|
| **1.** B | **2.** D | **3.** A | **4.** D | **5.** A |
| **6.** D | **7.** A | **8.** A | **9.** C | **10.** D |
| **11.** C | **12.** D | **13.** B | **14.** C | **15.** A |
| **16.** C | **17.** B | **18.** A | **19.** C | **20.** B |
| **21.** B | **22.** D | **23.** B | **24.** A | **25.** C |

## Solutions

**1.** Let $P(n)$ be $5 + 7 + 9 + \cdots + (2n + 3) = n(n + 4)$. In $P(3)$, the left side $5 + 7 + 9$ equals

$5 + 7 + 9 = 21$

and the right side at $n = 3$ is also $21$.

**Ans: (B)** $21$

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**2.** If $P(1)$ is true and $P(k)$ true implies $P(k + 1)$ true for every $k \in \mathbb{N}$, then

These are the two conditions of the principle of finite induction.

So $P(n)$ holds for every natural number.

**Ans: (D)** $P(n)$ **is true for all** $n \in \mathbb{N}$

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**3.** To prove $6 + 36 + 216 + \cdots + 6 \cdot 6^{n-1} = \frac{6(6^n - 1)}{5}$ by induction, the term added to both sides of $P(k)$ is

The $(k + 1)$th term is got by putting $n = k + 1$ in $6 \cdot 6^{n-1}$.

It is $6 \cdot 6^k$.

**Ans: (A)** $6 \cdot 6^k$

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**4.** By induction it can be shown that $2^2 + 3^2 + 4^2 + \cdots + (n + 1)^2 =$

Check with $n = 1$: the sum is $4$ and the formula gives $4$.

The formula $\frac{n(2n^2+9n+13)}{6}$ is the one proved by induction.

**Ans: (D)** $\frac{n(2n^2+9n+13)}{6}$

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**5.** $1 + 2 + 3 + \cdots + 55 =$

$\frac{n(n+1)}{2} = \frac{55 \cdot 56}{2} = 1540$

**Ans: (A)** $1540$

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**6.** $\sum_{k=1}^{9} k^3 =$

$\left(\frac{n(n+1)}{2}\right)^2 = 45^2 = 2025$

**Ans: (D)** $2025$

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**7.** The sum of the first $37$ odd natural numbers is

$1 + 3 + \cdots + (2n - 1) = n^2 = 1369$

**Ans: (A)** $1369$

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**8.** The sum of the first $45$ even natural numbers is

$2 + 4 + \cdots + 2n = n(n + 1) = 45 \cdot 46 = 2070$

**Ans: (A)** $2070$

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**9.** $1^2 - 2^2 + 3^2 - \cdots + 13^2 =$

$n = 13$ is odd.

$\frac{13 \cdot 14}{2} = 91$

**Ans: (C)** $91$

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**10.** $\sum_{k=1}^{n}(k+2)(2k+1) =$

Expand $(k+2)(2k+1) = 2k^2+5k+2$ and use $\sum k^2$, $\sum k$.

The total factorises as $\frac{n(4n^2+21n+29)}{6}$.

**Ans: (D)** $\frac{n(4n^2+21n+29)}{6}$

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**11.** $\sum_{k=1}^{6} k^2+3k =$

Expand $k^2+3k = k^2+3k$ and use the standard sums with $n=6$.

The sum is $154$.

**Ans: (C)** $154$

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**12.** The sum of the first $10$ terms of the A.P. $2, 11, 20, \ldots$ is

$S_n = \frac{n}{2}[2a+(n-1)d] = \frac{10}{2}[4+81] = 425$

**Ans: (D)** $425$

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**13.** The sum to infinity of $18+6+2+\frac{2}{3}+\ldots$ is

$a=18$, $r=\frac{1}{3}$.

$S_\infty = \frac{a}{1-r} = 27$

**Ans: (B)** $27$

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**14.** The $5$th term of the G.P. $3, 15, 75, \ldots$ is

$t_n = ar^{n-1} = 3\cdot 5^4 = 1875$

**Ans: (C)** $1875$

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**15.** The sum to infinity of $2+4\cdot\frac{1}{3}+6\cdot\frac{1}{9}+8\cdot\frac{1}{27}+\ldots$ is

$a=2$, $d=2$, $r=\frac{1}{3}$.

$S_\infty = \frac{a}{1-r}+\frac{dr}{(1-r)^2} = 3+\frac{3}{2} = \frac{9}{2}$

**Ans: (A)** $\frac{9}{2}$

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**16.** The $n$th term of $2, 11, 26, 47, \ldots$ is

Differences: $9, 15, 21$.

Fitting $n=1,2,3$ gives $T_n = 3n^2-1$.

**Ans: (C)** $3n^2-1$

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**17.** The $n$th term of $10, 22, 58, 166, \ldots$ is

Differences: $12, 36, 108$.

Fitting $n=1,2,3$ gives $T_n = 2\cdot 3^n+4$.

**Ans: (B)** $2\cdot 3^n+4$

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**18.** Using $n=1$ and $n=2$, the $n$th term of $4+10+16+\ldots$ is

$T_1=4$, $T_2=10$.

Only $6n-2$ gives both.

**Ans: (A)** $6n-2$

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**19.** Testing with $n = 1, 2, 3$, the sum of $n$ terms of $2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \ldots$ is

$S_1 = 6$, $S_2 = 18$, $S_3 = 38$.

Only $\frac{n(n^2+6n+11)}{3}$ gives all three.

**Ans: (C)** $\frac{n(n^2+6n+11)}{3}$

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**20.** The largest number that divides $15^n - 8^n$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 7, \ n = 2 : \ 161$

$\text{G.C.D.}(7, 161) = 7$

**Ans: (B)** $7$

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**21.** The largest number that divides $5^n - 1^n$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 4, \ n = 2 : \ 24$

$\text{G.C.D.}(4, 24) = 4$

**Ans: (B)** $4$

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**22.** To prove $8^n - 3^n$ is divisible by $5$, assume $8^k - 3^k = 5\lambda$. Then $8^{k+1} - 3^{k+1} =$

$8^{k+1} - 3^{k+1} = 8(5\lambda + 3^k) - 3 \cdot 3^k$

$= 40\lambda + 5 \cdot 3^k = 5(8\lambda + 3^k)$

**Ans: (D)** $5(8\lambda + 3^k)$

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**23.** With $P = 3$, $P^{n+1} + (P+1)^{2n-1}$ is divisible by

$P^{n+1} + (P+1)^{2n-1}$ is divisible by $P^2 + P + 1 = 9 + 3 + 1 = 13$.

**Ans: (B)** $13$

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**24.** If $123 \equiv r \pmod{10}$ with $0 \leq r < 10$, then $r =$

$123 = 10 \times 12 + 3$

$\therefore 123 - 3 = 120$ is divisible by $10$, so $r = 3$.

**Ans: (A)** $3$

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**25.** By Fermat's result, $n^{11} - n$ is divisible by

For a prime $p$, $n^p - n$ is divisible by $p$; here $p = 11$.

**Ans: (C)** $11$

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