EXPRESSWALA ACADEMY

Maths 1A: Mathematical Induction

MCQ Test, Set 03

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If $P(n)$: $5 + 9 + 13 + \cdots + (4n + 1) = n(2n + 3)$, then the value of each side of $P(4)$ is

A. 44 **B.** 27 **C.** 17 **D.** 65

2. In the principle of complete induction, to prove $P(k + 1)$ we may assume that

A. only $P(1)$ is true **B.** $P(1), P(2), \ldots, P(k)$ are all true

C. $P(k + 2)$ is true **D.** $P(k + 1)$ is true

3. To prove $3^2 + 5^2 + 7^2 + \cdots + (2n + 1)^2 = \frac{n(4n^2+12n+11)}{3}$ by induction, the term added to both sides of $P(k)$ is

A. $(2k + 3)^2$ **B.** $(2k + 1)^2$ **C.** $(2k + 5)^2$ **D.** $(2k + 1)^2 + 1$

4. By induction it can be shown that $6 + 13 + 20 + \cdots + (7n - 1) =$

A. $\frac{(n-1)(7n-2)}{2}$ **B.** $n(7n + 5)$

C. $\frac{(n+1)(7n+12)}{2}$ **D.** $\frac{n(7n+5)}{2}$

5. If $1 + 2 + 3 + \cdots + n = 136$, then $n =$

A. 32 **B.** 15 **C.** 16 **D.** 17

6. $1^2 + 2^2 + \cdots + 14^2 =$

A. 11025 **B.** 1015 **C.** 945 **D.** 2030

7. $1^3 + 3^3 + 5^3 + \ldots$ to 5 terms $=$

A. 2025 **B.** 225 **C.** 1275 **D.** 1225

8. $1^2 + 3^2 + 5^2 + \ldots$ to n terms $=$

A. n^2 **B.** $\frac{n}{3}(4n^2 - 1)$ **C.** $\frac{n}{6}(4n^2 - 1)$ **D.** $\frac{n}{3}(4n^2 + 1)$

9. $1^2 - 2^2 + 3^2 - 4^2 + \ldots$ to $2n$ terms $=$

A. $-\frac{n(n+1)}{2}$ **B.** $-n(2n + 1)$ **C.** $n(2n + 1)$ **D.** $-2n(2n + 1)$

10. $\sum_{k=1}^{n} k(2k + 3) =$

A. $\frac{n(n-1)(4n+7)}{6}$ **B.** $\frac{n(n+1)(4n+11)}{3}$

C. $\frac{n(n+1)(4n+11)}{6}$ **D.** $\frac{(n+1)(n+2)(4n+15)}{6}$

11. $\sum_{k=1}^{7} k^2 + 2k =$

A. 441 **B.** 196 **C.** 133 **D.** 276

12. The 31th term of the A.P. $10, 14, 18, \ldots$ is

A. 130 **B.** 126 **C.** 134 **D.** 124

13. The 9th term of the G.P. $5, 10, 20, \ldots$ is

A. 100000000 **B.** 2560 **C.** 640 **D.** 1280

14. The sum of the first 7 terms of $1 + 3 + 9 + \ldots$ is

A. 2187 **B.** 2186 **C.** 3280 **D.** 1093

15. The sum to infinity of $1 + 6 \cdot \frac{1}{4} + 11 \cdot \frac{1}{16} + 16 \cdot \frac{1}{64} + \ldots$ is

A. $\frac{4}{3}$ **B.** $\frac{32}{9}$ **C.** 3 **D.** $\frac{92}{9}$

16. The nth term of $5, 6, 9, 14, \ldots$ is

A. $n^2 + 5$ **B.** $n^2 - 2n$ **C.** $n^2 - 2n + 6$ **D.** $2n^2 - 3n + 5$

17. The nth term of $8, 28, 128, 628, \ldots$ is

A. $5^n - 3$ **B.** $5^{n+1} + 3$ **C.** 5^n **D.** $5^n + 3$

18. Using $n = 1$ and $n = 2$, the nth term of $2 \cdot 4 + 4 \cdot 6 + 6 \cdot 8 + \ldots$ is

A. $4n(n-1)$ **B.** $2(n+1)(2n+4)$ **C.** $2n(2n+2)$ **D.** $4n^2 + 5n - 1$

19. Testing with $n = 1, 2, 3$, the sum of n terms of $1 + 4 + 7 + \ldots$ is

A. $\frac{n(3n-1)}{2}$ **B.** $\frac{3n^2-7n+6}{2}$

C. $n(3n-2)$ **D.** $\frac{(n+1)(3n-2)}{2}$

20. The largest number that divides $10^n - 1$ for every $n \in \mathbb{N}$ is

A. 10 **B.** 18 **C.** 27 **D.** 9

21. The largest number that divides $2^{2n} - 1$ for every $n \in \mathbb{N}$ is

A. 4 **B.** 9 **C.** 6 **D.** 3

22. To prove $13^n - 4^n$ is divisible by 9, assume $13^k - 4^k = 9\lambda$. Then $13^{k+1} - 4^{k+1} =$

A. $9(4\lambda + 13^k)$ **B.** $9(13\lambda - 4^k)$ **C.** $9(13\lambda + 4^k)$ **D.** $9(\lambda + 4^k)$

23. If $65 \equiv r \pmod 9$ with $0 \le r < 9$, then $r =$

A. 2 **B.** 4 **C.** 3 **D.** 7

24. The sum of the cubes of three consecutive natural numbers is always divisible by

A. 27 **B.** 8 **C.** 9 **D.** 6

25. If $94 \equiv r \pmod{11}$ with $0 \le r < 11$, then $r =$

A. 5 **B.** 7 **C.** 8 **D.** 6

Key sheet

1. A	**2.** B	**3.** A	**4.** D	**5.** C
6. B	**7.** D	**8.** B	**9.** B	**10.** C
11. B	**12.** A	**13.** D	**14.** D	**15.** B
16. C	**17.** D	**18.** C	**19.** A	**20.** D
21. D	**22.** C	**23.** A	**24.** C	**25.** D

Solutions

1. If $P(n)$: $5 + 9 + 13 + \cdots + (4n + 1) = n(2n + 3)$, then the value of each side of $P(4)$ is

$5 + 9 + 13 + 17 = 44$

and the right side at $n = 4$ is also 44.

Ans: (A) 44

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2. In the principle of complete induction, to prove $P(k + 1)$ we may assume that

Complete induction lets us use every earlier case, $P(1)$ to $P(k)$.

Ans: (B) $P(1), P(2), \ldots, P(k)$ **are all true**

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3. To prove $3^2 + 5^2 + 7^2 + \cdots + (2n + 1)^2 = \frac{n(4n^2+12n+11)}{3}$ by induction, the term added to both sides of $P(k)$ is

The $(k + 1)$th term is got by putting $n = k + 1$ in $(2n + 1)^2$.

It is $(2k + 3)^2$.

Ans: (A) $(2k + 3)^2$

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4. By induction it can be shown that $6 + 13 + 20 + \cdots + (7n - 1) =$

Check with $n = 1$: the sum is 6 and the formula gives 6.

The formula $\frac{n(7n+5)}{2}$ is the one proved by induction.

Ans: (D) $\frac{n(7n+5)}{2}$

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5. If $1 + 2 + 3 + \cdots + n = 136$, then $n =$

$\frac{n(n+1)}{2} = 136 \Rightarrow n(n + 1) = 272 = 16 \cdot 17$

$\therefore n = 16$

Ans: (C) 16

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6. $1^2 + 2^2 + \cdots + 14^2 =$

$\frac{n(n+1)(2n+1)}{6} = \frac{14 \cdot 15 \cdot 29}{6} = 1015$

Ans: (B) 1015

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7. $1^3 + 3^3 + 5^3 + \ldots$ to 5 terms $=$

$n^2(2n^2 - 1) = 25(49) = 1225$

Ans: (D) 1225

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8. $1^2 + 3^2 + 5^2 + \ldots$ to n terms $=$

The standard result for the squares of odd numbers.

$\sum(2k - 1)^2 = \frac{n}{3}(4n^2 - 1)$

Ans: (B) $\frac{n}{3}(4n^2 - 1)$

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9. $1^2 - 2^2 + 3^2 - 4^2 + \ldots$ to $2n$ terms $=$

$2n$ is even, so the sum is $-\frac{2n(2n+1)}{2} = -n(2n + 1)$.

Ans: (B) $-n(2n + 1)$

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10. $\sum_{k=1}^{n} k(2k+3) =$

Expand $k(2k+3) = 2k^2 + 3k$ and use $\sum k^2$, $\sum k$.

The total factorises as $\frac{n(n+1)(4n+11)}{6}$.

Ans: (C) $\frac{n(n+1)(4n+11)}{6}$

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11. $\sum_{k=1}^{7} k^2 + 2k =$

Expand $k^2 + 2k = k^2 + 2k$ and use the standard sums with $n = 7$.

The sum is 196.

Ans: (B) 196

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12. The 31th term of the A.P. $10, 14, 18, \ldots$ is

$t_n = a + (n-1)d = 10 + 30(4) = 130$

Ans: (A) 130

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13. The 9th term of the G.P. $5, 10, 20, \ldots$ is

$t_n = ar^{n-1} = 5 \cdot 2^8 = 1280$

Ans: (D) 1280

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14. The sum of the first 7 terms of $1 + 3 + 9 + \ldots$ is

$S_n = a\frac{r^n - 1}{r - 1} = 1 \cdot \frac{2187 - 1}{2} = 1093$

Ans: (D) 1093

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15. The sum to infinity of $1 + 6 \cdot \frac{1}{4} + 11 \cdot \frac{1}{16} + 16 \cdot \frac{1}{64} + \ldots$ is

$a = 1$, $d = 5$, $r = \frac{1}{4}$.

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = \frac{4}{3} + \frac{20}{9} = \frac{32}{9}$

Ans: (B) $\frac{32}{9}$

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16. The nth term of $5, 6, 9, 14, \ldots$ is

Differences: $1, 3, 5$.

Fitting $n = 1, 2, 3$ gives $T_n = n^2 - 2n + 6$.

Ans: (C) $n^2 - 2n + 6$

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17. The nth term of $8, 28, 128, 628, \ldots$ is

Differences: $20, 100, 500$.

Fitting $n = 1, 2, 3$ gives $T_n = 5^n + 3$.

Ans: (D) $5^n + 3$

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18. Using $n = 1$ and $n = 2$, the nth term of $2 \cdot 4 + 4 \cdot 6 + 6 \cdot 8 + \ldots$ is

$T_1 = 8$, $T_2 = 24$.

Only $2n(2n+2)$ gives both.

Ans: (C) $2n(2n+2)$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $1 + 4 + 7 + \ldots$ is

$S_1 = 1$, $S_2 = 5$, $S_3 = 12$.

Only $\frac{n(3n-1)}{2}$ gives all three.

Ans: (A) $\frac{n(3n-1)}{2}$

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20. The largest number that divides $10^n - 1$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 9, \ n = 2 : \ 99$

$\text{G.C.D.}(9, 99) = 9$

Ans: (D) 9

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21. The largest number that divides $2^{2n} - 1$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 3, \ n = 2 : \ 15$

$\text{G.C.D.}(3, 15) = 3$

Ans: (D) 3

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22. To prove $13^n - 4^n$ is divisible by 9, assume $13^k - 4^k = 9\lambda$. Then $13^{k+1} - 4^{k+1} =$

$13^{k+1} - 4^{k+1} = 13(9\lambda + 4^k) - 4 \cdot 4^k$

$= 117\lambda + 9 \cdot 4^k = 9(13\lambda + 4^k)$

Ans: (C) $9(13\lambda + 4^k)$

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23. If $65 \equiv r \pmod 9$ with $0 \leq r < 9$, then $r =$

$65 = 9 \times 7 + 2$

$\therefore 65 - 2 = 63$ is divisible by 9, so $r = 2$.

Ans: (A) 2

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24. The sum of the cubes of three consecutive natural numbers is always divisible by

Standard result: divisible by 9.

For example $1 + 8 + 27 = 36 = 9 \times 4$.

Ans: (C) 9

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25. If $94 \equiv r \pmod{11}$ with $0 \leq r < 11$, then $r =$

$94 = 11 \times 8 + 6$

$\therefore 94 - 6 = 88$ is divisible by 11, so $r = 6$.

Ans: (D) 6
