EXPRESSWALA ACADEMY

Maths 1A: Mathematical Induction

MCQ Test, Set 02

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Let $P(n)$ be $5 + 12 + 19 + \cdots + (7n - 2) = \frac{n(7n+3)}{2}$. In $P(2)$, the left side $5 + 12$ equals

A. 5 **B.** 36 **C.** 17 **D.** 12

2. In an induction proof, the assumption that $P(k)$ is true is called

A. the induction hypothesis

B. the principle of complete induction

C. the base step

D. the conclusion

3. To prove $7 + 11 + 15 + \cdots + (4n + 3) = n(2n + 5)$ by induction, the term added to both sides of $P(k)$ is

A. $4k + 3$ **B.** $4k + 7$ **C.** $4k + 11$ **D.** $4k + 4$

4. By induction it can be shown that $\frac{1}{3\cdot7} + \frac{1}{7\cdot11} + \frac{1}{11\cdot15} + \cdots + \frac{1}{(4n-1)(4n+3)} =$

A. $\frac{n}{12n+9}$ **B.** $\frac{n+1}{3(4n+7)}$

C. $\frac{2n}{3(4n+3)}$ **D.** $\frac{n-1}{3(4n-1)}$

5. $\sum_{k=1}^{13} k^3 =$

A. 819 **B.** 16562 **C.** 6084 **D.** 8281

6. $1^4 + 2^4 + \cdots + 4^4 =$

A. 900 **B.** 354 **C.** 200 **D.** 98

7. $2^2 + 4^2 + \cdots + 14^2 =$

A. 1015 **B.** 280 **C.** 140 **D.** 560

8. $1^3 + 3^3 + 5^3 + \ldots$ to n terms $=$

A. $n^2(n + 1)^2$ **B.** $n^2(2n^2 + 1)$ **C.** $\frac{n}{3}(4n^2 - 1)$ **D.** $n^2(2n^2 - 1)$

9. $1^2 - 2^2 + 3^2 - \cdots - 20^2 =$

A. -210 **B.** 190 **C.** -420 **D.** 210

10. $\sum_{k=1}^{n}(k+1)(k+4) =$

A. $\frac{2n(n+4)(n+5)}{3}$ **B.** $\frac{n(n+4)(n+5)}{3}$

C. $\frac{(n+1)(n+5)(n+6)}{3}$ **D.** $\frac{(n-1)(n+3)(n+4)}{3}$

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11. $\sum_{k=1}^{9} k(k-1) =$

A. 648 **B.** 168 **C.** 330 **D.** 240

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12. The sum of the first 12 terms of the A.P. $3, 8, 13, \ldots$ is

A. 396 **B.** 696 **C.** 348 **D.** 366

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13. The sum of the first 5 terms of $3 + 12 + 48 + \ldots$ is

A. 3069 **B.** 3072 **C.** 1023 **D.** 4095

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14. The sum to infinity of $20 - 5 + \frac{5}{4} - \frac{5}{16} + \ldots$ is

A. $\frac{80}{3}$ **B.** 25 **C.** 16 **D.** -80

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15. The sum to infinity of $3 + 4 \cdot \frac{1}{3} + 5 \cdot \frac{1}{9} + 6 \cdot \frac{1}{27} + \ldots$ is

A. $\frac{9}{2}$ **B.** $\frac{21}{4}$ **C.** $\frac{27}{4}$ **D.** 5

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16. The nth term of $5, 11, 21, 35, \ldots$ is

A. $3n^2 - n + 2$ **B.** $2n^2 + 4n + 5$ **C.** $2n^2 + 3$ **D.** $2n^2$

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17. The nth term of $10, 28, 82, 244, \ldots$ is

A. $3 \cdot 3^n$ **B.** $3 \cdot 3^{n+1} + 1$ **C.** $3 \cdot 3^n - 1$ **D.** $3 \cdot 3^n + 1$

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18. Using $n = 1$ and $n = 2$, the nth term of $3^2 + 5^2 + 7^2 + \ldots$ is

A. $(2n+3)^2$ **B.** $(2n+1)^2$ **C.** $(2n-1)^2$ **D.** $4n^2 + 5n$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $1 \cdot 1 + 2 \cdot 3 + 3 \cdot 5 + \ldots$ is

A. $n^2(2n-1)$ **B.** $\frac{n(n+1)(4n-1)}{6}$

C. $\frac{4n^3+3n^2+5n-6}{6}$ **D.** $\frac{4n^3-9n^2+5n+6}{6}$

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20. The largest number that divides $12^n - 7^n$ for every $n \in \mathbb{N}$ is

A. 15 **B.** 6 **C.** 10 **D.** 5

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21. The largest number that divides $4^n - 1$ for every $n \in \mathbb{N}$ is

A. 4 **B.** 6 **C.** 3 **D.** 9

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22. To prove $9^n - 5^n$ is divisible by 4, assume $9^k - 5^k = 4\lambda$. Then $9^{k+1} - 5^{k+1} =$

A. $4(\lambda + 5^k)$ **B.** $4(9\lambda + 5^k)$ **C.** $4(5\lambda + 9^k)$ **D.** $4(9\lambda - 5^k)$

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23. If n is any odd integer, $n(n^2 - 1)$ is always divisible by

A. 36 **B.** 16 **C.** 48 **D.** 24

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24. If $100 \equiv r \pmod 7$ with $0 \le r < 7$, then $r =$

A. 2 **B.** 3 **C.** 5 **D.** 14

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25. The product of 4 consecutive natural numbers is always divisible by

A. 24 **B.** 12 **C.** 16 **D.** 48

Key sheet

1. C	**2.** A	**3.** B	**4.** A	**5.** D
6. B	**7.** D	**8.** D	**9.** A	**10.** B
11. D	**12.** D	**13.** C	**14.** C	**15.** B
16. C	**17.** D	**18.** B	**19.** B	**20.** D
21. C	**22.** B	**23.** D	**24.** A	**25.** A

Solutions

1. Let $P(n)$ be $5 + 12 + 19 + \cdots + (7n - 2) = \frac{n(7n+3)}{2}$. In $P(2)$, the left side $5 + 12$ equals

$5 + 12 = 17$

and the right side at $n = 2$ is also 17.

Ans: (C) 17

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2. In an induction proof, the assumption that $P(k)$ is true is called

Inside the inductive step we assume $P(k)$; this assumption is the induction hypothesis.

Ans: (A) the induction hypothesis

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3. To prove $7 + 11 + 15 + \cdots + (4n + 3) = n(2n + 5)$ by induction, the term added to both sides of $P(k)$ is

The $(k+1)$th term is got by putting $n = k + 1$ in $(4n + 3)$.

It is $4k + 7$.

Ans: (B) $4k + 7$

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4. By induction it can be shown that $\frac{1}{3\cdot7} + \frac{1}{7\cdot11} + \frac{1}{11\cdot15} + \cdots + \frac{1}{(4n-1)(4n+3)} =$

Check with $n = 1$: the sum is $\frac{1}{21}$ and the formula gives $\frac{1}{21}$.

The formula $\frac{n}{12n+9}$ is the one proved by induction.

Ans: (A) $\frac{n}{12n+9}$

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5. $\sum_{k=1}^{13} k^3 =$

$\left(\frac{n(n+1)}{2}\right)^2 = 91^2 = 8281$

Ans: (D) 8281

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6. $1^4 + 2^4 + \cdots + 4^4 =$

$\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$ with $n = 4$

$= 354$

Ans: (B) 354

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7. $2^2 + 4^2 + \cdots + 14^2 =$

$4\sum_{k=1}^{7} k^2 = 4 \cdot 140 = 560$

Ans: (D) 560

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8. $1^3 + 3^3 + 5^3 + \ldots$ to n terms $=$

The standard result for the cubes of odd numbers.

$\sum(2k - 1)^3 = n^2(2n^2 - 1)$

Ans: (D) $n^2(2n^2 - 1)$

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9. $1^2 - 2^2 + 3^2 - \cdots - 20^2 =$

$n = 20$ is even.

$-\frac{20 \cdot 21}{2} = -210$

Ans: (A) -210

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10. $\sum_{k=1}^{n}(k+1)(k+4) =$

Expand $(k+1)(k+4) = k^2 + 5k + 4$ and use $\sum k^2$, $\sum k$.

The total factorises as $\frac{n(n+4)(n+5)}{3}$.

Ans: (B) $\frac{n(n+4)(n+5)}{3}$

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11. $\sum_{k=1}^{9} k(k-1) =$

Expand $k(k-1) = k^2 - k$ and use the standard sums with $n = 9$.

The sum is 240.

Ans: (D) 240

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12. The sum of the first 12 terms of the A.P. $3, 8, 13, \ldots$ is

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{12}{2}[6 + 55] = 366$

Ans: (D) 366

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13. The sum of the first 5 terms of $3 + 12 + 48 + \ldots$ is

$S_n = a\frac{r^n - 1}{r - 1} = 3 \cdot \frac{1024 - 1}{3} = 1023$

Ans: (C) 1023

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14. The sum to infinity of $20 - 5 + \frac{5}{4} - \frac{5}{16} + \ldots$ is

$a = 20$, $r = -\frac{1}{4}$.

$S_\infty = \frac{a}{1 - r} = 16$

Ans: (C) 16

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15. The sum to infinity of $3 + 4 \cdot \frac{1}{3} + 5 \cdot \frac{1}{9} + 6 \cdot \frac{1}{27} + \ldots$ is

$a = 3$, $d = 1$, $r = \frac{1}{3}$.

$S_\infty = \frac{a}{1 - r} + \frac{dr}{(1 - r)^2} = \frac{9}{2} + \frac{3}{4} = \frac{21}{4}$

Ans: (B) $\frac{21}{4}$

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16. The nth term of $5, 11, 21, 35, \ldots$ is

Differences: $6, 10, 14$.

Fitting $n = 1, 2, 3$ gives $T_n = 2n^2 + 3$.

Ans: (C) $2n^2 + 3$

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17. The nth term of $10, 28, 82, 244, \ldots$ is

Differences: $18, 54, 162$.

Fitting $n = 1, 2, 3$ gives $T_n = 3 \cdot 3^n + 1$.

Ans: (D) $3 \cdot 3^n + 1$

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18. Using $n = 1$ and $n = 2$, the nth term of $3^2 + 5^2 + 7^2 + \ldots$ is

$T_1 = 9$, $T_2 = 25$.

Only $(2n + 1)^2$ gives both.

Ans: (B) $(2n + 1)^2$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $1 \cdot 1 + 2 \cdot 3 + 3 \cdot 5 + \ldots$ is

$S_1 = 1$, $S_2 = 7$, $S_3 = 22$.

Only $\frac{n(n+1)(4n-1)}{6}$ gives all three.

Ans: (B) $\frac{n(n+1)(4n-1)}{6}$

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20. The largest number that divides $12^n - 7^n$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 5, \ n = 2 : \ 95$

$\text{G.C.D.}(5, 95) = 5$

Ans: (D) 5

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21. The largest number that divides $4^n - 1$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 3, \ n = 2 : \ 15$

$\text{G.C.D.}(3, 15) = 3$

Ans: (C) 3

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22. To prove $9^n - 5^n$ is divisible by 4, assume $9^k - 5^k = 4\lambda$. Then $9^{k+1} - 5^{k+1} =$

$9^{k+1} - 5^{k+1} = 9(4\lambda + 5^k) - 5 \cdot 5^k$

$= 36\lambda + 4 \cdot 5^k = 4(9\lambda + 5^k)$

Ans: (B) $4(9\lambda + 5^k)$

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23. If n is any odd integer, $n(n^2 - 1)$ is always divisible by

Standard result: for odd n, $n(n^2 - 1)$ is divisible by 24.

For example $n = 5$ gives $120 = 24 \times 5$.

Ans: (D) 24

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24. If $100 \equiv r \pmod 7$ with $0 \leq r < 7$, then $r =$

$100 = 7 \times 14 + 2$

$\therefore 100 - 2 = 98$ is divisible by 7, so $r = 2$.

Ans: (A) 2

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25. The product of 4 consecutive natural numbers is always divisible by

The product of n consecutive natural numbers is divisible by $n!$; here $4! = 24$.

Ans: (A) 24
