EXPRESSWALA ACADEMY

Maths 1A: Mathematical Induction

MCQ Test, Set 10

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. Let $P(n)$ be $5 + 13 + 21 + \cdots + (8n - 3) = n(4n + 1)$. In $P(3)$, the left side $5 + 13 + 21$ equals

A. 68 **B.** 21 **C.** 18 **D.** 39

2. The last step of a proof by induction is

A. adding the $(k + 1)$th term

B. concluding that $P(n)$ holds for all $n \in \mathbb{N}$

C. assuming $P(k)$

D. verifying $P(1)$

3. To prove $8 + 13 + 18 + \cdots + (5n + 3) = \frac{n(5n+11)}{2}$ by induction, the term added to both sides of $P(k)$ is

A. $5k + 3$ **B.** $5k + 13$ **C.** $5k + 8$ **D.** $5k + 4$

4. By induction it can be shown that $3^2 + 7^2 + 11^2 + \cdots + (4n - 1)^2 =$

A. $\frac{n(16n^2+12n-1)}{3}$ **B.** $\frac{2n(16n^2+12n-1)}{3}$

C. $\frac{(n-1)(16n^2-20n+3)}{3}$ **D.** $\frac{(n+1)(16n^2+44n+27)}{3}$

5. $\sum_{k=1}^{11} k^3 =$

A. 3025 **B.** 4356 **C.** 506 **D.** 2178

6. $1^2 + 2^2 + \cdots + 9^2 =$

A. 285 **B.** 255 **C.** 570 **D.** 2025

7. $1^2 + 3^2 + 5^2 + \ldots$ to 14 terms $=$

A. 6930 **B.** 3682 **C.** 4060 **D.** 3654

8. $2^2 + 4^2 + \cdots + 28^2 =$

A. 4060 **B.** 1015 **C.** 2030 **D.** 7714

9. $1^2 - 2^2 + 3^2 - \cdots - 8^2 =$

A. -72 **B.** -36 **C.** 28 **D.** 36

10. $\sum_{k=1}^{n}(k+2)(k+4) =$

A. $\frac{n(2n^2+21n+67)}{6}$ **B.** $\frac{(n+1)(2n^2+25n+90)}{6}$

C. $\frac{n(2n^2+21n+67)}{3}$ **D.** $\frac{(n-1)(2n^2+17n+48)}{6}$

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11. $\sum_{k=1}^{7} 4k^2+1 =$

A. 824 **B.** 370 **C.** 567 **D.** 1379

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12. The sum of the first 11 terms of the A.P. $7, 11, 15, \ldots$ is

A. $\frac{517}{2}$ **B.** 517 **C.** 297 **D.** 319

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13. The sum to infinity of $24 + 12 + 6 + 3 + \ldots$ is

A. 24 **B.** 12 **C.** 16 **D.** 48

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14. The 8th term of the G.P. $9, 18, 36, \ldots$ is

A. 576 **B.** 612220032 **C.** 2304 **D.** 1152

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15. The sum to infinity of $1 + 7\cdot\frac{1}{2} + 13\cdot\frac{1}{4} + 19\cdot\frac{1}{8} + \ldots$ is

A. 14 **B.** 8 **C.** 2 **D.** 26

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16. The nth term of $6, 12, 20, 30, \ldots$ is

A. n^2+5n+6 **B.** n^2+3n **C.** $2n^2+2n+1$ **D.** n^2+3n+2

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17. The nth term of $9, 51, 345, 2403, \ldots$ is

A. 7^n **B.** 7^n+2 **C.** $7^{n+1}+2$ **D.** 7^n-2

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18. Using $n=1$ and $n=2$, the nth term of $3^2 + 7^2 + 11^2 + \ldots$ is

A. $(4n+3)^2$ **B.** $(4n-5)^2$ **C.** $(4n-1)^2$ **D.** $16n^2-7n$

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19. Testing with $n=1,2,3$, the sum of n terms of $1\cdot4 + 2\cdot5 + 3\cdot6 + \ldots$ is

A. $n^2(n+3)$ **B.** $\frac{n(n+1)(n+5)}{3}$

C. $\frac{(n+3)(n^2+3n-1)}{3}$ **D.** $\frac{n^3+3n^2-4n+12}{3}$

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20. The largest number that divides $2^{4n}-1$ for every $n\in\mathbb{N}$ is

A. 15 **B.** 30 **C.** 16 **D.** 45

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21. The largest number that divides $16^n - 9^n$ for every $n\in\mathbb{N}$ is

A. 8 **B.** 14 **C.** 7 **D.** 21

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22. To prove $17^n - 3^n$ is divisible by 14, assume $17^k - 3^k = 14\lambda$. Then $17^{k+1} - 3^{k+1} =$

A. $14(17\lambda - 3^k)$ **B.** $14(3\lambda + 17^k)$ **C.** $14(\lambda + 3^k)$ **D.** $14(17\lambda + 3^k)$

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23. If $58 \equiv r \pmod{11}$ with $0 \le r < 11$, then $r =$

A. 3 **B.** 5 **C.** 8 **D.** 4

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24. If $127 \equiv r \pmod{9}$ with $0 \le r < 9$, then $r =$

A. 14 **B.** 1 **C.** 2 **D.** 8

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25. If $91 \equiv r \pmod{16}$ with $0 \leq r < 16$, then $r =$

A. 5 **B.** 13 **C.** 12 **D.** 11

Key sheet

1. D	**2.** B	**3.** C	**4.** A	**5.** B
6. A	**7.** D	**8.** A	**9.** B	**10.** A
11. C	**12.** C	**13.** D	**14.** D	**15.** A
16. D	**17.** B	**18.** C	**19.** B	**20.** A
21. C	**22.** D	**23.** A	**24.** B	**25.** D

Solutions

1. Let $P(n)$ be $5 + 13 + 21 + \cdots + (8n - 3) = n(4n + 1)$. In $P(3)$, the left side $5 + 13 + 21$ equals

$5 + 13 + 21 = 39$

and the right side at $n = 3$ is also 39.

Ans: (D) 39

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2. The last step of a proof by induction is

After the base and inductive steps, we conclude by the principle of induction.

Ans: (B) concluding that $P(n)$ holds for all $n \in \mathbb{N}$

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3. To prove $8 + 13 + 18 + \cdots + (5n + 3) = \frac{n(5n+11)}{2}$ by induction, the term added to both sides of $P(k)$ is

The $(k+1)$th term is got by putting $n = k + 1$ in $(5n + 3)$.

It is $5k + 8$.

Ans: (C) $5k + 8$

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4. By induction it can be shown that $3^2 + 7^2 + 11^2 + \cdots + (4n - 1)^2 =$

Check with $n = 1$: the sum is 9 and the formula gives 9.

The formula $\frac{n(16n^2+12n-1)}{3}$ is the one proved by induction.

Ans: (A) $\frac{n(16n^2+12n-1)}{3}$

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5. $\sum_{k=1}^{11} k^3 =$

$\left(\frac{n(n+1)}{2}\right)^2 = 66^2 = 4356$

Ans: (B) 4356

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6. $1^2 + 2^2 + \cdots + 9^2 =$

$\frac{n(n+1)(2n+1)}{6} = \frac{9 \cdot 10 \cdot 19}{6} = 285$

Ans: (A) 285

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7. $1^2 + 3^2 + 5^2 + \ldots$ to 14 terms $=$

$\frac{n}{3}(4n^2 - 1) = \frac{14}{3}(783) = 3654$

Ans: (D) 3654

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8. $2^2 + 4^2 + \cdots + 28^2 =$

$4\sum_{k=1}^{14} k^2 = 4 \cdot 1015 = 4060$

Ans: (A) 4060

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9. $1^2 - 2^2 + 3^2 - \cdots - 8^2 =$

$n = 8$ is even.

$-\frac{8 \cdot 9}{2} = -36$

Ans: (B) -36

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10. $\sum_{k=1}^{n}(k+2)(k+4) =$

Expand $(k+2)(k+4) = k^2+6k+8$ and use $\sum k^2$, $\sum k$.

The total factorises as $\frac{n(2n^2+21n+67)}{6}$.

Ans: (A) $\frac{n(2n^2+21n+67)}{6}$

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11. $\sum_{k=1}^{7} 4k^2+1 =$

Expand $4k^2+1 = 4k^2+1$ and use the standard sums with $n=7$.

The sum is 567.

Ans: (C) 567

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12. The sum of the first 11 terms of the A.P. $7, 11, 15, \ldots$ is

$S_n = \frac{n}{2}[2a+(n-1)d] = \frac{11}{2}[14+40] = 297$

Ans: (C) 297

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13. The sum to infinity of $24+12+6+3+\ldots$ is

$a = 24$, $r = \frac{1}{2}$.

$S_\infty = \frac{a}{1-r} = 48$

Ans: (D) 48

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14. The 8th term of the G.P. $9, 18, 36, \ldots$ is

$t_n = ar^{n-1} = 9 \cdot 2^7 = 1152$

Ans: (D) 1152

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15. The sum to infinity of $1+7\cdot\frac{1}{2}+13\cdot\frac{1}{4}+19\cdot\frac{1}{8}+\ldots$ is

$a=1$, $d=6$, $r=\frac{1}{2}$.

$S_\infty = \frac{a}{1-r}+\frac{dr}{(1-r)^2} = 2+12 = 14$

Ans: (A) 14

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16. The nth term of $6, 12, 20, 30, \ldots$ is

Differences: $6, 8, 10$.

Fitting $n=1,2,3$ gives $T_n = n^2+3n+2$.

Ans: (D) n^2+3n+2

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17. The nth term of $9, 51, 345, 2403, \ldots$ is

Differences: $42, 294, 2058$.

Fitting $n=1,2,3$ gives $T_n = 7^n+2$.

Ans: (B) 7^n+2

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18. Using $n=1$ and $n=2$, the nth term of $3^2+7^2+11^2+\ldots$ is

$T_1 = 9$, $T_2 = 49$.

Only $(4n-1)^2$ gives both.

Ans: (C) $(4n-1)^2$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 + \ldots$ is

$S_1 = 4$, $S_2 = 14$, $S_3 = 32$.

Only $\frac{n(n+1)(n+5)}{3}$ gives all three.

Ans: (B) $\frac{n(n+1)(n+5)}{3}$

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20. The largest number that divides $2^{4n} - 1$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 15, \ n = 2 : \ 255$

$\text{G.C.D.}(15, 255) = 15$

Ans: (A) 15

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21. The largest number that divides $16^n - 9^n$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 7, \ n = 2 : \ 175$

$\text{G.C.D.}(7, 175) = 7$

Ans: (C) 7

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22. To prove $17^n - 3^n$ is divisible by 14, assume $17^k - 3^k = 14\lambda$. Then $17^{k+1} - 3^{k+1} =$

$17^{k+1} - 3^{k+1} = 17(14\lambda + 3^k) - 3 \cdot 3^k$

$= 238\lambda + 14 \cdot 3^k = 14(17\lambda + 3^k)$

Ans: (D) $14(17\lambda + 3^k)$

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23. If $58 \equiv r \pmod{11}$ with $0 \leq r < 11$, then $r =$

$58 = 11 \times 5 + 3$

$\therefore 58 - 3 = 55$ is divisible by 11, so $r = 3$.

Ans: (A) 3

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24. If $127 \equiv r \pmod{9}$ with $0 \leq r < 9$, then $r =$

$127 = 9 \times 14 + 1$

$\therefore 127 - 1 = 126$ is divisible by 9, so $r = 1$.

Ans: (B) 1

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25. If $91 \equiv r \pmod{16}$ with $0 \leq r < 16$, then $r =$

$91 = 16 \times 5 + 11$

$\therefore 91 - 11 = 80$ is divisible by 16, so $r = 11$.

Ans: (D) 11
