EXPRESSWALA ACADEMY

Maths 1A: Mathematical Induction

MCQ Test, Set 01

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. If $P(n)$: $1 + 7 + 13 + \cdots + (6n - 5) = n(3n - 2)$, then the value of each side of $P(3)$ is

A. 13 **B.** 8 **C.** 40 **D.** 21

2. In the principle of finite mathematical induction, the base step is to check that

A. $P(k + 1)$ is true **B.** $P(1)$ is true

C. $P(k)$ is true **D.** $P(n)$ is true for every n

3. To prove $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + n(n + 2) = \frac{n(n+1)(2n+7)}{6}$ by induction, the term added to both sides of $P(k)$ is

A. $(k + 1)(k + 3)$ **B.** $(k + 2)(k + 4)$ **C.** $k(k + 2) + 1$ **D.** $k(k + 2)$

4. By induction it can be shown that $2^2 + 5^2 + 8^2 + \cdots + (3n - 1)^2 =$

A. $\frac{(n+1)(6n^2+15n+8)}{2}$ **B.** $\frac{n(6n^2+3n-1)}{2}$

C. $\frac{(n-1)(6n^2-9n+2)}{2}$ **D.** $n(6n^2 + 3n - 1)$

5. $1 + 2 + 3 + \cdots + 40 =$

A. 780 **B.** 1640 **C.** 820 **D.** 861

6. $1^2 + 2^2 + \cdots + 11^2 =$

A. 1012 **B.** 4356 **C.** 506 **D.** 462

7. $1^2 + 3^2 + 5^2 + \ldots$ to 9 terms $=$

A. 1140 **B.** 1785 **C.** 975 **D.** 969

8. $2^2 + 4^2 + 6^2 + \cdots + (2n)^2 =$

A. $\frac{2}{3}n(n + 1)(2n + 1)$ **B.** $\frac{1}{6}n(n + 1)(2n + 1)$ **C.** $\frac{n}{3}(4n^2 - 1)$ **D.** $\frac{4}{3}n(n + 1)(2n + 1)$

9. $1^2 - 2^2 + 3^2 - \cdots + 11^2 =$

A. 66 **B.** 132 **C.** -66 **D.** -55

10. $\sum_{k=1}^{n} k(k + 5) =$

A. $\frac{(n+1)(n+2)(n+9)}{3}$ **B.** $\frac{2n(n+1)(n+8)}{3}$

C. $\frac{n(n+1)(n+8)}{3}$ **D.** $\frac{n(n-1)(n+7)}{3}$

11. $\sum_{k=1}^{8} k(k+1) =$

A. 576 **B.** 168 **C.** 330 **D.** 240

12. The 20th term of the A.P. $6, 13, 20, \ldots$ is

A. 139 **B.** 140 **C.** 132 **D.** 146

13. The 7th term of the G.P. $2, 6, 18, \ldots$ is

A. 4374 **B.** 1458 **C.** 46656 **D.** 486

14. The sum to infinity of $16 + 4 + 1 + \frac{1}{4} + \ldots$ is

A. $\frac{64}{5}$ **B.** 64 **C.** $\frac{64}{3}$ **D.** 12

15. The sum to infinity of $1 + 3 \cdot \frac{1}{2} + 5 \cdot \frac{1}{4} + 7 \cdot \frac{1}{8} + \ldots$ is

A. 4 **B.** 2 **C.** 6 **D.** 10

16. The nth term of $3, 7, 13, 21, \ldots$ is

A. $n^2 + n + 1$ **B.** $n^2 + n$ **C.** $n^2 + 3n + 3$ **D.** $2n^2$

17. The nth term of $1, 3, 7, 15, \ldots$ is

A. 2^n **B.** $2^n + 1$ **C.** $2^n - 1$ **D.** $2^{n+1} - 1$

18. Using $n = 1$ and $n = 2$, the nth term of $1 \cdot 4 + 4 \cdot 7 + 7 \cdot 10 + \ldots$ is

A. $(3n - 5)(3n - 2)$ **B.** $(3n - 2)(3n + 1)$ **C.** $9n^2 - 2n - 3$ **D.** $(3n + 1)(3n + 4)$

19. Testing with $n = 1, 2, 3$, the sum of n terms of $1 + 5 + 9 + \ldots$ is

A. $2n^2 - 1$ **B.** $n(4n - 3)$ **C.** $n(2n - 1)$ **D.** $2n^2 - 5n + 4$

20. The largest number that divides $10^n - 4^n$ for every $n \in \mathbb{N}$ is

A. 7 **B.** 42 **C.** 12 **D.** 6

21. The largest number that divides $3^{2n} - 1$ for every $n \in \mathbb{N}$ is

A. 9 **B.** 8 **C.** 40 **D.** 16

22. To prove $7^n - 2^n$ is divisible by 5, assume $7^k - 2^k = 5\lambda$. Then $7^{k+1} - 2^{k+1} =$

A. $5(2\lambda + 7^k)$ **B.** $5(\lambda + 2^k)$ **C.** $5(7\lambda - 2^k)$ **D.** $5(7\lambda + 2^k)$

23. If $53 \equiv r \pmod 6$ with $0 \le r < 6$, then $r =$

A. 5 **B.** 0 **C.** 1 **D.** 8

24. For every positive integer n, $x^{2n+1} + y^{2n+1}$ is divisible by

A. $x - y$ **B.** $x^2 + y^2$ **C.** $x + y$ **D.** xy

25. If $79 \equiv r \pmod 8$ with $0 \le r < 8$, then $r =$

A. 0 **B.** 7 **C.** 1 **D.** 9

Key sheet

1. D	**2.** B	**3.** A	**4.** B	**5.** C
6. C	**7.** D	**8.** A	**9.** A	**10.** C
11. D	**12.** A	**13.** B	**14.** C	**15.** C
16. A	**17.** C	**18.** B	**19.** C	**20.** D
21. B	**22.** D	**23.** A	**24.** C	**25.** B

Solutions

1. If $P(n)$: $1 + 7 + 13 + \cdots + (6n - 5) = n(3n - 2)$, then the value of each side of $P(3)$ is

$1 + 7 + 13 = 21$

and the right side at $n = 3$ is also 21.

Ans: (D) 21

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2. In the principle of finite mathematical induction, the base step is to check that

The base step verifies the first case, $P(1)$, by direct calculation.

Ans: (B) $P(1)$ **is true**

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3. To prove $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + n(n + 2) = \frac{n(n+1)(2n+7)}{6}$ by induction, the term added to both sides of $P(k)$ is

The $(k + 1)$th term is got by putting $n = k + 1$ in $n(n + 2)$.

It is $(k + 1)(k + 3)$.

Ans: (A) $(k + 1)(k + 3)$

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4. By induction it can be shown that $2^2 + 5^2 + 8^2 + \cdots + (3n - 1)^2 =$

Check with $n = 1$: the sum is 4 and the formula gives 4.

The formula $\frac{n(6n^2+3n-1)}{2}$ is the one proved by induction.

Ans: (B) $\frac{n(6n^2+3n-1)}{2}$

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5. $1 + 2 + 3 + \cdots + 40 =$

$\frac{n(n+1)}{2} = \frac{40 \cdot 41}{2} = 820$

Ans: (C) 820

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6. $1^2 + 2^2 + \cdots + 11^2 =$

$\frac{n(n+1)(2n+1)}{6} = \frac{11 \cdot 12 \cdot 23}{6} = 506$

Ans: (C) 506

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7. $1^2 + 3^2 + 5^2 + \ldots$ to 9 terms $=$

$\frac{n}{3}(4n^2 - 1) = \frac{9}{3}(323) = 969$

Ans: (D) 969

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8. $2^2 + 4^2 + 6^2 + \cdots + (2n)^2 =$

$(2k)^2 = 4k^2$

$4 \cdot \frac{n(n+1)(2n+1)}{6} = \frac{2}{3}n(n + 1)(2n + 1)$

Ans: (A) $\frac{2}{3}n(n + 1)(2n + 1)$

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9. $1^2 - 2^2 + 3^2 - \cdots + 11^2 =$

$n = 11$ is odd.

$\frac{11 \cdot 12}{2} = 66$

Ans: (A) 66

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10. $\sum_{k=1}^{n} k(k+5) =$

Expand $k(k+5) = k^2 + 5k$ and use $\sum k^2$, $\sum k$.

The total factorises as $\frac{n(n+1)(n+8)}{3}$.

Ans: (C) $\frac{n(n+1)(n+8)}{3}$

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11. $\sum_{k=1}^{8} k(k+1) =$

Expand $k(k+1) = k^2 + k$ and use the standard sums with $n = 8$.

The sum is 240.

Ans: (D) 240

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12. The 20th term of the A.P. $6, 13, 20, \ldots$ is

$t_n = a + (n-1)d = 6 + 19(7) = 139$

Ans: (A) 139

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13. The 7th term of the G.P. $2, 6, 18, \ldots$ is

$t_n = ar^{n-1} = 2 \cdot 3^6 = 1458$

Ans: (B) 1458

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14. The sum to infinity of $16 + 4 + 1 + \frac{1}{4} + \ldots$ is

$a = 16$, $r = \frac{1}{4}$.

$S_\infty = \frac{a}{1-r} = \frac{64}{3}$

Ans: (C) $\frac{64}{3}$

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15. The sum to infinity of $1 + 3 \cdot \frac{1}{2} + 5 \cdot \frac{1}{4} + 7 \cdot \frac{1}{8} + \ldots$ is

$a = 1$, $d = 2$, $r = \frac{1}{2}$.

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = 2 + 4 = 6$

Ans: (C) 6

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16. The nth term of $3, 7, 13, 21, \ldots$ is

Differences: $4, 6, 8$.

Fitting $n = 1, 2, 3$ gives $T_n = n^2 + n + 1$.

Ans: (A) $n^2 + n + 1$

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17. The nth term of $1, 3, 7, 15, \ldots$ is

Differences: $2, 4, 8$.

Fitting $n = 1, 2, 3$ gives $T_n = 2^n - 1$.

Ans: (C) $2^n - 1$

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18. Using $n = 1$ and $n = 2$, the nth term of $1 \cdot 4 + 4 \cdot 7 + 7 \cdot 10 + \ldots$ is

$T_1 = 4$, $T_2 = 28$.

Only $(3n-2)(3n+1)$ gives both.

Ans: (B) $(3n-2)(3n+1)$

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19. Testing with $n = 1, 2, 3$, the sum of n terms of $1 + 5 + 9 + \ldots$ is

$S_1 = 1$, $S_2 = 6$, $S_3 = 15$.

Only $n(2n - 1)$ gives all three.

Ans: (C) $n(2n - 1)$

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20. The largest number that divides $10^n - 4^n$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 6, \ n = 2 : \ 84$

$\text{G.C.D.}(6, 84) = 6$

Ans: (D) 6

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21. The largest number that divides $3^{2n} - 1$ for every $n \in \mathbb{N}$ is

$n = 1 : \ 8, \ n = 2 : \ 80$

$\text{G.C.D.}(8, 80) = 8$

Ans: (B) 8

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22. To prove $7^n - 2^n$ is divisible by 5, assume $7^k - 2^k = 5\lambda$. Then $7^{k+1} - 2^{k+1} =$

$7^{k+1} - 2^{k+1} = 7(5\lambda + 2^k) - 2 \cdot 2^k$

$= 35\lambda + 5 \cdot 2^k = 5(7\lambda + 2^k)$

Ans: (D) $5(7\lambda + 2^k)$

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23. If $53 \equiv r \pmod 6$ with $0 \leq r < 6$, then $r =$

$53 = 6 \times 8 + 5$

$\therefore 53 - 5 = 48$ is divisible by 6, so $r = 5$.

Ans: (A) 5

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24. For every positive integer n, $x^{2n+1} + y^{2n+1}$ is divisible by

An odd power sum always has the factor $x + y$.

Ans: (C) $x + y$

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25. If $79 \equiv r \pmod 8$ with $0 \leq r < 8$, then $r =$

$79 = 8 \times 9 + 7$

$\therefore 79 - 7 = 72$ is divisible by 8, so $r = 7$.

Ans: (B) 7
