

Logarithms

Question Paper, Set 09

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $x = \log_3 \frac{1}{27}$, find x . [1 mark]
2. Is $\log_1 1$ defined? Give a reason. [1 mark]
3. Write $10^5 = 100000$ in logarithmic form. [1 mark]
4. Find $9 - 3 \log_4 4$. [1 mark]
5. Find the value of $\log_5 15 + \log_5 10 - \log_5 6$. [2 marks]
6. Find the value of $\log_{81} 9$. [2 marks]
7. If $\log_7 2 = t$, write $\log_2 7$ in terms of t . [1 mark]
8. Find the value of $\log_2 7 \cdot \log_3 2 \cdot \log_7 3$. [2 marks]
9. Write $\frac{\log 36}{\log 216}$ as a single logarithm and find its value. [1 mark]
10. Find the value of $4^{\log_4 3} \times 7^{\log_7 3} \times 2^{\log_2 5}$. [2 marks]
11. Find the value of $\frac{5^{\log_6 13}}{13^{\log_6 5}}$. [2 marks]
12. If $\log_{11}(5x - 1) = \log_{11} 24$, find x . [2 marks]
13. If $\log_4(x^2 + 3x) = 1$, find x . [3 marks]
14. Solve $\log_5 x - \log_5(x + 4) = 1$. [5 marks]
15. Find the value of $\log_{0.01} 0.001 + \log_5 \sqrt{125}$. [3 marks]
16. Show that $\log_{q^3} p^2 \cdot \log_{r^2} q \cdot \log_{p^5} r^3 = \frac{1}{5}$. [5 marks]
17. Find the value of $4^{\log_{16} 49}$. [3 marks]
18. If $a^2 + b^2 = 47ab$, show that $2 \log(a + b) = \log a + \log b + 2 \log 7$. [5 marks]
19. Solve $\log_6(x + 4) = \log_{36}(5x + 26)$. [5 marks]
20. Complete the table of $y = \log_{10} x$ for $x = \frac{1}{100}, \frac{1}{10}, 1, 10, 100, 1000$. [3 marks]

Key sheet

Q	Answer
1	-3
2	Not defined
3	$5 = \log_{10} 100000$
4	6
5	2
6	$\frac{1}{2}$
7	$\frac{1}{t}$
8	1
9	$\log_{216} 36 = \frac{2}{3}$
10	45
11	1
12	$x = 5$
13	$x = 1$ (or) $x = -4$
14	No real x
15	3
16	$\frac{1}{5}$
17	7
18	Shown: $2 \log(a + b) = \log a + \log b + 2 \log 7$
19	$x = 2$
20	$y = -2, -1, 0, 1, 2, 3$

Solutions

1. If $x = \log_3 \frac{1}{27}$, find x . [1]

$$\frac{1}{27} = \frac{1}{3^3} = 3^{-3}$$

$$a^x = N \Rightarrow x = \log_a N$$

$$\therefore \log_3 \frac{1}{27} = -3$$

Ans: -3

2. Is $\log_1 1$ defined? Give a reason. [1]

$$1^3 = 1 \Rightarrow 3 = \log_1 1$$

$$1^8 = 1 \Rightarrow 8 = \log_1 1$$

So $3 = 8$, which has no meaning.

$\therefore \log_1 1$ is not defined (the base cannot be 1).

Ans: Not defined

3. Write $10^5 = 100000$ in logarithmic form. [1]

$$a^x = N \Leftrightarrow x = \log_a N$$

Here $a = 10$, $x = 5$, $N = 100000$.

$$\therefore 5 = \log_{10} 100000$$

Ans: $5 = \log_{10} 100000$

4. Find $9 - 3 \log_4 4$. [1]

$$\log_4 4 = 1$$

$$= 9 - 3(1) = 6$$

Ans: 6

5. Find the value of $\log_5 15 + \log_5 10 - \log_5 6$. [2]

$$\log_a m + \log_a n - \log_a p = \log_a \frac{m \times n}{p}$$

$$= \log_5 \frac{15 \times 10}{6} = \log_5 25$$

$$5^2 = 25 \Rightarrow \log_5 25 = 2$$

Ans: 2

6. Find the value of $\log_{81} 9$. [2]

$$\log_{81} 9 = \log_{3^4} 3^2$$

$$= \frac{2}{4} \log_3 3$$

$$= \frac{2}{4}(1) = \frac{1}{2}$$

Ans: $\frac{1}{2}$

7. If $\log_7 2 = t$, write $\log_2 7$ in terms of t . [1]

$$\log_b a = \frac{1}{\log_a b}$$

$$\log_2 7 = \frac{1}{\log_7 2} = \frac{1}{t}$$

Ans: $\frac{1}{t}$

8. Find the value of $\log_2 7 \cdot \log_3 2 \cdot \log_7 3$. [2]

$$\begin{aligned}\log_b a \cdot \log_c b \cdot \log_d c &= \log_d a \\ &= \log_7 7 \\ &= 1\end{aligned}$$

Ans: 1

9. Write $\frac{\log 36}{\log 216}$ as a single logarithm and find its value. [1]

$$\begin{aligned}\frac{\log a}{\log b} &= \log_b a \\ &= \log_{216} 36 \\ &= \frac{2}{3}\end{aligned}$$

Ans: $\log_{216} 36 = \frac{2}{3}$

10. Find the value of $4^{\log_4 3} \times 7^{\log_7 3} \times 2^{\log_2 5}$. [2]

$$\begin{aligned}a^{\log_a N} &= N \\ &= 3 \times 3 \times 5 = 45\end{aligned}$$

Ans: 45

11. Find the value of $\frac{5^{\log_6 13}}{13^{\log_6 5}}$. [2]

$$\begin{aligned}a^{\log_c b} &= b^{\log_c a} \\ 5^{\log_6 13} &= 13^{\log_6 5}\end{aligned}$$

$\therefore$ the numerator and denominator are equal, and the value is 1.

Ans: 1

12. If $\log_{11}(5x - 1) = \log_{11} 24$, find x . [2]

$$\begin{aligned}\log_a m &= \log_a n \Rightarrow m = n \\ \Rightarrow 5x - 1 &= 24 \\ \Rightarrow 5x &= 25 \\ \Rightarrow x &= 5\end{aligned}$$

Ans: $x = 5$

13. If $\log_4(x^2 + 3x) = 1$, find x . [3]

$$\begin{aligned}\log_a N &= x \Rightarrow N = a^x \\ \Rightarrow x^2 + 3x &= 4 \\ \Rightarrow x^2 + 3x - 4 &= 0 \\ \Rightarrow x^2 + 4x - x - 4 &= 0 \\ \Rightarrow x(x + 4) - 1(x + 4) &= 0 \\ \Rightarrow (x - 1)(x + 4) &= 0 \\ \Rightarrow x = 1 \text{ (or) } x = -4\end{aligned}$$

Ans: $x = 1$ (or) $x = -4$

14. Solve $\log_5 x - \log_5(x+4) = 1$. [5]

$$\log_a m - \log_a n = \log_a \frac{m}{n}$$

$$\Rightarrow \log_5 \frac{x}{x+4} = 1$$

$$\Rightarrow \frac{x}{x+4} = 5$$

$$\Rightarrow x = 5x + 20$$

$$\Rightarrow 4x = -20$$

$$\Rightarrow x = -5$$

For $x = -5$, $\log_5 x$ is not defined.

$\therefore$ there is no real x .

Ans: No real x

15. Find the value of $\log_{0.01} 0.001 + \log_5 \sqrt{125}$. [3]

$$\log_{0.01} 0.001 = \log_{(0.1)^2} (0.1)^3 = \frac{3}{2}$$

$$\log_5 \sqrt{125} = \log_5 (5^3)^{\frac{1}{2}} = \frac{3}{2}$$

$$= \frac{3}{2} + \frac{3}{2} = 3$$

Ans: 3

16. Show that $\log_{q^3} p^2 \cdot \log_{r^2} q \cdot \log_{p^5} r^3 = \frac{1}{5}$. [5]

$$\log_{b^n} a^m = \frac{m}{n} \log_b a$$

$$LHS = \frac{2}{3} \log_q p \cdot \frac{1}{2} \log_r q \cdot \frac{3}{5} \log_p r$$

$$= \frac{2 \times 1 \times 3}{3 \times 2 \times 5} \log_q p \log_r q \log_p r$$

$$= \frac{1}{5} \log_p p$$

$$= \frac{1}{5} (1) = \frac{1}{5} = RHS$$

Ans: $\frac{1}{5}$

17. Find the value of $4^{\log_{16} 49}$. [3]

$$\log_{16} 49 = \log_{4^2} 7^2 = \log_4 7$$

$$4^{\log_{16} 49} = 4^{\log_4 7}$$

$$= 7$$

Ans: 7

18. If $a^2 + b^2 = 47ab$, show that $2 \log(a+b) = \log a + \log b + 2 \log 7$. [5]

$$\text{Given } a^2 + b^2 = 47ab$$

$$LHS = 2 \log(a+b) = \log(a+b)^2$$

$$= \log(a^2 + b^2 + 2ab)$$

$$= \log(47ab + 2ab) = \log(49ab)$$

$$= \log 49 + \log a + \log b$$

$$= \log 7^2 + \log a + \log b = 2 \log 7 + \log a + \log b$$

$$= RHS$$

Ans: Shown: $2 \log(a+b) = \log a + \log b + 2 \log 7$

19. Solve $\log_6(x+4) = \log_{36}(5x+26)$. [5]

$$\log_{36}(5x+26) = \log_{6^2}(5x+26) = \frac{1}{2} \log_6(5x+26)$$

$$\Rightarrow 2 \log_6(x+4) = \log_6(5x+26)$$

$$\Rightarrow \log_6(x+4)^2 = \log_6(5x+26)$$

$$\Rightarrow (x+4)^2 = 5x+26$$

$$\Rightarrow x^2 + 8x + 16 = 5x + 26$$

$$\Rightarrow x^2 + 3x - 10 = 0$$

$$\Rightarrow x^2 + 5x - 2x - 10 = 0$$

$$\Rightarrow x(x+5) - 2(x+5) = 0$$

$$\Rightarrow (x-2)(x+5) = 0$$

$$\Rightarrow x = 2 \text{ (or) } x = -5$$

For $x = -5$, $\log_6(x+4)$ is not defined.

$$\therefore x = 2$$

Ans: $x = 2$

20. Complete the table of $y = \log_{10} x$ for $x = \frac{1}{100}, \frac{1}{10}, 1, 10, 100, 1000$. [3]

$$y = \log_{10} x$$

$$x = \frac{1}{100} : y = \log_{10} \frac{1}{100} = -2$$

$$x = \frac{1}{10} : y = \log_{10} \frac{1}{10} = -1$$

$$x = 1 : y = \log_{10} 1 = 0$$

$$x = 10 : y = \log_{10} 10 = 1$$

$$x = 100 : y = \log_{10} 100 = 2$$

$$x = 1000 : y = \log_{10} 1000 = 3$$

Ans: $y = -2, -1, 0, 1, 2, 3$
