EXPRESSWALA ACADEMY

Logarithms

Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the logarithm of 169 to the base 13. [1 mark]

2. Write the conditions on a and N for $\log_a N$ to be defined. [1 mark]

3. Write $\log_{0.5} 0.125 = 3$ in exponential form. [1 mark]

4. Find $\log_{11} 11 + \log_{14} 14 + \log_{17} 1$. [1 mark]

5. Find the value of $\log_7 98 - \log_7 2$. [2 marks]

6. Find the value of $\log_{\frac{1}{5}} 125$. [2 marks]

7. Find the value of $3\log_5 17 \times \log_{17} 5$. [1 mark]

8. Find the value of $\log_{13} 27 \cdot \log_9 13$. [2 marks]

9. Write $\frac{\log_{10} 625}{\log_{10} 25}$ as a single logarithm and find its value. [1 mark]

10. Find the value of $8^{\log_8 0.5} + 3^{\log_3 2.5}$. [2 marks]

11. Write $10^{\log_4 9}$ as a power of 9. [2 marks]

12. If $\log_9(x+4) = \log_9(2x)$, find x. [2 marks]

13. If $\log_5 \log_3 \log_2(x-3) = 0$, find x. [3 marks]

14. Solve $\log_4(x-3) + \log_4 x = 1$. [5 marks]

15. Find the value of $\log_3 \log_2 \log_7 49^4$. [3 marks]

16. Find the value of $\frac{1}{\log_{25} 5} + \frac{1}{\log_{125} 5} + \frac{1}{\log_{625} 5} + \frac{1}{\log_5 5}$. [5 marks]

17. Find the value of $10^{-2+\log_{10} 3} \times 7^{\log_{49} 25}$. [3 marks]

18. If $x^3 = y^3 + 125$, show that $\log_5(x-y) + \log_5(x^2+xy+y^2) = 3$. [5 marks]

19. Solve $\log_6(5x-4) = \log_{\frac{1}{6}} x$. [5 marks]

20. Complete the table of $y = \log_{\frac{1}{4}} x$ for $x = \frac{1}{16}, \frac{1}{4}, 1, 4, 16, 64$ and say whether the graph rises or falls from left to right. [3 marks]

Key sheet

Q	Answer
1	2
2	$a > 0$, $a \neq 1$ and $N > 0$
3	$0.125 = 0.5^3$
4	2
5	2
6	-3
7	3
8	$\frac{3}{2}$
9	$\log_{25} 625 = 2$
10	3
11	$9^{\log_4 10}$
12	$x = 4$
13	$x = 11$
14	$x = 4$
15	1
16	10
17	$\frac{3}{20}$
18	Shown: the sum is 3
19	$x = 1$
20	$y = 2, 1, 0, -1, -2, -3$; the graph falls

Solutions

1. Find the logarithm of 169 to the base 13. [1]

$13^2 = 169$

$a^x = N \Rightarrow x = \log_a N$

$\therefore \log_{13} 169 = 2$

Ans: 2

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2. Write the conditions on a and N for $\log_a N$ to be defined. [1]

$a^x = N \Leftrightarrow x = \log_a N$

The base cannot be 0, 1 or a negative number: $a > 0$, $a \neq 1$.

a^x is always positive, so $N > 0$.

$\therefore a > 0,\ a \neq 1,\ N > 0$

Ans: $a > 0$, $a \neq 1$ **and** $N > 0$

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3. Write $\log_{0.5} 0.125 = 3$ in exponential form. [1]

$\log_a N = x \Leftrightarrow N = a^x$

Here $a = 0.5$, $N = 0.125$, $x = 3$.

$\therefore 0.125 = 0.5^3$

Ans: $0.125 = 0.5^3$

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4. Find $\log_{11} 11 + \log_{14} 14 + \log_{17} 1$. [1]

$\log_{11} 11 = 1,\ \log_{14} 14 = 1,\ \log_{17} 1 = 0$

$= 1 + 1 + 0 = 2$

Ans: 2

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5. Find the value of $\log_7 98 - \log_7 2$. [2]

$\log_a m - \log_a n = \log_a \frac{m}{n}$

$= \log_7 \frac{98}{2} = \log_7 49$

$7^2 = 49 \Rightarrow \log_7 49 = 2$

Ans: 2

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6. Find the value of $\log_{\frac{1}{5}} 125$. [2]

$\frac{1}{5} = 5^{-1},\ 125 = 5^3$

$\log_{\frac{1}{5}} 125 = \log_{5^{-1}} 5^3$

$= \frac{3}{-1} \log_5 5 = -3$

Ans: -3

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7. Find the value of $3 \log_5 17 \times \log_{17} 5$. [1]

$\log_b a \cdot \log_a b = 1$

$= 3(1) = 3$

Ans: 3

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8. Find the value of $\log_{13} 27 \cdot \log_9 13$. [2]

$\log_b a \cdot \log_c b = \log_c a$

$= \log_9 27$

$= \frac{3}{2}$

Ans: $\frac{3}{2}$

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9. Write $\frac{\log_{10} 625}{\log_{10} 25}$ as a single logarithm and find its value. [1]

$\frac{\log a}{\log b} = \log_b a$

$= \log_{25} 625$

$= 2$

Ans: $\log_{25} 625 = 2$

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10. Find the value of $8^{\log_8 0.5} + 3^{\log_3 2.5}$. [2]

$a^{\log_a N} = N$

$= 0.5 + 2.5 = 3$

Ans: 3

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11. Write $10^{\log_4 9}$ as a power of 9. [2]

$a^{\log_c b} = b^{\log_c a}$

$\therefore 10^{\log_4 9} = 9^{\log_4 10}$

Ans: $9^{\log_4 10}$

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12. If $\log_9(x+4) = \log_9(2x)$, find x. [2]

$\log_a m = \log_a n \Rightarrow m = n$

$\Rightarrow x + 4 = 2x$

$\Rightarrow x = 4$

$\Rightarrow x = 4$

Ans: $x = 4$

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13. If $\log_5 \log_3 \log_2(x-3) = 0$, find x. [3]

$\log_a N = 0 \Rightarrow N = 1$

$\Rightarrow \log_3 \log_2(x-3) = 1$

$\Rightarrow \log_2(x-3) = 3^1 = 3$

$\Rightarrow x - 3 = 2^3 = 8$

$\Rightarrow x = 11$

Ans: $x = 11$

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14. Solve $\log_4(x-3)+\log_4 x=1$. [5]

$\log_a m+\log_a n=\log_a mn$

$\Rightarrow \log_4(x-3)x=1$

$\Rightarrow \log_4(x^2-3x)=1$

$\Rightarrow x^2-3x=4$

$\Rightarrow x^2-3x-4=0$

$\Rightarrow x^2-4x+x-4=0$

$\Rightarrow x(x-4)+1(x-4)=0$

$\Rightarrow (x+1)(x-4)=0$

$\Rightarrow x=-1$ (or) $x=4$

If $x=-1$, $\log_4 x$ is not defined.

$\therefore x\neq -1$

$\therefore x=4$

Ans: $x=4$

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15. Find the value of $\log_3\log_2\log_7 49^4$. [3]

$\log_7 49^4=\log_7(7^2)^4=\log_7 7^8=8$

$\log_2 8=\log_2 2^3=3$

$\log_3 3=1$

$\therefore \log_3\log_2\log_7 49^4=1$

Ans: 1

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16. Find the value of $\frac{1}{\log_{25}5}+\frac{1}{\log_{125}5}+\frac{1}{\log_{625}5}+\frac{1}{\log_5 5}$. [5]

$\log_b a=\frac{1}{\log_a b}$

$=\log_5 25+\log_5 125+\log_5 625+\log_5 5$

$=\log_5(25\times 125\times 625\times 5)$

$=\log_5(5^2\times 5^3\times 5^4\times 5^1)=\log_5 5^{10}$

$=10\log_5 5=10$

Ans: 10

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17. Find the value of $10^{-2+\log_{10}3}\times 7^{\log_{49}25}$. [3]

$10^{-2+\log_{10}3}=10^{-2}\times 10^{\log_{10}3}=\frac{1}{100}\times 3=\frac{3}{100}$

$7^{\log_{49}25}=7^{\log_{7^2}5^2}=7^{\log_7 5}=5$

$=\frac{3}{100}\times 5=\frac{3}{20}$

Ans: $\frac{3}{20}$

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18. If $x^3 = y^3 + 125$, show that $\log_5(x-y) + \log_5(x^2+xy+y^2) = 3$. [5]

Given $x^3 = y^3 + 125 \Rightarrow x^3 - y^3 = 125$

$LHS = \log_5(x-y) + \log_5(x^2+xy+y^2)$

$= \log_5[(x-y)(x^2+xy+y^2)]$

$(a-b)(a^2+ab+b^2) = a^3 - b^3$

$= \log_5(x^3 - y^3) = \log_5 125$

$= \log_5 5^3 = 3 = RHS$

Ans: Shown: the sum is 3

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19. Solve $\log_6(5x-4) = \log_{\frac{1}{6}} x$. [5]

$\frac{1}{6} = 6^{-1}$

$\log_6(5x-4) = \log_{6^{-1}} x = \frac{1}{-1}\log_6 x = \log_6 x^{-1}$

$\Rightarrow 5x - 4 = x^{-1} = \frac{1}{x}$

$\Rightarrow 5x^2 - 4x = 1$

$\Rightarrow 5x^2 - 4x - 1 = 0$

$\Rightarrow 5x^2 + x - 5x - 1 = 0$

$\Rightarrow x(5x+1) - 1(5x+1) = 0$

$\Rightarrow (x-1)(5x+1) = 0$

$\Rightarrow x = 1$ (or) $x = -\frac{1}{5}$

For $x = -\frac{1}{5}$, $\log_{\frac{1}{6}} x$ is not defined.

$\therefore x = 1$

Ans: $x = 1$

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20. Complete the table of $y = \log_{\frac{1}{4}} x$ for $x = \frac{1}{16}, \frac{1}{4}, 1, 4, 16, 64$ and say whether the graph rises or falls from left to right. [3]

$y = \log_{\frac{1}{4}} x$

$x = \frac{1}{16}:\ y = \log_{\frac{1}{4}} \frac{1}{16} = 2$

$x = \frac{1}{4}:\ y = \log_{\frac{1}{4}} \frac{1}{4} = 1$

$x = 1:\ y = \log_{\frac{1}{4}} 1 = 0$

$x = 4:\ y = \log_{\frac{1}{4}} 4 = -1$

$x = 16:\ y = \log_{\frac{1}{4}} 16 = -2$

$x = 64:\ y = \log_{\frac{1}{4}} 64 = -3$

The base is between 0 and 1, so the graph falls from left to right.

Ans: $y = 2, 1, 0, -1, -2, -3$**; the graph falls**