

Logarithms

Question Paper, Set 07

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the value of $\log_6 \frac{1}{36}$. [1 mark]
2. Of $\log_0 0$ and $\log_8 8$, which one is defined? Find its value. [1 mark]
3. Write $9^{\frac{1}{2}} = 3$ in logarithmic form. [1 mark]
4. Find $(\log_7 7)^2 + 3\log_{25} 1$. [1 mark]
5. Find the value of $\log_3 54 + \log_3 \frac{1}{2}$. [2 marks]
6. Find the value of $\log_4 128$. [2 marks]
7. Write $\frac{1}{\log_{12} 7}$ as a single logarithm. [1 mark]
8. Find the value of $\log_6 16 \cdot \log_{11} 6 \cdot \log_8 11$. [2 marks]
9. Write $\frac{\log 128}{\log 8}$ as a single logarithm and find its value. [1 mark]
10. Find the value of $(13^{\log_{13} 4})^2$. [2 marks]
11. Find the value of $2^{\log_7 9} - 9^{\log_7 2}$. [2 marks]
12. If $\log_{10}(4x - 3) = \log_{10}(2x + 9)$, find x . [2 marks]
13. If $\log_2(x^2 - 2x - 7) = 3$, find x . [3 marks]
14. Solve $\log_2(x^2 + 2x + 4) + \log_2(x - 2) = 4$. [5 marks]
15. Find the value of $\log_{25} 125 + \log_{125} 25$. [3 marks]
16. Find the value of $\frac{\log 2^{\sqrt[3]{2}} + \log 5^{\sqrt[3]{5}}}{\log 10}$. [5 marks]
17. Find the value of $6^{\log_6 \frac{4}{3}} \cdot 3^{\log_3 \frac{3}{2}} \cdot 5^{\log_{25} 16}$. [3 marks]
18. If $\log(a + 3b) = \log a - \log b$, find a in terms of b . [5 marks]
19. Solve $\log_7(2x + 1) = \log_{\frac{1}{7}} x$. [5 marks]
20. Complete the table of $y = \log_5 x$ for $x = \frac{1}{25}, \frac{1}{5}, 1, 5, 25, 125$. [3 marks]

Key sheet

Q	Answer
1	-2
2	$\log_8 8 = 1$ ($\log_0 0$ is not defined)
3	$\frac{1}{2} = \log_9 3$
4	1
5	3
6	$\frac{7}{2}$
7	$\log_7 12$
8	$\frac{4}{3}$
9	$\log_8 128 = \frac{7}{3}$
10	16
11	0
12	$x = 6$
13	$x = -3$ (or) $x = 5$
14	$x = 2\sqrt[3]{3}$
15	$\frac{13}{6}$
16	$\frac{4}{3}$
17	8
18	$a = \frac{3b^2}{1-b}$
19	$x = \frac{1}{2}$
20	$y = -2, -1, 0, 1, 2, 3$

Solutions

1. Find the value of $\log_6 \frac{1}{36}$. [1]

$$\frac{1}{36} = \frac{1}{6^2} = 6^{-2}$$

$$a^x = N \Rightarrow x = \log_a N$$

$$\therefore \log_6 \frac{1}{36} = -2$$

Ans: -2

2. Of $\log_0 0$ and $\log_8 8$, which one is defined? Find its value. [1]

The base cannot be 0, so $\log_0 0$ is not defined.

$$8^1 = 8 \Rightarrow 1 = \log_8 8$$

$$\therefore \log_8 8 = 1$$

Ans: $\log_8 8 = 1$ ($\log_0 0$ is not defined)

3. Write $9^{\frac{1}{2}} = 3$ in logarithmic form. [1]

$$a^x = N \Leftrightarrow x = \log_a N$$

Here $a = 9$, $x = \frac{1}{2}$, $N = 3$.

$$\therefore \frac{1}{2} = \log_9 3$$

Ans: $\frac{1}{2} = \log_9 3$

4. Find $(\log_7 7)^2 + 3 \log_{25} 1$. [1]

$$\log_7 7 = 1, \log_{25} 1 = 0$$

$$= 1^2 + 3(0) = 1$$

Ans: 1

5. Find the value of $\log_3 54 + \log_3 \frac{1}{2}$. [2]

$$\log_a m + \log_a n = \log_a mn$$

$$= \log_3 (54 \times \frac{1}{2}) = \log_3 27$$

$$3^3 = 27 \Rightarrow \log_3 27 = 3$$

Ans: 3

6. Find the value of $\log_4 128$. [2]

$$\log_4 128 = \log_{2^2} 2^7$$

$$= \frac{7}{2} \log_2 2$$

$$= \frac{7}{2}(1) = \frac{7}{2}$$

Ans: $\frac{7}{2}$

7. Write $\frac{1}{\log_{12} 7}$ as a single logarithm. [1]

$$\log_b a = \frac{1}{\log_a b}$$

$$\therefore \frac{1}{\log_{12} 7} = \log_7 12$$

Ans: $\log_7 12$

8. Find the value of $\log_6 16 \cdot \log_{11} 6 \cdot \log_8 11$. [2]

$$\begin{aligned}\log_b a \cdot \log_c b \cdot \log_d c &= \log_d a \\ &= \log_8 16 \\ &= \frac{4}{3}\end{aligned}$$

Ans: $\frac{4}{3}$

9. Write $\frac{\log 128}{\log 8}$ as a single logarithm and find its value. [1]

$$\begin{aligned}\frac{\log a}{\log b} &= \log_b a \\ &= \log_8 128 \\ &= \frac{7}{3}\end{aligned}$$

Ans: $\log_8 128 = \frac{7}{3}$

10. Find the value of $(13^{\log_{13} 4})^2$. [2]

$$\begin{aligned}13^{\log_{13} 4} &= 4 \text{ since } a^{\log_a N} = N \\ &= 4^2 = 16\end{aligned}$$

Ans: 16

11. Find the value of $2^{\log_7 9} - 9^{\log_7 2}$. [2]

$$\begin{aligned}a^{\log_c b} &= b^{\log_c a} \\ 2^{\log_7 9} &= 9^{\log_7 2} \\ &= 9^{\log_7 2} - 9^{\log_7 2} = 0\end{aligned}$$

Ans: 0

12. If $\log_{10}(4x - 3) = \log_{10}(2x + 9)$, find x . [2]

$$\begin{aligned}\log_a m &= \log_a n \Rightarrow m = n \\ \Rightarrow 4x - 3 &= 2x + 9 \\ \Rightarrow 2x &= 12 \\ \Rightarrow x &= 6\end{aligned}$$

Ans: $x = 6$

13. If $\log_2(x^2 - 2x - 7) = 3$, find x . [3]

$$\begin{aligned}\log_a N &= x \Rightarrow N = a^x \\ \Rightarrow x^2 - 2x - 7 &= 2^3 = 8 \\ \Rightarrow x^2 - 2x - 15 &= 0 \\ \Rightarrow x^2 - 5x + 3x - 15 &= 0 \\ \Rightarrow x(x - 5) + 3(x - 5) &= 0 \\ \Rightarrow (x + 3)(x - 5) &= 0 \\ \Rightarrow x &= -3 \text{ (or) } x = 5\end{aligned}$$

Ans: $x = -3$ (or) $x = 5$

14. Solve $\log_2(x^2 + 2x + 4) + \log_2(x - 2) = 4$. [5]

$$\log_a m + \log_a n = \log_a mn$$

$$\Rightarrow \log_2[(x^2 + 2x + 4)(x - 2)] = 4$$

$$(a - b)(a^2 + ab + b^2) = a^3 - b^3$$

$$\Rightarrow \log_2(x^3 - 8) = 4$$

$$\Rightarrow x^3 - 8 = 2^4 = 16$$

$$\Rightarrow x^3 = 24$$

$$\Rightarrow x = \sqrt[3]{24} = 2\sqrt[3]{3}$$

Ans: $x = 2\sqrt[3]{3}$

15. Find the value of $\log_{25} 125 + \log_{125} 25$. [3]

$$= \log_{5^2} 5^3 + \log_{5^3} 5^2$$

$$= \frac{3}{2} \log_5 5 + \frac{2}{3} \log_5 5$$

$$= \frac{3}{2} + \frac{2}{3}$$

$$= \frac{13}{6}$$

Ans: $\frac{13}{6}$

16. Find the value of $\frac{\log 2^{\sqrt[3]{2}} + \log 5^{\sqrt[3]{5}}}{\log 10}$. [5]

$$2^{\sqrt[3]{2}} = 2^1 \times 2^{\frac{1}{3}} = 2^{\frac{4}{3}} \text{ (and the same for the others)}$$

$$= \frac{\log(2 \times 5)^{\frac{4}{3}}}{\log 10}$$

$$= \log_{10}(10)^{\frac{4}{3}} \text{ since } 2 \times 5 = 10$$

$$= \frac{4}{3} \log_{10} 10 = \frac{4}{3}$$

Ans: $\frac{4}{3}$

17. Find the value of $6^{\log_6 \frac{4}{3}} \cdot 3^{\log_3 \frac{3}{2}} \cdot 5^{\log_{25} 16}$. [3]

$$a^{\log_a N} = N$$

$$5^{\log_{25} 16} = 5^{\log_{5^2} 4^2} = 5^{\log_5 4} = 4$$

$$= \frac{4}{3} \times \frac{3}{2} \times 4 = 8$$

Ans: 8

18. If $\log(a + 3b) = \log a - \log b$, find a in terms of b . [5]

$$\log a - \log b = \log \frac{a}{b}$$

$$\Rightarrow \log(a + 3b) = \log \frac{a}{b}$$

$$\Rightarrow a + 3b = \frac{a}{b}$$

$$\Rightarrow a - \frac{a}{b} = -3b$$

$$\Rightarrow a \left(1 - \frac{1}{b}\right) = -3b$$

$$\Rightarrow a \left(\frac{b-1}{b}\right) = -3b$$

$$\Rightarrow a = \frac{-3b^2}{b-1} = \frac{3b^2}{1-b}$$

Ans: $a = \frac{3b^2}{1-b}$

19. Solve $\log_7(2x + 1) = \log_{\frac{1}{7}} x$. [5]

$$\frac{1}{7} = 7^{-1}$$

$$\log_7(2x + 1) = \log_{7^{-1}} x = \frac{1}{-1} \log_7 x = \log_7 x^{-1}$$

$$\Rightarrow 2x + 1 = x^{-1} = \frac{1}{x}$$

$$\Rightarrow 2x^2 + x = 1$$

$$\Rightarrow 2x^2 + x - 1 = 0$$

$$\Rightarrow 2x^2 - x + 2x - 1 = 0$$

$$\Rightarrow x(2x - 1) + 1(2x - 1) = 0$$

$$\Rightarrow (x + 1)(2x - 1) = 0$$

$$\Rightarrow x = -1 \text{ (or) } x = \frac{1}{2}$$

For $x = -1$, $\log_{\frac{1}{7}} x$ is not defined.

$$\therefore x = \frac{1}{2}$$

$$\text{Ans: } x = \frac{1}{2}$$

20. Complete the table of $y = \log_5 x$ for $x = \frac{1}{25}, \frac{1}{5}, 1, 5, 25, 125$. [3]

$$y = \log_5 x$$

$$x = \frac{1}{25} : y = \log_5 \frac{1}{25} = -2$$

$$x = \frac{1}{5} : y = \log_5 \frac{1}{5} = -1$$

$$x = 1 : y = \log_5 1 = 0$$

$$x = 5 : y = \log_5 5 = 1$$

$$x = 25 : y = \log_5 25 = 2$$

$$x = 125 : y = \log_5 125 = 3$$

$$\text{Ans: } y = -2, -1, 0, 1, 2, 3$$
