

Logarithms

Question Paper, Set 06

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $x = \log_2 128$, find x . [1 mark]
2. Why can 0 not be the base of a logarithm? Explain using 0^2 and 0^5 . [1 mark]
3. Write $\log_{11} 1331 = 3$ in exponential form. [1 mark]
4. Find $\frac{\log_{20} 20}{5} + \log_3 1$. [1 mark]
5. Find the value of $\log_2 12 + \log_2 10 - \log_2 15$. [2 marks]
6. Write $\log_8 11^4$ in terms of $\log_2 11$. [2 marks]
7. Find the value of $\frac{1}{\log_{36} 6}$. [1 mark]
8. Find the value of $\log_5 64 \cdot \log_4 5$. [2 marks]
9. Write $\log_3 20$ as a quotient of logarithms to the base 10. [1 mark]
10. Find the value of $5^{\log_5 14} - 6^{\log_6 9}$. [2 marks]
11. Find the value of $\frac{10^{\log_3 4}}{4^{\log_3 10}}$. [2 marks]
12. If $\log_6 x^2 = \log_6 (5x - 6)$, find x . [2 marks]
13. If $\log_9 (x^2 + x - 5) = 0$, find x . [3 marks]
14. Solve $\log_{10} (x + 15) + \log_{10} x = 2$. [5 marks]
15. Find the value of $\log_{0.5} 0.0625 - \log_3 \sqrt[3]{243}$. [3 marks]
16. Show that $\frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz} = 2$. [5 marks]
17. Find the value of $4^{-2+\log_7 1} + 3^{1+\log_3 2}$. [3 marks]
18. If $\log \frac{a-b}{4} = \frac{1}{2}(\log a + \log b)$, find the value of $\frac{a}{b} + \frac{b}{a}$. [5 marks]
19. Solve $\log_7 (x - 2) = \log_{49} (x + 10)$. [5 marks]
20. Complete the table of $y = \log_{\frac{1}{5}} x$ for $x = \frac{1}{25}, \frac{1}{5}, 1, 5, 125$. [3 marks]

Key sheet

Q	Answer
1	7
2	$0^2 = 0$ and $0^5 = 0$ would give $2 = 5$, which has no meaning
3	$1331 = 11^3$
4	$\frac{1}{5}$
5	3
6	$\frac{4}{3} \log_2 11$
7	2
8	3
9	$\frac{\log_{10} 20}{\log_{10} 3}$
10	5
11	1
12	$x = 2$ (or) $x = 3$
13	$x = 2$ (or) $x = -3$
14	$x = 5$
15	$\frac{7}{3}$
16	2
17	$\frac{97}{16}$
18	18
19	$x = 6$
20	$y = 2, 1, 0, -1, -3$

Solutions

1. If $x = \log_2 128$, find x . [1]

$$2^7 = 128$$

$$a^x = N \Rightarrow x = \log_a N$$

$$\therefore \log_2 128 = 7$$

Ans: 7

2. Why can 0 not be the base of a logarithm? Explain using 0^2 and 0^5 . [1]

$$0^2 = 0 \Rightarrow 2 = \log_0 0$$

$$0^5 = 0 \Rightarrow 5 = \log_0 0$$

So $2 = 5$, which has no meaning.

$$\therefore a \neq 0$$

Ans: $0^2 = 0$ and $0^5 = 0$ would give $2 = 5$, which has no meaning

3. Write $\log_{11} 1331 = 3$ in exponential form. [1]

$$\log_a N = x \Leftrightarrow N = a^x$$

Here $a = 11$, $N = 1331$, $x = 3$.

$$\therefore 1331 = 11^3$$

Ans: $1331 = 11^3$

4. Find $\frac{\log_{20} 20}{5} + \log_3 1$. [1]

$$\log_{20} 20 = 1, \log_3 1 = 0$$

$$= \frac{1}{5} + 0 = \frac{1}{5}$$

Ans: $\frac{1}{5}$

5. Find the value of $\log_2 12 + \log_2 10 - \log_2 15$. [2]

$$\log_a m + \log_a n - \log_a p = \log_a \frac{m \times n}{p}$$

$$= \log_2 \frac{12 \times 10}{15} = \log_2 8$$

$$2^3 = 8 \Rightarrow \log_2 8 = 3$$

Ans: 3

6. Write $\log_8 11^4$ in terms of $\log_2 11$. [2]

$$\log_8 11^4 = \log_{2^3} 11^4$$

$$= \frac{4}{3} \log_2 11$$

Ans: $\frac{4}{3} \log_2 11$

7. Find the value of $\frac{1}{\log_{36} 6}$. [1]

$$\frac{1}{\log_{36} 6} = \log_6 36$$

$$= \log_6 6^2 = 2 \log_6 6 = 2$$

Ans: 2

8. Find the value of $\log_5 64 \cdot \log_4 5$. [2]

$$\begin{aligned}\log_b a \cdot \log_c b &= \log_c a \\ &= \log_4 64 \\ &= 3\end{aligned}$$

Ans: 3

9. Write $\log_3 20$ as a quotient of logarithms to the base 10. [1]

$$\begin{aligned}\log_b a &= \frac{\log_x a}{\log_x b} \text{ with } x = 10 \\ \therefore \log_3 20 &= \frac{\log_{10} 20}{\log_{10} 3}\end{aligned}$$

Ans: $\frac{\log_{10} 20}{\log_{10} 3}$

10. Find the value of $5^{\log_5 14} - 6^{\log_6 9}$. [2]

$$\begin{aligned}a^{\log_a N} &= N \\ &= 14 - 9 = 5\end{aligned}$$

Ans: 5

11. Find the value of $\frac{10^{\log_3 4}}{4^{\log_3 10}}$. [2]

$$\begin{aligned}a^{\log_c b} &= b^{\log_c a} \\ 10^{\log_3 4} &= 4^{\log_3 10} \\ \therefore \text{the numerator and denominator are equal, and the value is 1.}\end{aligned}$$

Ans: 1

12. If $\log_6 x^2 = \log_6 (5x - 6)$, find x . [2]

$$\begin{aligned}\log_a m &= \log_a n \Rightarrow m = n \\ \Rightarrow x^2 &= 5x - 6 \\ \Rightarrow x^2 - 5x + 6 &= 0 \\ \Rightarrow x^2 - 3x - 2x + 6 &= 0 \\ \Rightarrow x(x - 3) - 2(x - 3) &= 0 \\ \Rightarrow (x - 2)(x - 3) &= 0 \\ \Rightarrow x = 2 \text{ (or) } x = 3\end{aligned}$$

Ans: $x = 2$ (or) $x = 3$

13. If $\log_9 (x^2 + x - 5) = 0$, find x . [3]

$$\begin{aligned}\log_a N &= x \Rightarrow N = a^x \\ \Rightarrow x^2 + x - 5 &= 9^0 = 1 \\ \Rightarrow x^2 + x - 6 &= 0 \\ \Rightarrow x^2 + 3x - 2x - 6 &= 0 \\ \Rightarrow x(x + 3) - 2(x + 3) &= 0 \\ \Rightarrow (x - 2)(x + 3) &= 0 \\ \Rightarrow x = 2 \text{ (or) } x = -3\end{aligned}$$

Ans: $x = 2$ (or) $x = -3$

14. Solve $\log_{10}(x + 15) + \log_{10} x = 2$. [5]

$$\log_a m + \log_a n = \log_a mn$$

$$\Rightarrow \log_{10}(x + 15)x = 2$$

$$\Rightarrow \log_{10}(x^2 + 15x) = 2$$

$$\Rightarrow x^2 + 15x = 10^2 = 100$$

$$\Rightarrow x^2 + 15x - 100 = 0$$

$$\Rightarrow x^2 + 20x - 5x - 100 = 0$$

$$\Rightarrow x(x + 20) - 5(x + 20) = 0$$

$$\Rightarrow (x - 5)(x + 20) = 0$$

$$\Rightarrow x = 5 \text{ (or) } x = -20$$

If $x = -20$, $\log_{10} x$ is not defined.

$$\therefore x \neq -20$$

$$\therefore x = 5$$

Ans: $x = 5$

15. Find the value of $\log_{0.5} 0.0625 - \log_3 \sqrt[3]{243}$. [3]

$$\log_{0.5} 0.0625 = \log_{0.5} (0.5)^4 = 4$$

$$\log_3 \sqrt[3]{243} = \log_3 (3^5)^{\frac{1}{3}} = \log_3 3^{\frac{5}{3}} = \frac{5}{3}$$

$$= 4 - \frac{5}{3} = \frac{7}{3}$$

Ans: $\frac{7}{3}$

16. Show that $\frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz} = 2$. [5]

$$\frac{1}{\log_a b} = \log_b a$$

$$LHS = \log_{xyz} xy + \log_{xyz} yz + \log_{xyz} zx$$

$$= \log_{xyz} (xy \cdot yz \cdot zx)$$

$$= \log_{xyz} (xyz)^2$$

$$= 2 \log_{xyz} xyz = 2(1) = 2 = RHS$$

Ans: 2

17. Find the value of $4^{-2+\log_7 1} + 3^{1+\log_3 2}$. [3]

$$\log_7 1 = 0 \Rightarrow 4^{-2+0} = 4^{-2} = \frac{1}{16}$$

$$3^{1+\log_3 2} = 3^1 \times 3^{\log_3 2} = 3 \times 2 = 6$$

$$= \frac{1}{16} + 6 = \frac{97}{16}$$

Ans: $\frac{97}{16}$

18. If $\log \frac{a-b}{4} = \frac{1}{2}(\log a + \log b)$, find the value of $\frac{a}{b} + \frac{b}{a}$. [5]

$$\log \frac{a-b}{4} = \frac{1}{2} \log ab = \log \sqrt{ab}$$

$$\Rightarrow \frac{a-b}{4} = \sqrt{ab}$$

$$\Rightarrow a - b = 4\sqrt{ab}$$

Squaring on both sides,

$$\Rightarrow a^2 - 2ab + b^2 = 16ab$$

$$\Rightarrow a^2 + b^2 = 18ab$$

$$\Rightarrow \frac{a^2}{ab} + \frac{b^2}{ab} = 18$$

$$\Rightarrow \frac{a}{b} + \frac{b}{a} = 18$$

Ans: 18

19. Solve $\log_7(x - 2) = \log_{49}(x + 10)$. [5]

$$\log_{49}(x + 10) = \log_{7^2}(x + 10) = \frac{1}{2} \log_7(x + 10)$$

$$\Rightarrow 2 \log_7(x - 2) = \log_7(x + 10)$$

$$\Rightarrow \log_7(x - 2)^2 = \log_7(x + 10)$$

$$\Rightarrow (x - 2)^2 = x + 10$$

$$\Rightarrow x^2 - 4x + 4 = x + 10$$

$$\Rightarrow x^2 - 5x - 6 = 0$$

$$\Rightarrow x^2 - 6x + x - 6 = 0$$

$$\Rightarrow x(x - 6) + 1(x - 6) = 0$$

$$\Rightarrow (x + 1)(x - 6) = 0$$

$$\Rightarrow x = -1 \text{ (or) } x = 6$$

For $x = -1$, $\log_7(x - 2)$ is not defined.

$$\therefore x = 6$$

Ans: $x = 6$

20. Complete the table of $y = \log_{\frac{1}{5}} x$ for $x = \frac{1}{25}, \frac{1}{5}, 1, 5, 125$. [3]

$$y = \log_{\frac{1}{5}} x$$

$$x = \frac{1}{25} : y = \log_{\frac{1}{5}} \frac{1}{25} = 2$$

$$x = \frac{1}{5} : y = \log_{\frac{1}{5}} \frac{1}{5} = 1$$

$$x = 1 : y = \log_{\frac{1}{5}} 1 = 0$$

$$x = 5 : y = \log_{\frac{1}{5}} 5 = -1$$

$$x = 125 : y = \log_{\frac{1}{5}} 125 = -3$$

Ans: $y = 2, 1, 0, -1, -3$