

Logarithms

Question Paper, Set 05

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the logarithm of $\frac{1}{100}$ to the base 10. [1 mark]
2. Of $\log_5(-25)$ and $\log_5 25$, which one is defined? Find its value. [1 mark]
3. Write $16^{\frac{3}{4}} = 8$ in logarithmic form. [1 mark]
4. Find $\log_{0.7} 0.7 \times \log_{12} 12$. [1 mark]
5. Find the value of $\log_5 250 - \log_5 2$. [2 marks]
6. Find the value of $\log_{625} 125$. [2 marks]
7. Find the value of $\log_9 20 \times \log_{20} 9$. [1 mark]
8. Find the value of $\log_7 27 \cdot \log_5 7 \cdot \log_3 5$. [2 marks]
9. Write $\frac{\log_{10} 2401}{\log_{10} 49}$ as a single logarithm and find its value. [1 mark]
10. Find the value of $10^{\log_{10} 7} \times 2^{\log_2 3}$. [2 marks]
11. Write $8^{\log_3 7}$ as a power of 7. [2 marks]
12. If $\log_5 x = \log_5(12 - x)$, find x . [2 marks]
13. If $\log_2 \log_5 \log_3 x = 0$, find x . [3 marks]
14. Solve $\log_2(x + 6) - \log_2(x - 1) = 3$. [5 marks]
15. Find the value of $\log_2 \log_3 \log_4 64^3$. [3 marks]
16. If $\log_{\sqrt[4]{b}} a \cdot \log_{\sqrt[3]{c}} b \cdot \log_{\sqrt[5]{d}} c = k \log_d a$, find k . [5 marks]
17. Find the value of $3^{\log_5 7} + 2^{\log_3 9} - 9^{\log_3 2} - 7^{\log_5 3}$. [3 marks]
18. If $x^2 + y^2 = 11xy$ and $x > y$, show that $2 \log(x - y) = \log x + \log y + 2 \log 3$. [5 marks]
19. Solve $\log_5(4x + 3) = \log_{\frac{1}{5}} x$. [5 marks]

- 20.** Complete the table of $y = \log_4 x$ for $x = \frac{1}{16}, \frac{1}{4}, 1, 4, 16, 64$ and say whether the graph rises or falls from left to right. [3 marks]
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Key sheet

Q	Answer
1	-2
2	$\log_5 25 = 2$ ($\log_5(-25)$ is not defined)
3	$\frac{3}{4} = \log_{16} 8$
4	1
5	3
6	$\frac{3}{4}$
7	1
8	3
9	$\log_{49} 2401 = 2$
10	21
11	$7^{\log_3 8}$
12	$x = 6$
13	$x = 243$
14	$x = 2$
15	1
16	$k = 60$
17	0
18	Shown: $2 \log(x - y) = \log x + \log y + 2 \log 3$
19	$x = \frac{1}{4}$
20	$y = -2, -1, 0, 1, 2, 3$; the graph rises

Solutions

1. Find the logarithm of $\frac{1}{100}$ to the base 10. [1]

$$\frac{1}{100} = \frac{1}{10^2} = 10^{-2}$$

$$a^x = N \Rightarrow x = \log_a N$$

$$\therefore \log_{10} \frac{1}{100} = -2$$

Ans: -2

2. Of $\log_5(-25)$ and $\log_5 25$, which one is defined? Find its value. [1]

The number must be positive ($N > 0$), so $\log_5(-25)$ is not defined.

$$5^2 = 25 \Rightarrow 2 = \log_5 25$$

$$\therefore \log_5 25 = 2$$

Ans: $\log_5 25 = 2$ ($\log_5(-25)$ is not defined)

3. Write $16^{\frac{3}{4}} = 8$ in logarithmic form. [1]

$$a^x = N \Leftrightarrow x = \log_a N$$

Here $a = 16$, $x = \frac{3}{4}$, $N = 8$.

$$\therefore \frac{3}{4} = \log_{16} 8$$

Ans: $\frac{3}{4} = \log_{16} 8$

4. Find $\log_{0.7} 0.7 \times \log_{12} 12$. [1]

$$\log_{0.7} 0.7 = 1, \log_{12} 12 = 1$$

$$= 1 \times 1 = 1$$

Ans: 1

5. Find the value of $\log_5 250 - \log_5 2$. [2]

$$\log_a m - \log_a n = \log_a \frac{m}{n}$$

$$= \log_5 \frac{250}{2} = \log_5 125$$

$$5^3 = 125 \Rightarrow \log_5 125 = 3$$

Ans: 3

6. Find the value of $\log_{625} 125$. [2]

$$\log_{625} 125 = \log_{5^4} 5^3$$

$$= \frac{3}{4} \log_5 5$$

$$= \frac{3}{4}(1) = \frac{3}{4}$$

Ans: $\frac{3}{4}$

7. Find the value of $\log_9 20 \times \log_{20} 9$. [1]

$$\log_b a \cdot \log_a b = 1$$

$$\therefore \log_9 20 \times \log_{20} 9 = 1$$

Ans: 1

8. Find the value of $\log_7 27 \cdot \log_5 7 \cdot \log_3 5$. [2]

$$\begin{aligned}\log_b a \cdot \log_c b \cdot \log_d c &= \log_d a \\ &= \log_3 27 \\ &= 3\end{aligned}$$

Ans: 3

9. Write $\frac{\log_{10} 2401}{\log_{10} 49}$ as a single logarithm and find its value. [1]

$$\begin{aligned}\frac{\log a}{\log b} &= \log_b a \\ &= \log_{49} 2401 \\ &= 2\end{aligned}$$

Ans: $\log_{49} 2401 = 2$

10. Find the value of $10^{\log_{10} 7} \times 2^{\log_2 3}$. [2]

$$\begin{aligned}a^{\log_a N} &= N \\ &= 7 \times 3 = 21\end{aligned}$$

Ans: 21

11. Write $8^{\log_3 7}$ as a power of 7. [2]

$$\begin{aligned}a^{\log_c b} &= b^{\log_c a} \\ \therefore 8^{\log_3 7} &= 7^{\log_3 8}\end{aligned}$$

Ans: $7^{\log_3 8}$

12. If $\log_5 x = \log_5(12 - x)$, find x . [2]

$$\begin{aligned}\log_a m &= \log_a n \Rightarrow m = n \\ \Rightarrow x &= 12 - x \\ \Rightarrow 2x &= 12 \\ \Rightarrow x &= 6\end{aligned}$$

Ans: $x = 6$

13. If $\log_2 \log_5 \log_3 x = 0$, find x . [3]

$$\begin{aligned}\log_a N &= 0 \Rightarrow N = 1 \\ \Rightarrow \log_5 \log_3 x &= 1 \\ \Rightarrow \log_3 x &= 5^1 = 5 \\ \Rightarrow x &= 3^5 = 243\end{aligned}$$

Ans: $x = 243$

14. Solve $\log_2(x+6) - \log_2(x-1) = 3$. [5]

$$\log_a m - \log_a n = \log_a \frac{m}{n}$$

$$\Rightarrow \log_2 \frac{x+6}{x-1} = 3$$

$$\Rightarrow \frac{x+6}{x-1} = 2^3 = 8$$

$$\Rightarrow x+6 = 8x-8$$

$$\Rightarrow 7x = 14$$

$$\Rightarrow x = 2$$

For $x = 2$ both logarithms are defined.

Ans: $x = 2$

15. Find the value of $\log_2 \log_3 \log_4 64^3$. [3]

$$\log_4 64^3 = \log_4 (4^3)^3 = \log_4 4^9 = 9$$

$$\log_3 9 = \log_3 3^2 = 2$$

$$\log_2 2 = 1$$

$$\therefore \log_2 \log_3 \log_4 64^3 = 1$$

Ans: 1

16. If $\log_{\sqrt[n]{b}} a \cdot \log_{\sqrt[m]{c}} b \cdot \log_{\sqrt[d]{a}} c = k \log_d a$, find k . [5]

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$= \log_{b^{\frac{1}{4}}} a \cdot \log_{c^{\frac{1}{3}}} b \cdot \log_{d^{\frac{1}{5}}} c$$

$$= 4 \log_b a \cdot 3 \log_c b \cdot 5 \log_d c$$

$$= 4 \times 3 \times 5 \cdot \log_b a \log_c b \log_d c$$

$$= 60 \log_d a$$

$$\therefore k = 60$$

Ans: $k = 60$

17. Find the value of $3^{\log_5 7} + 2^{\log_3 9} - 9^{\log_3 2} - 7^{\log_5 3}$. [3]

$$a^{\log_c b} = b^{\log_c a}$$

$$3^{\log_5 7} = 7^{\log_5 3}, 2^{\log_3 9} = 9^{\log_3 2}$$

$$= 7^{\log_5 3} + 9^{\log_3 2} - 9^{\log_3 2} - 7^{\log_5 3} = 0$$

Ans: 0

18. If $x^2 + y^2 = 11xy$ and $x > y$, show that $2 \log(x-y) = \log x + \log y + 2 \log 3$. [5]

$$\text{Given } x^2 + y^2 = 11xy$$

$$LHS = 2 \log(x-y) = \log(x-y)^2$$

$$= \log(x^2 + y^2 - 2xy)$$

$$= \log(11xy - 2xy) = \log(9xy)$$

$$= \log 9 + \log x + \log y$$

$$= \log 3^2 + \log x + \log y = 2 \log 3 + \log x + \log y$$

$$= RHS$$

Ans: Shown: $2 \log(x-y) = \log x + \log y + 2 \log 3$

19. Solve $\log_5(4x + 3) = \log_{\frac{1}{5}} x$. [5]

$$\frac{1}{5} = 5^{-1}$$

$$\log_5(4x + 3) = \log_{5^{-1}} x = \frac{1}{-1} \log_5 x = \log_5 x^{-1}$$

$$\Rightarrow 4x + 3 = x^{-1} = \frac{1}{x}$$

$$\Rightarrow 4x^2 + 3x = 1$$

$$\Rightarrow 4x^2 + 3x - 1 = 0$$

$$\Rightarrow 4x^2 - x + 4x - 1 = 0$$

$$\Rightarrow x(4x - 1) + 1(4x - 1) = 0$$

$$\Rightarrow (x + 1)(4x - 1) = 0$$

$$\Rightarrow x = -1 \text{ (or) } x = \frac{1}{4}$$

For $x = -1$, $\log_{\frac{1}{5}} x$ is not defined.

$$\therefore x = \frac{1}{4}$$

$$\text{Ans: } x = \frac{1}{4}$$

20. Complete the table of $y = \log_4 x$ for $x = \frac{1}{16}, \frac{1}{4}, 1, 4, 16, 64$ and say whether the graph rises or falls from left to right. [3]

$$y = \log_4 x$$

$$x = \frac{1}{16} : y = \log_4 \frac{1}{16} = -2$$

$$x = \frac{1}{4} : y = \log_4 \frac{1}{4} = -1$$

$$x = 1 : y = \log_4 1 = 0$$

$$x = 4 : y = \log_4 4 = 1$$

$$x = 16 : y = \log_4 16 = 2$$

$$x = 64 : y = \log_4 64 = 3$$

The base is more than 1, so the graph rises from left to right.

Ans: $y = -2, -1, 0, 1, 2, 3$; the graph rises
