EXPRESSWALA ACADEMY

Logarithms

Question Paper, Set 04

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the value of $\log_{11} 121$. [1 mark]

2. Why can 1 not be the base of a logarithm? Explain using 1^4 and 1^7. [1 mark]

3. Write $\log_8 \frac{1}{64} = -2$ in exponential form. [1 mark]

4. Find $6\log_{13} 13 - 2\log_8 8$. [1 mark]

5. Find the value of $\log_{12} 8 + \log_{12} 18$. [2 marks]

6. Find the value of $\log_{\frac{1}{3}} 81$. [2 marks]

7. If $\log_3 10 = p$, write $\log_{10} 3$ in terms of p. [1 mark]

8. Find the value of $\log_{11} 8 \cdot \log_{16} 11$. [2 marks]

9. Write $\frac{\log 64}{\log 16}$ as a single logarithm and find its value. [1 mark]

10. Find the value of $\frac{11^{\log_{11} 45}}{3^{\log_3 9}}$. [2 marks]

11. Find the value of $6^{\log_5 11} - 11^{\log_5 6}$. [2 marks]

12. If $\log_2 \sqrt[3]{x} = \log_2 4$, find x. [2 marks]

13. If $\log_5(x^2 + 6x + 13) = 1$, find x. [3 marks]

14. Solve $\log_3(x - 6) + \log_3 x = 3$. [5 marks]

15. Find the value of $\log_4 32 - \log_{32} 4$. [3 marks]

16. Show that $\log_{b^2} a \cdot \log_{c^3} b \cdot \log_{a^4} c = \frac{1}{24}$. [5 marks]

17. Find the value of $2^{4+\log_2 3} - 9^{\log_3 5}$. [3 marks]

18. If $c^3 - b^3 = 27$, find $\log_3(c - b) + \log_3(c^2 + cb + b^2)$. [5 marks]

19. Solve $\log_3(x - 1) = \log_9(x + 5)$. [5 marks]

20. Complete the table of $y = \log_{\frac{1}{2}} x$ for $x = \frac{1}{8}, \frac{1}{4}, 1, 2, 8, 16$. [3 marks]

Key sheet

Q	Answer
1	2
2	$1^4 = 1$ and $1^7 = 1$ would give $4 = 7$, which has no meaning
3	$\frac{1}{64} = 8^{-2}$
4	4
5	2
6	-4
7	$\frac{1}{p}$
8	$\frac{3}{4}$
9	$\log_{16} 64 = \frac{3}{2}$
10	5
11	0
12	$x = 64$
13	$x = -2$ (or) $x = -4$
14	$x = 9$
15	$\frac{21}{10}$
16	$\frac{1}{24}$
17	23
18	3
19	$x = 4$
20	$y = 3, 2, 0, -1, -3, -4$

Solutions

1. Find the value of $\log_{11} 121$. [1]

$11^2 = 121$

$a^x = N \Rightarrow x = \log_a N$

$\therefore \log_{11} 121 = 2$

Ans: 2

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2. Why can 1 not be the base of a logarithm? Explain using 1^4 and 1^7. [1]

$1^4 = 1 \Rightarrow 4 = \log_1 1$

$1^7 = 1 \Rightarrow 7 = \log_1 1$

So $4 = 7$, which has no meaning.

$\therefore a \neq 1$

Ans: $1^4 = 1$ **and** $1^7 = 1$ **would give** $4 = 7$**, which has no meaning**

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3. Write $\log_8 \frac{1}{64} = -2$ in exponential form. [1]

$\log_a N = x \Leftrightarrow N = a^x$

Here $a = 8$, $N = \frac{1}{64}$, $x = -2$.

$\therefore \frac{1}{64} = 8^{-2}$

Ans: $\frac{1}{64} = 8^{-2}$

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4. Find $6 \log_{13} 13 - 2 \log_8 8$. [1]

$\log_{13} 13 = 1,\ \log_8 8 = 1$

$= 6(1) - 2(1) = 4$

Ans: 4

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5. Find the value of $\log_{12} 8 + \log_{12} 18$. [2]

$\log_a m + \log_a n = \log_a mn$

$= \log_{12}(8 \times 18) = \log_{12} 144$

$12^2 = 144 \Rightarrow \log_{12} 144 = 2$

Ans: 2

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6. Find the value of $\log_{\frac{1}{3}} 81$. [2]

$\frac{1}{3} = 3^{-1},\ 81 = 3^4$

$\log_{\frac{1}{3}} 81 = \log_{3^{-1}} 3^4$

$= \frac{4}{-1} \log_3 3 = -4$

Ans: -4

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7. If $\log_3 10 = p$, write $\log_{10} 3$ in terms of p. [1]

$\log_b a = \frac{1}{\log_a b}$

$\log_{10} 3 = \frac{1}{\log_3 10} = \frac{1}{p}$

Ans: $\frac{1}{p}$

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8. Find the value of $\log_{11} 8 \cdot \log_{16} 11$. [2]

$\log_b a \cdot \log_c b = \log_c a$

$= \log_{16} 8$

$= \frac{3}{4}$

Ans: $\frac{3}{4}$

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9. Write $\frac{\log 64}{\log 16}$ as a single logarithm and find its value. [1]

$\frac{\log a}{\log b} = \log_b a$

$= \log_{16} 64$

$= \frac{3}{2}$

Ans: $\log_{16} 64 = \frac{3}{2}$

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10. Find the value of $\frac{11^{\log_{11} 45}}{3^{\log_3 9}}$. [2]

$a^{\log_a N} = N$

$= \frac{45}{9} = 5$

Ans: 5

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11. Find the value of $6^{\log_5 11} - 11^{\log_5 6}$. [2]

$a^{\log_c b} = b^{\log_c a}$

$6^{\log_5 11} = 11^{\log_5 6}$

$= 11^{\log_5 6} - 11^{\log_5 6} = 0$

Ans: 0

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12. If $\log_2 \sqrt[3]{x} = \log_2 4$, find x. [2]

$\log_a m = \log_a n \Rightarrow m = n$

$\Rightarrow \sqrt[3]{x} = 4$

$\Rightarrow x = 4^3 = 64$

Ans: $x = 64$

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13. If $\log_5(x^2 + 6x + 13) = 1$, find x. [3]

$\log_a N = x \Rightarrow N = a^x$

$\Rightarrow x^2 + 6x + 13 = 5$

$\Rightarrow x^2 + 6x + 8 = 0$

$\Rightarrow x^2 + 4x + 2x + 8 = 0$

$\Rightarrow x(x + 4) + 2(x + 4) = 0$

$\Rightarrow (x + 2)(x + 4) = 0$

$\Rightarrow x = -2$ (or) $x = -4$

Ans: $x = -2$ **(or)** $x = -4$

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14. Solve $\log_3(x-6)+\log_3 x=3$. [5]

$\log_a m+\log_a n=\log_a mn$

$\Rightarrow \log_3(x-6)x=3$

$\Rightarrow \log_3(x^2-6x)=3$

$\Rightarrow x^2-6x=3^3=27$

$\Rightarrow x^2-6x-27=0$

$\Rightarrow x^2-9x+3x-27=0$

$\Rightarrow x(x-9)+3(x-9)=0$

$\Rightarrow (x+3)(x-9)=0$

$\Rightarrow x=-3$ (or) $x=9$

If $x=-3$, $\log_3 x$ is not defined.

$\therefore x\neq -3$

$\therefore x=9$

Ans: $x=9$

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15. Find the value of $\log_4 32-\log_{32} 4$. [3]

$=\log_{2^2} 2^5-\log_{2^5} 2^2$

$=\frac{5}{2}\log_2 2-\frac{2}{5}\log_2 2$

$=\frac{5}{2}-\frac{2}{5}$

$=\frac{21}{10}$

Ans: $\frac{21}{10}$

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16. Show that $\log_{b^2} a\cdot\log_{c^3} b\cdot\log_{a^4} c=\frac{1}{24}$. [5]

$\log_{b^n} a^m=\frac{m}{n}\log_b a$

$LHS=\frac{1}{2}\log_b a\cdot\frac{1}{3}\log_c b\cdot\frac{1}{4}\log_a c$

$=\frac{1}{24}\log_b a\log_c b\log_a c$

$\log_b a\log_c b\log_a c=\log_a a=1$

$=\frac{1}{24}(1)=\frac{1}{24}=RHS$

Ans: $\frac{1}{24}$

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17. Find the value of $2^{4+\log_2 3}-9^{\log_3 5}$. [3]

$2^{4+\log_2 3}=2^4\times 2^{\log_2 3}=16\times 3=48$

$9^{\log_3 5}=(3^2)^{\log_3 5}=(3^{\log_3 5})^2=5^2=25$

$=48-25=23$

Ans: 23

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18. If $c^3-b^3=27$, find $\log_3(c-b)+\log_3(c^2+cb+b^2)$. [5]

$\log_a m+\log_a n=\log_a mn$

$=\log_3[(c-b)(c^2+cb+b^2)]$

$(a-b)(a^2+ab+b^2)=a^3-b^3$

$=\log_3(c^3-b^3)=\log_3 27$

$=\log_3 3^3=3$

Ans: 3

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19. Solve $\log_3(x-1) = \log_9(x+5)$. [5]

$\log_9(x+5) = \log_{3^2}(x+5) = \frac{1}{2}\log_3(x+5)$

$\Rightarrow 2\log_3(x-1) = \log_3(x+5)$

$\Rightarrow \log_3(x-1)^2 = \log_3(x+5)$

$\Rightarrow (x-1)^2 = x+5$

$\Rightarrow x^2 - 2x + 1 = x + 5$

$\Rightarrow x^2 - 3x - 4 = 0$

$\Rightarrow x^2 - 4x + x - 4 = 0$

$\Rightarrow x(x-4) + 1(x-4) = 0$

$\Rightarrow (x+1)(x-4) = 0$

$\Rightarrow x = -1$ (or) $x = 4$

For $x = -1$, $\log_3(x-1)$ is not defined.

$\therefore x = 4$

Ans: $x = 4$

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20. Complete the table of $y = \log_{\frac{1}{2}} x$ for $x = \frac{1}{8}, \frac{1}{4}, 1, 2, 8, 16$. [3]

$y = \log_{\frac{1}{2}} x$

$x = \frac{1}{8}: \ y = \log_{\frac{1}{2}} \frac{1}{8} = 3$

$x = \frac{1}{4}: \ y = \log_{\frac{1}{2}} \frac{1}{4} = 2$

$x = 1: \ y = \log_{\frac{1}{2}} 1 = 0$

$x = 2: \ y = \log_{\frac{1}{2}} 2 = -1$

$x = 8: \ y = \log_{\frac{1}{2}} 8 = -3$

$x = 16: \ y = \log_{\frac{1}{2}} 16 = -4$

Ans: $y = 3, 2, 0, -1, -3, -4$