EXPRESSWALA ACADEMY

Logarithms

Question Paper, Set 03

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $x = \log_5 125$, find x. [1 mark]

2. Of $\log_{-3} 9$ and $\log_3 9$, which one is defined? Find its value. [1 mark]

3. Write $5^{-3} = \frac{1}{125}$ in logarithmic form. [1 mark]

4. Find $\log_{\frac{3}{5}} \frac{3}{5} + \log_{\frac{3}{5}} 1$. [1 mark]

5. Find the value of $\log_{10} 8 + \log_{10} 25 - \log_{10} 2$. [2 marks]

6. Find the value of $\log_9 243$. [2 marks]

7. Find the value of $\frac{1}{\log_{32} 2}$. [1 mark]

8. Find the value of $\log_7 243 \cdot \log_3 7$. [2 marks]

9. Write $\frac{\log_{10} 49}{\log_{10} 343}$ as a single logarithm and find its value. [1 mark]

10. Find the value of $9^{\log_9 \frac{2}{3}} \times 6^{\log_6 12}$. [2 marks]

11. Find the value of $\frac{7^{\log_2 3}}{3^{\log_2 7}}$. [2 marks]

12. If $\log_4(2x + 1) = \log_4(x + 6)$, find x. [2 marks]

13. If $\log_6(x^2 - 7x + 13) = 0$, find x. [3 marks]

14. Solve $\log_3 x - \log_3(x + 2) = 2$. [5 marks]

15. Find the value of $\log_{0.2} 0.008 + \log_2 \sqrt{32}$. [3 marks]

16. Find the value of $\frac{\log 6\sqrt{6} + \log 5\sqrt{5} - \log 4\sqrt{4}}{\log 7.5}$. [5 marks]

17. Find the value of $5^{\log_{25} 64}$. [3 marks]

18. If $\log(3a - 2b) = \log a - \log b$, find a in terms of b. [5 marks]

19. Solve $\log_3(3x + 2) = \log_{\frac{1}{3}} x$. [5 marks]

20. Complete the table of $y = \log_3 x$ for $x = \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27$ and say whether the graph rises or falls from left to right. [3 marks]

Key sheet

Q	Answer
1	3
2	$\log_3 9 = 2$ ($\log_{-3} 9$ is not defined)
3	$-3 = \log_5 \frac{1}{125}$
4	1
5	2
6	$\frac{5}{2}$
7	5
8	5
9	$\log_{343} 49 = \frac{2}{3}$
10	8
11	1
12	$x = 5$
13	$x = 3$ (or) $x = 4$
14	No real x
15	$\frac{11}{2}$
16	$\frac{3}{2}$
17	8
18	$a = \frac{2b^2}{3b-1}$
19	$x = \frac{1}{3}$
20	$y = -2, -1, 0, 1, 2, 3$; the graph rises

Solutions

1. If $x = \log_5 125$, find x. [1]

$5^3 = 125$

$a^x = N \Rightarrow x = \log_a N$

$\therefore \log_5 125 = 3$

Ans: 3

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2. Of $\log_{-3} 9$ and $\log_3 9$, which one is defined? Find its value. [1]

The base cannot be negative, so $\log_{-3} 9$ is not defined.

$3^2 = 9 \Rightarrow 2 = \log_3 9$

$\therefore \log_3 9 = 2$

Ans: $\log_3 9 = 2$ **($\log_{-3} 9$ is not defined)**

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3. Write $5^{-3} = \frac{1}{125}$ in logarithmic form. [1]

$a^x = N \Leftrightarrow x = \log_a N$

Here $a = 5$, $x = -3$, $N = \frac{1}{125}$.

$\therefore -3 = \log_5 \frac{1}{125}$

Ans: $-3 = \log_5 \frac{1}{125}$

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4. Find $\log_{\frac{3}{5}} \frac{3}{5} + \log_{\frac{3}{5}} 1$. [1]

$\log_a a = 1,\ \log_a 1 = 0$ with $a = \frac{3}{5}$

$= 1 + 0 = 1$

Ans: 1

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5. Find the value of $\log_{10} 8 + \log_{10} 25 - \log_{10} 2$. [2]

$\log_a m + \log_a n - \log_a p = \log_a \frac{m \times n}{p}$

$= \log_{10} \frac{8 \times 25}{2} = \log_{10} 100$

$10^2 = 100 \Rightarrow \log_{10} 100 = 2$

Ans: 2

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6. Find the value of $\log_9 243$. [2]

$\log_9 243 = \log_{3^2} 3^5$

$= \frac{5}{2} \log_3 3$

$= \frac{5}{2}(1) = \frac{5}{2}$

Ans: $\frac{5}{2}$

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7. Find the value of $\frac{1}{\log_{32} 2}$. [1]

$\frac{1}{\log_{32} 2} = \log_2 32$

$= \log_2 2^5 = 5 \log_2 2 = 5$

Ans: 5

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8. Find the value of $\log_7 243 \cdot \log_3 7$. [2]

$\log_b a \cdot \log_c b = \log_c a$

$= \log_3 243$

$= 5$

Ans: 5

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9. Write $\frac{\log_{10} 49}{\log_{10} 343}$ as a single logarithm and find its value. [1]

$\frac{\log a}{\log b} = \log_b a$

$= \log_{343} 49$

$= \frac{2}{3}$

Ans: $\log_{343} 49 = \frac{2}{3}$

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10. Find the value of $9^{\log_9 \frac{2}{3}} \times 6^{\log_6 12}$. [2]

$a^{\log_a N} = N$

$= \frac{2}{3} \times 12 = 8$

Ans: 8

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11. Find the value of $\frac{7^{\log_2 3}}{3^{\log_2 7}}$. [2]

$a^{\log_c b} = b^{\log_c a}$

$7^{\log_2 3} = 3^{\log_2 7}$

$\therefore$ the numerator and denominator are equal, and the value is 1.

Ans: 1

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12. If $\log_4(2x + 1) = \log_4(x + 6)$, find x. [2]

$\log_a m = \log_a n \Rightarrow m = n$

$\Rightarrow 2x + 1 = x + 6$

$\Rightarrow x = 5$

$\Rightarrow x = 5$

Ans: $x = 5$

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13. If $\log_6(x^2 - 7x + 13) = 0$, find x. [3]

$\log_a N = x \Rightarrow N = a^x$

$\Rightarrow x^2 - 7x + 13 = 6^0 = 1$

$\Rightarrow x^2 - 7x + 12 = 0$

$\Rightarrow x^2 - 4x - 3x + 12 = 0$

$\Rightarrow x(x - 4) - 3(x - 4) = 0$

$\Rightarrow (x - 3)(x - 4) = 0$

$\Rightarrow x = 3$ (or) $x = 4$

Ans: $x = 3$ **(or)** $x = 4$

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14. Solve $\log_3 x - \log_3(x+2) = 2$. [5]

$\log_a m - \log_a n = \log_a \frac{m}{n}$

$\Rightarrow \log_3 \frac{x}{x+2} = 2$

$\Rightarrow \frac{x}{x+2} = 3^2 = 9$

$\Rightarrow x = 9x + 18$

$\Rightarrow 8x = -18$

$\Rightarrow x = -\frac{9}{4}$

For $x = -\frac{9}{4}$, $\log_3 x$ is not defined.

$\therefore$ there is no real x.

Ans: No real x

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15. Find the value of $\log_{0.2} 0.008 + \log_2 \sqrt{32}$. [3]

$\log_{0.2} 0.008 = \log_{0.2}(0.2)^3 = 3\log_{0.2} 0.2 = 3$

$\log_2 \sqrt{32} = \log_2 (2^5)^{\frac{1}{2}} = \log_2 2^{\frac{5}{2}} = \frac{5}{2}$

$= 3 + \frac{5}{2} = \frac{11}{2}$

Ans: $\frac{11}{2}$

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16. Find the value of $\frac{\log 6\sqrt{6} + \log 5\sqrt{5} - \log 4\sqrt{4}}{\log 7.5}$. [5]

$6\sqrt{6} = 6^1 \times 6^{\frac{1}{2}} = 6^{\frac{3}{2}}$ (and the same for the others)

$= \frac{\log(\frac{6\times5}{4})^{\frac{3}{2}}}{\log 7.5}$

$= \log_{7.5}(7.5)^{\frac{3}{2}}$ since $\frac{6\times5}{4} = 7.5$

$= \frac{3}{2}\log_{7.5} 7.5 = \frac{3}{2}$

Ans: $\frac{3}{2}$

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17. Find the value of $5^{\log_{25} 64}$. [3]

$\log_{25} 64 = \log_{5^2} 8^2 = \frac{2}{2}\log_5 8 = \log_5 8$

$5^{\log_{25} 64} = 5^{\log_5 8}$

$= 8$

Ans: 8

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18. If $\log(3a - 2b) = \log a - \log b$, find a in terms of b. [5]

$\log a - \log b = \log \frac{a}{b}$

$\Rightarrow \log(3a - 2b) = \log \frac{a}{b}$

$\Rightarrow 3a - 2b = \frac{a}{b}$

$\Rightarrow 3a - \frac{a}{b} = 2b$

$\Rightarrow a\left(3 - \frac{1}{b}\right) = 2b$

$\Rightarrow a\left(\frac{3b-1}{b}\right) = 2b$

$\Rightarrow a = \frac{2b^2}{3b-1}$

Ans: $a = \frac{2b^2}{3b-1}$

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19. Solve $\log_3(3x+2) = \log_{\frac{1}{3}} x$. [5]

$\frac{1}{3} = 3^{-1}$

$\log_3(3x+2) = \log_{3^{-1}} x = \frac{1}{-1}\log_3 x = \log_3 x^{-1}$

$\Rightarrow 3x+2 = x^{-1} = \frac{1}{x}$

$\Rightarrow 3x^2 + 2x = 1$

$\Rightarrow 3x^2 + 2x - 1 = 0$

$\Rightarrow 3x^2 - x + 3x - 1 = 0$

$\Rightarrow x(3x-1) + 1(3x-1) = 0$

$\Rightarrow (x+1)(3x-1) = 0$

$\Rightarrow x = -1$ (or) $x = \frac{1}{3}$

For $x = -1$, $\log_{\frac{1}{3}} x$ is not defined.

$\therefore x = \frac{1}{3}$

Ans: $x = \frac{1}{3}$

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20. Complete the table of $y = \log_3 x$ for $x = \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27$ and say whether the graph rises or falls from left to right. [3]

$y = \log_3 x$

$x = \frac{1}{9}:\ y = \log_3 \frac{1}{9} = -2$

$x = \frac{1}{3}:\ y = \log_3 \frac{1}{3} = -1$

$x = 1:\ y = \log_3 1 = 0$

$x = 3:\ y = \log_3 3 = 1$

$x = 9:\ y = \log_3 9 = 2$

$x = 27:\ y = \log_3 27 = 3$

The base is more than 1, so the graph rises from left to right.

Ans: $y = -2, -1, 0, 1, 2, 3$; **the graph rises**