

Logarithms

Question Paper, Set 02

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the logarithm of $\frac{1}{32}$ to the base 2. [1 mark]
2. Is $\log_{-4} 16$ defined? Give a reason. [1 mark]
3. Write $\log_{27} 9 = \frac{2}{3}$ in exponential form. [1 mark]
4. Find $\log_{15} 1 - \log_6 6$. [1 mark]
5. Find the value of $\log_2 96 - \log_2 3$. [2 marks]
6. Write $\log_{25} 7^6$ in terms of $\log_5 7$. [2 marks]
7. Write $\frac{1}{\log_5 9}$ as a single logarithm. [1 mark]
8. Find the value of $\log_3 4 \cdot \log_5 3 \cdot \log_2 5$. [2 marks]
9. Write $\log_7 12$ as a quotient of logarithms to the base 10. [1 mark]
10. Find the value of $2 \times 7^{\log_7 6} - 5^{\log_5 4}$. [2 marks]
11. Write $5^{\log_2 3}$ as a power of 3. [2 marks]
12. If $\log_3 \sqrt{x} = \log_3 6$, find x . [2 marks]
13. If $\log_3 \log_2 \log_4 x = 0$, find x . [3 marks]
14. Solve $\log_3(x^2 + x + 1) + \log_3(x - 1) = 2$. [5 marks]
15. Find the value of $\log_2 \log_2 \log_3 81$. [3 marks]
16. Find the value of $\frac{1}{\log_9 3} + \frac{1}{\log_{81} 3} + \frac{1}{\log_{243} 3}$. [5 marks]
17. Find the value of $10^{\log_{10} \frac{2}{5}} \cdot 7^{\log_7 \frac{5}{2}} \cdot 2^{\log_4 9}$. [3 marks]
18. If $\log \frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$, find the value of $\frac{a}{b} + \frac{b}{a}$. [5 marks]
19. Solve $\log_2(x + 2) = \log_4(x + 14)$. [5 marks]

- 20.** Complete the table of $y = \log_{\frac{1}{3}} x$ for $x = \frac{1}{27}, \frac{1}{3}, 1, 3, 9$ and say whether the graph rises or falls from left to right. [3 marks]
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Key sheet

Q	Answer
1	-5
2	Not defined: the base cannot be negative
3	$9 = 27^{\frac{2}{3}}$
4	-1
5	5
6	$3 \log_5 7$
7	$\log_9 5$
8	2
9	$\frac{\log_{10} 12}{\log_{10} 7}$
10	8
11	$3^{\log_2 5}$
12	$x = 36$
13	$x = 16$
14	$x = \sqrt[3]{10}$
15	1
16	11
17	3
18	7
19	$x = 2$
20	$y = 3, 1, 0, -1, -2$; the graph falls

Solutions

1. Find the logarithm of $\frac{1}{32}$ to the base 2. [1]

$$\frac{1}{32} = \frac{1}{2^5} = 2^{-5}$$

$$a^x = N \Rightarrow x = \log_a N$$

$$\therefore \log_2 \frac{1}{32} = -5$$

Ans: -5

2. Is $\log_{-4} 16$ defined? Give a reason. [1]

$$(-4)^2 = 16 \Rightarrow 2 = \log_{-4} 16$$

$$4^2 = 16 \Rightarrow 2 = \log_4 16$$

This makes $\log_{-4} 16 = \log_4 16$, which has no meaning.

$\therefore$ the base cannot be negative, and $\log_{-4} 16$ is not defined.

Ans: Not defined: the base cannot be negative

3. Write $\log_{27} 9 = \frac{2}{3}$ in exponential form. [1]

$$\log_a N = x \Leftrightarrow N = a^x$$

Here $a = 27$, $N = 9$, $x = \frac{2}{3}$.

$$\therefore 9 = 27^{\frac{2}{3}}$$

Ans: $9 = 27^{\frac{2}{3}}$

4. Find $\log_{15} 1 - \log_6 6$. [1]

$$\log_{15} 1 = 0, \log_6 6 = 1$$

$$= 0 - 1 = -1$$

Ans: -1

5. Find the value of $\log_2 96 - \log_2 3$. [2]

$$\log_a m - \log_a n = \log_a \frac{m}{n}$$

$$= \log_2 \frac{96}{3} = \log_2 32$$

$$2^5 = 32 \Rightarrow \log_2 32 = 5$$

Ans: 5

6. Write $\log_{25} 7^6$ in terms of $\log_5 7$. [2]

$$\log_{25} 7^6 = \log_{5^2} 7^6$$

$$= \frac{6}{2} \log_5 7$$

$$= 3 \log_5 7$$

Ans: $3 \log_5 7$

7. Write $\frac{1}{\log_5 9}$ as a single logarithm. [1]

$$\log_b a = \frac{1}{\log_a b}$$

$$\therefore \frac{1}{\log_5 9} = \log_9 5$$

Ans: $\log_9 5$

8. Find the value of $\log_3 4 \cdot \log_5 3 \cdot \log_2 5$. [2]

$$\begin{aligned}\log_b a \cdot \log_c b \cdot \log_d c &= \log_d a \\ &= \log_2 4 \\ &= 2\end{aligned}$$

Ans: 2

9. Write $\log_7 12$ as a quotient of logarithms to the base 10. [1]

$$\begin{aligned}\log_b a &= \frac{\log_x a}{\log_x b} \text{ with } x = 10 \\ \therefore \log_7 12 &= \frac{\log_{10} 12}{\log_{10} 7}\end{aligned}$$

Ans: $\frac{\log_{10} 12}{\log_{10} 7}$

10. Find the value of $2 \times 7^{\log_7 6} - 5^{\log_5 4}$. [2]

$$\begin{aligned}a^{\log_a N} &= N \\ &= 2 \times 6 - 4 = 8\end{aligned}$$

Ans: 8

11. Write $5^{\log_2 3}$ as a power of 3. [2]

$$\begin{aligned}a^{\log_c b} &= b^{\log_c a} \\ \therefore 5^{\log_2 3} &= 3^{\log_2 5}\end{aligned}$$

Ans: $3^{\log_2 5}$

12. If $\log_3 \sqrt{x} = \log_3 6$, find x . [2]

$$\begin{aligned}\log_a m &= \log_a n \Rightarrow m = n \\ \Rightarrow \sqrt{x} &= 6 \\ \Rightarrow x &= 6^2 = 36\end{aligned}$$

Ans: $x = 36$

13. If $\log_3 \log_2 \log_4 x = 0$, find x . [3]

$$\begin{aligned}\log_a N &= 0 \Rightarrow N = 1 \\ \Rightarrow \log_2 \log_4 x &= 1 \\ \Rightarrow \log_4 x &= 2^1 = 2 \\ \Rightarrow x &= 4^2 = 16\end{aligned}$$

Ans: $x = 16$

14. Solve $\log_3(x^2 + x + 1) + \log_3(x - 1) = 2$. [5]

$$\begin{aligned}\log_a m + \log_a n &= \log_a mn \\ \Rightarrow \log_3[(x^2 + x + 1)(x - 1)] &= 2 \\ (a - b)(a^2 + ab + b^2) &= a^3 - b^3 \\ \Rightarrow \log_3(x^3 - 1) &= 2 \\ \Rightarrow x^3 - 1 &= 3^2 = 9 \\ \Rightarrow x^3 &= 10 \\ \Rightarrow x &= \sqrt[3]{10}\end{aligned}$$

Ans: $x = \sqrt[3]{10}$

15. Find the value of $\log_2 \log_2 \log_3 81$. [3]

$$\log_3 81 = \log_3 3^4 = 4 \log_3 3 = 4$$

$$\log_2 4 = \log_2 2^2 = 2$$

$$\log_2 2 = 1$$

$$\therefore \log_2 \log_2 \log_3 81 = 1$$

Ans: 1

16. Find the value of $\frac{1}{\log_9 3} + \frac{1}{\log_{81} 3} + \frac{1}{\log_{243} 3}$. [5]

$$\log_b a = \frac{1}{\log_a b}$$

$$= \log_3 9 + \log_3 81 + \log_3 243$$

$$= \log_3 (9 \times 81 \times 243)$$

$$= \log_3 (3^2 \times 3^4 \times 3^5) = \log_3 3^{11}$$

$$= 11 \log_3 3 = 11$$

Ans: 11

17. Find the value of $10^{\log_{10} \frac{2}{5}} \cdot 7^{\log_7 \frac{5}{2}} \cdot 2^{\log_4 9}$. [3]

$$a^{\log_a N} = N$$

$$2^{\log_4 9} = 2^{\log_{2^2} 3^2} = 2^{\log_2 3} = 3$$

$$= \frac{2}{5} \times \frac{5}{2} \times 3 = 3$$

Ans: 3

18. If $\log \frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$, find the value of $\frac{a}{b} + \frac{b}{a}$. [5]

$$\log \frac{a+b}{3} = \frac{1}{2} \log ab = \log \sqrt{ab}$$

$$\Rightarrow \frac{a+b}{3} = \sqrt{ab}$$

$$\Rightarrow a + b = 3\sqrt{ab}$$

Squaring on both sides,

$$\Rightarrow a^2 + 2ab + b^2 = 9ab$$

$$\Rightarrow a^2 + b^2 = 7ab$$

$$\Rightarrow \frac{a^2}{ab} + \frac{b^2}{ab} = 7$$

$$\Rightarrow \frac{a}{b} + \frac{b}{a} = 7$$

Ans: 7

19. Solve $\log_2(x+2) = \log_4(x+14)$. [5]

$$\log_4(x+14) = \log_{2^2}(x+14) = \frac{1}{2} \log_2(x+14)$$

$$\Rightarrow 2 \log_2(x+2) = \log_2(x+14)$$

$$\Rightarrow \log_2(x+2)^2 = \log_2(x+14)$$

$$\Rightarrow (x+2)^2 = x+14$$

$$\Rightarrow x^2 + 4x + 4 = x + 14$$

$$\Rightarrow x^2 + 3x - 10 = 0$$

$$\Rightarrow x^2 + 5x - 2x - 10 = 0$$

$$\Rightarrow x(x+5) - 2(x+5) = 0$$

$$\Rightarrow (x-2)(x+5) = 0$$

$$\Rightarrow x = 2 \text{ (or) } x = -5$$

For $x = -5$, $\log_2(x+2)$ is not defined.

$$\therefore x = 2$$

Ans: $x = 2$

20. Complete the table of $y = \log_{\frac{1}{3}} x$ for $x = \frac{1}{27}, \frac{1}{3}, 1, 3, 9$ and say whether the graph rises or falls from left to right. [3]

$$y = \log_{\frac{1}{3}} x$$

$$x = \frac{1}{27} : y = \log_{\frac{1}{3}} \frac{1}{27} = 3$$

$$x = \frac{1}{3} : y = \log_{\frac{1}{3}} \frac{1}{3} = 1$$

$$x = 1 : y = \log_{\frac{1}{3}} 1 = 0$$

$$x = 3 : y = \log_{\frac{1}{3}} 3 = -1$$

$$x = 9 : y = \log_{\frac{1}{3}} 9 = -2$$

The base is between 0 and 1, so the graph falls from left to right.

Ans: $y = 3, 1, 0, -1, -2$; the graph falls
