

Logarithms

Question Paper, Set 10

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the value of $\log_{21} 441$. [1 mark]
2. Is $\log_{-2}(-8)$ defined? Give a reason. [1 mark]
3. Write $\log_3 2187 = 7$ in exponential form. [1 mark]
4. Find $\log_{\sqrt{2}} \sqrt{2} - 5 \log_{\sqrt{2}} 1$. [1 mark]
5. Find the value of $\log_{20} 16 + \log_{20} 25$. [2 marks]
6. Find the value of $\log_{27} 81$. [2 marks]
7. Find the value of $\frac{1}{\log_{1000} 10}$. [1 mark]
8. Find the value of $\log_{10} 32 \cdot \log_2 10$. [2 marks]
9. Write $\frac{\log 243}{\log 81}$ as a single logarithm and find its value. [1 mark]
10. Find the value of $\frac{1}{2} \cdot 12^{\log_{12} 30}$. [2 marks]
11. Find the value of $12^{\log_{10} 5} - 5^{\log_{10} 12}$. [2 marks]
12. If $\log_7 \sqrt[4]{x} = \log_7 2$, find x . [2 marks]
13. If $\log_7(x^2 - 5x + 13) = 1$, find x . [3 marks]
14. Solve $\log_2(x + 2) + \log_2(x + 6) = 5$. [5 marks]
15. Find the value of $\log_{27} 243 - \log_{243} 27$. [3 marks]
16. If $\log_{b^3} a \cdot \log_{\sqrt{c}} b \cdot \log_{d^2} c = k \log_d a$, find k . [5 marks]
17. Find the value of $5^{1+\log_5 7} - 3^{\log_9 4}$. [3 marks]
18. If $m^2 + n^2 = 14mn$, show that $2 \log(m + n) = \log m + \log n + 4 \log 2$. [5 marks]
19. Solve $\log_{10}(5x + 4) = \log_{0.1} x$. [5 marks]

- 20.** Complete the table of $y = \log_6 x$ for $x = \frac{1}{36}, \frac{1}{6}, 1, 6, 36$ and say whether the graph rises or falls from left to right. [3 marks]
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Key sheet

Q	Answer
1	2
2	Not defined: the base and the number are both negative
3	$2187 = 3^7$
4	1
5	2
6	$\frac{4}{3}$
7	3
8	5
9	$\log_{81} 243 = \frac{5}{4}$
10	15
11	0
12	$x = 16$
13	$x = 2$ (or) $x = 3$
14	$x = 2$
15	$\frac{16}{15}$
16	$k = \frac{1}{3}$
17	33
18	Shown: $2 \log(m + n) = \log m + \log n + 4 \log 2$
19	$x = \frac{1}{5}$
20	$y = -2, -1, 0, 1, 2$; the graph rises

Solutions

1. Find the value of $\log_{21} 441$. [1]

$$21^2 = 441$$

$$a^x = N \Rightarrow x = \log_a N$$

$$\therefore \log_{21} 441 = 2$$

Ans: 2

2. Is $\log_{-2}(-8)$ defined? Give a reason. [1]

The base must satisfy $a > 0$, $a \neq 1$; here $a = -2 < 0$.

The number must satisfy $N > 0$; here $N = -8 < 0$.

$\therefore \log_{-2}(-8)$ is not defined.

Ans: Not defined: the base and the number are both negative

3. Write $\log_3 2187 = 7$ in exponential form. [1]

$$\log_a N = x \Leftrightarrow N = a^x$$

Here $a = 3$, $N = 2187$, $x = 7$.

$$\therefore 2187 = 3^7$$

Ans: $2187 = 3^7$

4. Find $\log_{\sqrt{2}} \sqrt{2} - 5 \log_{\sqrt{2}} 1$. [1]

$$\log_a a = 1, \log_a 1 = 0 \text{ with } a = \sqrt{2}$$

$$= 1 - 5(0) = 1$$

Ans: 1

5. Find the value of $\log_{20} 16 + \log_{20} 25$. [2]

$$\log_a m + \log_a n = \log_a mn$$

$$= \log_{20}(16 \times 25) = \log_{20} 400$$

$$20^2 = 400 \Rightarrow \log_{20} 400 = 2$$

Ans: 2

6. Find the value of $\log_{27} 81$. [2]

$$\log_{27} 81 = \log_{3^3} 3^4$$

$$= \frac{4}{3} \log_3 3$$

$$= \frac{4}{3}(1) = \frac{4}{3}$$

Ans: $\frac{4}{3}$

7. Find the value of $\frac{1}{\log_{1000} 10}$. [1]

$$\frac{1}{\log_{1000} 10} = \log_{10} 1000$$

$$= \log_{10} 10^3 = 3 \log_{10} 10 = 3$$

Ans: 3

8. Find the value of $\log_{10} 32 \cdot \log_2 10$. [2]

$$\begin{aligned}\log_b a \cdot \log_c b &= \log_c a \\ &= \log_2 32 \\ &= 5\end{aligned}$$

Ans: 5

9. Write $\frac{\log 243}{\log 81}$ as a single logarithm and find its value. [1]

$$\begin{aligned}\frac{\log a}{\log b} &= \log_b a \\ &= \log_{81} 243 \\ &= \frac{5}{4}\end{aligned}$$

Ans: $\log_{81} 243 = \frac{5}{4}$

10. Find the value of $\frac{1}{2} \cdot 12^{\log_{12} 30}$. [2]

$$\begin{aligned}12^{\log_{12} 30} &= 30 \text{ since } a^{\log_a N} = N \\ &= \frac{1}{2} \times 30 = 15\end{aligned}$$

Ans: 15

11. Find the value of $12^{\log_{10} 5} - 5^{\log_{10} 12}$. [2]

$$\begin{aligned}a^{\log_c b} &= b^{\log_c a} \\ 12^{\log_{10} 5} &= 5^{\log_{10} 12} \\ &= 5^{\log_{10} 12} - 5^{\log_{10} 12} = 0\end{aligned}$$

Ans: 0

12. If $\log_7 \sqrt[4]{x} = \log_7 2$, find x . [2]

$$\begin{aligned}\log_a m &= \log_a n \Rightarrow m = n \\ \Rightarrow \sqrt[4]{x} &= 2 \\ \Rightarrow x &= 2^4 = 16\end{aligned}$$

Ans: $x = 16$

13. If $\log_7(x^2 - 5x + 13) = 1$, find x . [3]

$$\begin{aligned}\log_a N &= x \Rightarrow N = a^x \\ \Rightarrow x^2 - 5x + 13 &= 7 \\ \Rightarrow x^2 - 5x + 6 &= 0 \\ \Rightarrow x^2 - 3x - 2x + 6 &= 0 \\ \Rightarrow x(x - 3) - 2(x - 3) &= 0 \\ \Rightarrow (x - 2)(x - 3) &= 0 \\ \Rightarrow x &= 2 \text{ (or) } x = 3\end{aligned}$$

Ans: $x = 2$ (or) $x = 3$

14. Solve $\log_2(x+2) + \log_2(x+6) = 5$. [5]

$$\log_a m + \log_a n = \log_a mn$$

$$\Rightarrow \log_2(x+2)(x+6) = 5$$

$$\Rightarrow \log_2(x^2 + 8x + 12) = 5$$

$$\Rightarrow x^2 + 8x + 12 = 2^5 = 32$$

$$\Rightarrow x^2 + 8x - 20 = 0$$

$$\Rightarrow x^2 + 10x - 2x - 20 = 0$$

$$\Rightarrow x(x+10) - 2(x+10) = 0$$

$$\Rightarrow (x-2)(x+10) = 0$$

$$\Rightarrow x = 2 \text{ (or) } x = -10$$

If $x = -10$, $\log_2(x+2)$ is not defined.

$$\therefore x \neq -10$$

$$\therefore x = 2$$

Ans: $x = 2$

15. Find the value of $\log_{27} 243 - \log_{243} 27$. [3]

$$= \log_{3^3} 3^5 - \log_{3^5} 3^3$$

$$= \frac{5}{3} \log_3 3 - \frac{3}{5} \log_3 3$$

$$= \frac{5}{3} - \frac{3}{5}$$

$$= \frac{16}{15}$$

Ans: $\frac{16}{15}$

16. If $\log_{b^3} a \cdot \log_{\sqrt{c}} b \cdot \log_{d^2} c = k \log_d a$, find k . [5]

$$\sqrt{c} = c^{\frac{1}{2}}$$

$$\log_{b^3} a = \frac{1}{3} \log_b a, \log_{c^{\frac{1}{2}}} b = 2 \log_c b, \log_{d^2} c = \frac{1}{2} \log_d c$$

$$= \frac{1}{3} \times 2 \times \frac{1}{2} \log_b a \log_c b \log_d c$$

$$= \frac{1}{3} \log_d a$$

$$\therefore k = \frac{1}{3}$$

Ans: $k = \frac{1}{3}$

17. Find the value of $5^{1+\log_5 7} - 3^{\log_9 4}$. [3]

$$5^{1+\log_5 7} = 5 \times 7 = 35$$

$$3^{\log_9 4} = 3^{\log_{3^2} 2^2} = 3^{\log_3 2} = 2$$

$$= 35 - 2 = 33$$

Ans: 33

18. If $m^2 + n^2 = 14mn$, show that $2 \log(m + n) = \log m + \log n + 4 \log 2$. [5]

Given $m^2 + n^2 = 14mn$

$$LHS = 2 \log(m + n) = \log(m + n)^2$$

$$= \log(m^2 + n^2 + 2mn)$$

$$= \log(14mn + 2mn) = \log(16mn)$$

$$= \log 16 + \log m + \log n$$

$$= \log 2^4 + \log m + \log n = 4 \log 2 + \log m + \log n$$

$$= RHS$$

Ans: Shown: $2 \log(m + n) = \log m + \log n + 4 \log 2$

19. Solve $\log_{10}(5x + 4) = \log_{0.1} x$. [5]

$$0.1 = 10^{-1}$$

$$\log_{10}(5x + 4) = \log_{10^{-1}} x = \frac{1}{-1} \log_{10} x = \log_{10} x^{-1}$$

$$\Rightarrow 5x + 4 = x^{-1} = \frac{1}{x}$$

$$\Rightarrow 5x^2 + 4x = 1$$

$$\Rightarrow 5x^2 + 4x - 1 = 0$$

$$\Rightarrow 5x^2 - x + 5x - 1 = 0$$

$$\Rightarrow x(5x - 1) + 1(5x - 1) = 0$$

$$\Rightarrow (x + 1)(5x - 1) = 0$$

$$\Rightarrow x = -1 \text{ (or) } x = \frac{1}{5}$$

For $x = -1$, $\log_{0.1} x$ is not defined.

$$\therefore x = \frac{1}{5}$$

Ans: $x = \frac{1}{5}$

20. Complete the table of $y = \log_6 x$ for $x = \frac{1}{36}, \frac{1}{6}, 1, 6, 36$ and say whether the graph rises or falls from left to right. [3]

$$y = \log_6 x$$

$$x = \frac{1}{36} : y = \log_6 \frac{1}{36} = -2$$

$$x = \frac{1}{6} : y = \log_6 \frac{1}{6} = -1$$

$$x = 1 : y = \log_6 1 = 0$$

$$x = 6 : y = \log_6 6 = 1$$

$$x = 36 : y = \log_6 36 = 2$$

The base is more than 1, so the graph rises from left to right.

Ans: $y = -2, -1, 0, 1, 2$; the graph rises
