

Logarithms

Question Paper, Set 01

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the value of $\log_3 81$. [1 mark]
2. Of $\log_1 6$ and $\log_6 1$, which one is defined? Find its value. [1 mark]
3. Write $2^{10} = 1024$ in logarithmic form. [1 mark]
4. Find $4\log_9 9 + 7\log_9 1$. [1 mark]
5. Find the value of $\log_6 4 + \log_6 9$. [2 marks]
6. Find the value of $\log_8 32$. [2 marks]
7. Find the value of $\log_{11} 4 \times \log_4 11$. [1 mark]
8. Find the value of $\log_5 9 \cdot \log_3 5$. [2 marks]
9. Write $\frac{\log 3125}{\log 5}$ as a single logarithm and find its value. [1 mark]
10. Find the value of $3^{\log_3 8} + 4^{\log_4 5}$. [2 marks]
11. Find the value of $3^{\log_5 7} - 7^{\log_5 3}$. [2 marks]
12. If $\log_7(3x - 2) = \log_7 13$, find x . [2 marks]
13. If $\log_3(x^2 - 4x + 6) = 1$, find x . [3 marks]
14. Solve $\log_6 x + \log_6(x + 5) = 2$. [5 marks]
15. Find the value of $\log_8 16 - \log_{16} 8$. [3 marks]
16. If $\log_{\sqrt[3]{b}} a \cdot \log_{\sqrt[5]{c}} b \cdot \log_{\sqrt{d}} c = k \log_d a$, find k . [5 marks]
17. Find the value of $3^{2+\log_3 5}$. [3 marks]
18. If $x^2 + y^2 = 7xy$, show that $2\log(x + y) = \log x + \log y + 2\log 3$. [5 marks]
19. Solve $\log_2(6x - 1) = \log_{\frac{1}{2}} x$. [5 marks]
20. Complete the table of $y = \log_2 x$ for $x = \frac{1}{8}, \frac{1}{2}, 1, 4, 16$. [3 marks]

Key sheet

Q	Answer
1	4
2	$\log_6 1 = 0$ ($\log_1 6$ is not defined)
3	$10 = \log_2 1024$
4	4
5	2
6	$\frac{5}{3}$
7	1
8	2
9	$\log_5 3125 = 5$
10	13
11	0
12	$x = 5$
13	$x = 1$ (or) $x = 3$
14	$x = 4$
15	$\frac{7}{12}$
16	$k = 30$
17	45
18	Shown: $2 \log(x + y) = \log x + \log y + 2 \log 3$
19	$x = \frac{1}{2}$
20	$y = -3, -1, 0, 2, 4$

Solutions

1. Find the value of $\log_3 81$. [1]

$$3^4 = 81$$

$$a^x = N \Rightarrow x = \log_a N$$

$$\therefore \log_3 81 = 4$$

Ans: 4

2. Of $\log_1 6$ and $\log_6 1$, which one is defined? Find its value. [1]

The base cannot be 1, so $\log_1 6$ is not defined.

$$6^0 = 1 \Rightarrow 0 = \log_6 1$$

$$\therefore \log_6 1 = 0$$

Ans: $\log_6 1 = 0$ ($\log_1 6$ is not defined)

3. Write $2^{10} = 1024$ in logarithmic form. [1]

$$a^x = N \Leftrightarrow x = \log_a N$$

Here $a = 2$, $x = 10$, $N = 1024$.

$$\therefore 10 = \log_2 1024$$

Ans: $10 = \log_2 1024$

4. Find $4\log_9 9 + 7\log_9 1$. [1]

$$\log_9 9 = 1, \log_9 1 = 0$$

$$= 4(1) + 7(0) = 4$$

Ans: 4

5. Find the value of $\log_6 4 + \log_6 9$. [2]

$$\log_a m + \log_a n = \log_a mn$$

$$= \log_6 (4 \times 9) = \log_6 36$$

$$6^2 = 36 \Rightarrow \log_6 36 = 2$$

Ans: 2

6. Find the value of $\log_8 32$. [2]

$$\log_8 32 = \log_{2^3} 2^5$$

$$= \frac{5}{3} \log_2 2$$

$$= \frac{5}{3} (1) = \frac{5}{3}$$

Ans: $\frac{5}{3}$

7. Find the value of $\log_{11} 4 \times \log_4 11$. [1]

$$\log_b a \cdot \log_a b = 1$$

$$\therefore \log_{11} 4 \times \log_4 11 = 1$$

Ans: 1

8. Find the value of $\log_5 9 \cdot \log_3 5$. [2]

$$\begin{aligned}\log_b a \cdot \log_c b &= \log_c a \\ &= \log_3 9 \\ &= 2\end{aligned}$$

Ans: 2

9. Write $\frac{\log 3125}{\log 5}$ as a single logarithm and find its value. [1]

$$\begin{aligned}\frac{\log a}{\log b} &= \log_b a \\ &= \log_5 3125 \\ &= 5\end{aligned}$$

Ans: $\log_5 3125 = 5$

10. Find the value of $3^{\log_3 8} + 4^{\log_4 5}$. [2]

$$\begin{aligned}a^{\log_a N} &= N \\ &= 8 + 5 = 13\end{aligned}$$

Ans: 13

11. Find the value of $3^{\log_5 7} - 7^{\log_5 3}$. [2]

$$\begin{aligned}a^{\log_c b} &= b^{\log_c a} \\ 3^{\log_5 7} &= 7^{\log_5 3} \\ &= 7^{\log_5 3} - 7^{\log_5 3} = 0\end{aligned}$$

Ans: 0

12. If $\log_7(3x - 2) = \log_7 13$, find x . [2]

$$\begin{aligned}\log_a m &= \log_a n \Rightarrow m = n \\ \Rightarrow 3x - 2 &= 13 \\ \Rightarrow 3x &= 15 \\ \Rightarrow x &= 5\end{aligned}$$

Ans: $x = 5$

13. If $\log_3(x^2 - 4x + 6) = 1$, find x . [3]

$$\begin{aligned}\log_a N &= x \Rightarrow N = a^x \\ \Rightarrow x^2 - 4x + 6 &= 3 \\ \Rightarrow x^2 - 4x + 3 &= 0 \\ \Rightarrow x^2 - 3x - x + 3 &= 0 \\ \Rightarrow x(x - 3) - 1(x - 3) &= 0 \\ \Rightarrow (x - 1)(x - 3) &= 0 \\ \Rightarrow x = 1 \text{ (or) } x &= 3\end{aligned}$$

Ans: $x = 1$ (or) $x = 3$

14. Solve $\log_6 x + \log_6(x + 5) = 2$. [5]

$$\log_a m + \log_a n = \log_a mn$$

$$\Rightarrow \log_6 x(x + 5) = 2$$

$$\Rightarrow \log_6(x^2 + 5x) = 2$$

$$\Rightarrow x^2 + 5x = 6^2 = 36$$

$$\Rightarrow x^2 + 5x - 36 = 0$$

$$\Rightarrow x^2 + 9x - 4x - 36 = 0$$

$$\Rightarrow x(x + 9) - 4(x + 9) = 0$$

$$\Rightarrow (x - 4)(x + 9) = 0$$

$$\Rightarrow x = 4 \text{ (or) } x = -9$$

If $x = -9$, $\log_6 x$ is not defined.

$$\therefore x \neq -9$$

$$\therefore x = 4$$

Ans: $x = 4$

15. Find the value of $\log_8 16 - \log_{16} 8$. [3]

$$= \log_{2^3} 2^4 - \log_{2^4} 2^3$$

$$= \frac{4}{3} \log_2 2 - \frac{3}{4} \log_2 2$$

$$= \frac{4}{3} - \frac{3}{4}$$

$$= \frac{7}{12}$$

Ans: $\frac{7}{12}$

16. If $\log_{\sqrt[3]{b}} a \cdot \log_{\sqrt[5]{c}} b \cdot \log_{\sqrt{d}} c = k \log_d a$, find k . [5]

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$= \log_{b^{\frac{1}{3}}} a \cdot \log_{c^{\frac{1}{5}}} b \cdot \log_{d^{\frac{1}{2}}} c$$

$$= 3 \log_b a \cdot 5 \log_c b \cdot 2 \log_d c$$

$$= 3 \times 5 \times 2 \cdot \log_b a \log_c b \log_d c$$

$$= 30 \log_d a$$

$$\therefore k = 30$$

Ans: $k = 30$

17. Find the value of $3^{2+\log_3 5}$. [3]

$$a^{m+n} = a^m \cdot a^n$$

$$= 3^2 \times 3^{\log_3 5}$$

$$= 9 \times 5 = 45$$

Ans: 45

18. If $x^2 + y^2 = 7xy$, show that $2 \log(x + y) = \log x + \log y + 2 \log 3$. [5]

$$\text{Given } x^2 + y^2 = 7xy$$

$$LHS = 2 \log(x + y) = \log(x + y)^2$$

$$= \log(x^2 + y^2 + 2xy)$$

$$= \log(7xy + 2xy) = \log(9xy)$$

$$= \log 9 + \log x + \log y$$

$$= \log 3^2 + \log x + \log y = 2 \log 3 + \log x + \log y$$

$$= RHS$$

$$\text{Ans: Shown: } 2 \log(x + y) = \log x + \log y + 2 \log 3$$

19. Solve $\log_2(6x - 1) = \log_{\frac{1}{2}} x$. [5]

$$\frac{1}{2} = 2^{-1}$$

$$\log_2(6x - 1) = \log_{2^{-1}} x = \frac{1}{-1} \log_2 x = \log_2 x^{-1}$$

$$\Rightarrow 6x - 1 = x^{-1} = \frac{1}{x}$$

$$\Rightarrow 6x^2 - x = 1$$

$$\Rightarrow 6x^2 - x - 1 = 0$$

$$\Rightarrow 6x^2 + 2x - 3x - 1 = 0$$

$$\Rightarrow 2x(3x + 1) - 1(3x + 1) = 0$$

$$\Rightarrow (2x - 1)(3x + 1) = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ (or) } x = -\frac{1}{3}$$

For $x = -\frac{1}{3}$, $\log_{\frac{1}{2}} x$ is not defined.

$$\therefore x = \frac{1}{2}$$

$$\text{Ans: } x = \frac{1}{2}$$

20. Complete the table of $y = \log_2 x$ for $x = \frac{1}{8}, \frac{1}{2}, 1, 4, 16$. [3]

$$y = \log_2 x$$

$$x = \frac{1}{8} : y = \log_2 \frac{1}{8} = -3$$

$$x = \frac{1}{2} : y = \log_2 \frac{1}{2} = -1$$

$$x = 1 : y = \log_2 1 = 0$$

$$x = 4 : y = \log_2 4 = 2$$

$$x = 16 : y = \log_2 16 = 4$$

$$\text{Ans: } y = -3, -1, 0, 2, 4$$
