EXPRESSWALA ACADEMY

Maths 1A: Trigonometric Ratios

Question Paper, Set 09

Time: 1 hour 30 minutes Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. In which quadrant does the point $(11, -1)$ lie? Give the reason. [1 mark]

2. Express 216° in grades. [1 mark]

3. Express $131^\circ 15'$ in radians. [2 marks]

4. An arc of a circle of radius 42 cm subtends 210° at the centre. Find the length of the arc (take $\pi = \frac{22}{7}$). [2 marks]

5. The terminal side of an angle θ passes through $(-60, -11)$. Find $\sin\theta$, $\cos\theta$ and $\tan\theta$. [2 marks]

6. If $\cot\theta = \frac{33}{56}$ and θ lies in the third quadrant, find the other five trigonometric ratios. [3 marks]

7. If $\sec\theta + \tan\theta = \frac{8}{5}$, find $\sec\theta$ and $\tan\theta$. [3 marks]

8. Prove that $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$. [5 marks]

9. In which quadrant does θ lie if $\tan\theta > 0$ and $\cos\theta < 0$? [1 mark]

10. Evaluate $5\cos^2 60^\circ - 2\,\mathrm{cosec}^2\, 60^\circ + 3\sec^2 0^\circ$. [2 marks]

11. Find the value of $\sin 210^\circ$. [1 mark]

12. Evaluate $\cos(930^\circ) + \mathrm{cosec}(930^\circ) - \sec(840^\circ)$. [3 marks]

13. Simplify $\dfrac{\tan(90^\circ + \theta)\sec(360^\circ - \theta)\tan(180^\circ - \theta)}{\sec(180^\circ - \theta)\sec(270^\circ + \theta)}$. [5 marks]

14. Find the range of $3\cos x + 9$. [1 mark]

15. Arrange $\sin(-\frac{2}{5})$, $-\frac{2}{5}$ and $\tan(-\frac{2}{5})$ in increasing order, giving the reason. [2 marks]

16. Three angles of a quadrilateral are 69°, 52° and 126°. Find the fourth angle in radians. [1 mark]

17. If $\sin\theta + \cos\theta = \frac{6}{5}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5 marks]

18. If $\tan\theta = \frac{7}{24}$, find the value of $\dfrac{-\sin\theta + 5\cos\theta}{2\sin\theta - 3\cos\theta}$. [3 marks]

19. If $x = 2\,\text{cosec}\,\theta + 10\cot\theta$ and $y = 2\cot\theta + 10\,\text{cosec}\,\theta$, prove that $x^2 - y^2 = -96$. Hence find y^2 when $x = 10$. [5 marks]

20. Solve $6x - 3y - 9 = 0$ and $2x + 2y + 12 = 0$ by cross multiplication. [2 marks]

Key sheet

Q	Answer
1	The fourth quadrant
2	240^g
3	$\frac{35\pi}{48}$
4	154 cm
5	$\sin\theta = -\frac{11}{61}$, $\cos\theta = -\frac{60}{61}$, $\tan\theta = \frac{11}{60}$
6	$\sin\theta = -\frac{56}{65}$, $\cos\theta = -\frac{33}{65}$, $\tan\theta = \frac{56}{33}$, $\sec\theta = -\frac{65}{33}$, $\operatorname{cosec}\theta = -\frac{65}{56}$
7	$\sec\theta = \frac{89}{80}$, $\tan\theta = \frac{39}{80}$
8	Proved
9	The third quadrant
10	$\frac{19}{12}$
11	$-\frac{1}{2}$
12	$-\frac{\sqrt{3}}{2}$
13	$-\sin\theta$
14	$[6, 12]$
15	$\tan(-\frac{2}{5}) < -\frac{2}{5} < \sin(-\frac{2}{5})$
16	$\frac{113\pi}{180}$
17	$\frac{11}{50}$, $\frac{1129}{1250}$, $\frac{2137}{2500}$
18	$-\frac{113}{58}$
19	$x^2 - y^2 = -96$; $y^2 = 196$
20	$x = -1$, $y = -5$

Solutions

1. In which quadrant does the point $(11, -1)$ lie? Give the reason. [1]

Here $x > 0$ and $y < 0$.

So the point is in the fourth quadrant.

Ans: The fourth quadrant

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2. Express 216° in grades. [1]

$90^\circ = 100^g \Rightarrow 1^\circ = \frac{10}{9}\,^g$

$216^\circ = 216 \times \frac{10}{9} = 240^g$

Ans: 240^g

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3. Express $131^\circ 15'$ in radians. [2]

$15' = \frac{15}{60}\,^\circ = \frac{1}{4}^\circ$

$131^\circ 15' = \frac{525}{4}^\circ$

$= \frac{525}{4} \times \frac{\pi}{180} = \frac{35\pi}{48}$

Ans: $\frac{35\pi}{48}$

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4. An arc of a circle of radius 42 cm subtends 210° at the centre. Find the length of the arc (take $\pi = \frac{22}{7}$). [2]

$\theta = 210^\circ = 210 \times \frac{\pi}{180}$

$l = r\theta = 42 \times \frac{7}{6} \times \frac{22}{7} = 154$ cm

Ans: 154 **cm**

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5. The terminal side of an angle θ passes through $(-60, -11)$. Find $\sin\theta$, $\cos\theta$ and $\tan\theta$. [2]

$r = \sqrt{(-60)^2 + (-11)^2} = 61$

$\sin\theta = -\frac{11}{61}$

$\cos\theta = -\frac{60}{61}$

$\tan\theta = \frac{11}{60}$

Ans: $\sin\theta = -\frac{11}{61}$, $\cos\theta = -\frac{60}{61}$, $\tan\theta = \frac{11}{60}$

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6. If $\cot\theta = \frac{33}{56}$ and θ lies in the third quadrant, find the other five trigonometric ratios. [3]

Take $x = -33$, $y = -56$, $r = 65$, with the signs of x and y fixed by the third quadrant.

$\sin\theta = -\frac{56}{65}$, $\cos\theta = -\frac{33}{65}$, $\tan\theta = \frac{56}{33}$, $\sec\theta = -\frac{65}{33}$, $\operatorname{cosec}\theta = -\frac{65}{56}$

Ans: $\sin\theta = -\frac{56}{65}$, $\cos\theta = -\frac{33}{65}$, $\tan\theta = \frac{56}{33}$, $\sec\theta = -\frac{65}{33}$, $\operatorname{cosec}\theta = -\frac{65}{56}$

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7. If $\sec\theta + \tan\theta = \frac{8}{5}$, find $\sec\theta$ and $\tan\theta$. [3]

$\sec^2\theta - \tan^2\theta = 1 \Rightarrow \sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta} = \frac{5}{8}$

Adding: $2\sec\theta = \frac{8}{5} + \frac{5}{8} = \frac{89}{40}$, so $\sec\theta = \frac{89}{80}$

Subtracting: $2\tan\theta = \frac{39}{40}$, so $\tan\theta = \frac{39}{80}$

Ans: $\sec\theta = \frac{89}{80}$, $\tan\theta = \frac{39}{80}$

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8. Prove that $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$. [5]

LHS $= \sec^2\theta(\sec^2\theta - 1) = (1 + \tan^2\theta)\tan^2\theta$

$= \tan^2\theta + \tan^4\theta =$ RHS

Ans: Proved

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9. In which quadrant does θ lie if $\tan\theta > 0$ and $\cos\theta < 0$? [1]

$\tan\theta > 0$ in quadrants 1 and 3.

$\cos\theta < 0$ in quadrants 2 and 3.

Both hold only in the third quadrant.

Ans: The third quadrant

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10. Evaluate $5\cos^2 60^\circ - 2\,\mathrm{cosec}^2 60^\circ + 3\sec^2 0^\circ$. [2]

$\cos 60^\circ = \frac{1}{2}$, $\mathrm{cosec}\,60^\circ = \frac{2\sqrt{3}}{3}$, $\sec 0^\circ = 1$

$5\cos^2 60^\circ - 2\,\mathrm{cosec}^2 60^\circ + 3\sec^2 0^\circ = 5\left(\frac{1}{4}\right) - 2\left(\frac{4}{3}\right) + 3\,(1)$

$= \frac{19}{12}$

Ans: $\frac{19}{12}$

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11. Find the value of $\sin 210^\circ$. [1]

$\sin 210^\circ = \sin(180^\circ + 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$

Ans: $-\frac{1}{2}$

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12. Evaluate $\cos(930^\circ) + \mathrm{cosec}(930^\circ) - \sec(840^\circ)$. [3]

$\cos(930^\circ) = -\frac{\sqrt{3}}{2}$

$\mathrm{cosec}(930^\circ) = -2$

$\sec(840^\circ) = -2$

Value $= -\frac{\sqrt{3}}{2}$

Ans: $-\frac{\sqrt{3}}{2}$

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13. Simplify $\dfrac{\tan(90^\circ + \theta)\sec(360^\circ - \theta)\tan(180^\circ - \theta)}{\sec(180^\circ - \theta)\sec(270^\circ + \theta)}$. [5]

Reduce each ratio (the ratio changes for $90^\circ \pm \theta$ and $270^\circ \pm \theta$; the sign is that of the quadrant):

$\tan(90^\circ + \theta) = -\cot\theta$

$\sec(360^\circ - \theta) = \sec\theta$

$\tan(180^\circ - \theta) = -\tan\theta$

$\sec(180^\circ - \theta) = -\sec\theta$

$\sec(270^\circ + \theta) = \mathrm{cosec}\,\theta$

$$\text{Expression} = \frac{(-\cot\theta)\cdot(\sec\theta)\cdot(-\tan\theta)}{(-\sec\theta)\cdot(\mathrm{cosec}\,\theta)}$$

$= -\sin\theta$

Ans: $-\sin\theta$

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14. Find the range of $3\cos x + 9$. [1]

$-1 \le \cos x \le 1$

$6 \le 3\cos x + 9 \le 12$

Ans: $[6, 12]$

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15. Arrange $\sin(-\frac{2}{5})$, $-\frac{2}{5}$ and $\tan(-\frac{2}{5})$ in increasing order, giving the reason. [2]

$-\frac{2}{5}$ lies in $\left(-\frac{\pi}{2}, 0\right)$ since $\frac{\pi}{2} \approx 1.57$.

For such x, $\tan x < x < \sin x$.

So $\tan(-\frac{2}{5}) < -\frac{2}{5} < \sin(-\frac{2}{5})$.

Ans: $\tan(-\frac{2}{5}) < -\frac{2}{5} < \sin(-\frac{2}{5})$

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16. Three angles of a quadrilateral are 69°, 52° and 126°. Find the fourth angle in radians. [1]

The angles add up to 360°: fourth $= 360^\circ - 247^\circ = 113^\circ$.

$113^\circ = 113 \times \frac{\pi}{180} = \frac{113\pi}{180}$

Ans: $\frac{113\pi}{180}$

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17. If $\sin\theta + \cos\theta = \frac{6}{5}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5]

$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta = \frac{36}{25}$

$\sin\theta\cos\theta = \frac{11}{50}$

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta = 1 - 2\left(\frac{11}{50}\right)^2 = \frac{1129}{1250}$

$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta = 1 - 3\left(\frac{11}{50}\right)^2 = \frac{2137}{2500}$

Ans: $\frac{11}{50}$, $\frac{1129}{1250}$, $\frac{2137}{2500}$

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18. If $\tan\theta = \frac{7}{24}$, find the value of $\dfrac{-\sin\theta + 5\cos\theta}{2\sin\theta - 3\cos\theta}$. [3]

Divide numerator and denominator by $\cos\theta$:

$\frac{-\sin\theta+5\cos\theta}{2\sin\theta-3\cos\theta} = \frac{-\tan\theta+5}{2\tan\theta-3}$

$= \frac{\frac{113}{24}}{-\frac{29}{12}} = -\frac{113}{58}$

Ans: $-\frac{113}{58}$

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19. If $x = 2\operatorname{cosec}\theta + 10\cot\theta$ and $y = 2\cot\theta + 10\operatorname{cosec}\theta$, prove that $x^2 - y^2 = -96$. Hence find y^2 when $x = 10$. [5]

$x^2 = 4\operatorname{cosec}^2\theta + 40\operatorname{cosec}\theta\cot\theta + 100\cot^2\theta$

$y^2 = 4\cot^2\theta + 40\operatorname{cosec}\theta\cot\theta + 100\operatorname{cosec}^2\theta$

Subtracting, the middle terms cancel: $x^2 - y^2 = 4(\operatorname{cosec}^2\theta - \cot^2\theta) - 100(\operatorname{cosec}^2\theta - \cot^2\theta) = 4 - 100 = -96$

$x = 10 \Rightarrow y^2 = 100 - (-96) = 196$

Ans: $x^2 - y^2 = -96$; $y^2 = 196$

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20. Solve $6x - 3y - 9 = 0$ and $2x + 2y + 12 = 0$ by cross multiplication. [2]

$a_1 = 6, b_1 = -3, c_1 = -9;\ a_2 = 2, b_2 = 2, c_2 = 12$

$b_1c_2 - b_2c_1 = -18,\ c_1a_2 - c_2a_1 = -90,\ a_1b_2 - a_2b_1 = 18$

$x = \frac{-18}{18} = -1, \quad y = \frac{-90}{18} = -5$

Ans: $x = -1$, $y = -5$
