EXPRESSWALA ACADEMY

Maths 1A: Trigonometric Ratios

Question Paper, Set 07

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. In which quadrant does the point $(-10, 10)$ lie? Give the reason. [1 mark]

2. Express 40° in radians. [1 mark]

3. Express $17^\circ 12'$ in radians. [2 marks]

4. A wire of length 22 cm is bent into an arc of a circle of radius 14 cm. Find the angle subtended at the centre in degrees (take $\pi = \frac{22}{7}$). [2 marks]

5. The terminal side of an angle θ passes through $(16, 63)$. Find $\sin\theta$, $\cos\theta$ and $\tan\theta$. [2 marks]

6. If $\cos\theta = -\frac{55}{73}$ and θ lies in the third quadrant, find the other five trigonometric ratios. [3 marks]

7. If $\sec\theta + \tan\theta = \frac{7}{4}$, find $\sec\theta$ and $\tan\theta$. [3 marks]

8. Prove that $\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \frac{1+\cos\theta}{\sin\theta}$. [5 marks]

9. In which quadrant does θ lie if $\operatorname{cosec}\theta > 0$ and $\cot\theta > 0$? [1 mark]

10. Evaluate $5\sin^2 30^\circ + 3\sin^2 45^\circ - \sin^2 60^\circ$. [2 marks]

11. Find the value of $\sec 330^\circ$. [1 mark]

12. Evaluate $\sec(495^\circ) + \sec(570^\circ) - \sec(210^\circ)$. [3 marks]

13. Simplify $\dfrac{\cos(180^\circ + \theta)\sin(270^\circ + \theta)}{\cos(180^\circ - \theta)\cos(-\theta)\sin(-\theta)}$. [5 marks]

14. Find the range of $6\sin x - 9$. [1 mark]

15. Arrange $\sin(\frac{4}{5})$, $\frac{4}{5}$ and $\tan(\frac{4}{5})$ in increasing order, giving the reason. [2 marks]

16. If $A + B = 90^\circ$ and $\sin A = \frac{11}{61}$, find $\cos B$. [1 mark]

17. If $\sin\theta + \cos\theta = \frac{8}{7}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5 marks]

18. If $\tan\theta = \frac{4}{3}$, find the value of $\dfrac{-2\sin\theta + 4\cos\theta}{4\sin\theta - 5\cos\theta}$. [3 marks]

19. If $x = 10\cos\theta + 9\sin\theta$ and $y = 10\sin\theta - 9\cos\theta$, prove that $x^2 + y^2 = 181$. Hence find y^2 when $x = 8$. [5 marks]

20. Solve $-3x - 4y - 2 = 0$ and $-2x + 2y + 8 = 0$ by cross multiplication. [2 marks]

Key sheet

Q	Answer
1	The second quadrant
2	$\frac{2\pi}{9}$
3	$\frac{43\pi}{450}$
4	90°
5	$\sin\theta = \frac{63}{65}$, $\cos\theta = \frac{16}{65}$, $\tan\theta = \frac{63}{16}$
6	$\sin\theta = -\frac{48}{73}$, $\tan\theta = \frac{48}{55}$, $\cot\theta = \frac{55}{48}$, $\sec\theta = -\frac{73}{55}$, $\operatorname{cosec}\theta = -\frac{73}{48}$
7	$\sec\theta = \frac{65}{56}$, $\tan\theta = \frac{33}{56}$
8	Proved
9	The first quadrant
10	2
11	$\frac{2\sqrt{3}}{3}$
12	$-\sqrt{2}$
13	$\operatorname{cosec}\theta$
14	$[-15, -3]$
15	$\sin\frac{4}{5} < \frac{4}{5} < \tan\frac{4}{5}$
16	$\frac{11}{61}$
17	$\frac{15}{98}$, $\frac{4577}{4802}$, $\frac{8929}{9604}$
18	4
19	$x^2 + y^2 = 181$; $y^2 = 117$
20	$x = 2$, $y = -2$

Solutions

1. In which quadrant does the point $(-10, 10)$ lie? Give the reason. [1]

Here $x < 0$ and $y > 0$.

So the point is in the second quadrant.

Ans: The second quadrant

---

2. Express 40° in radians. [1]

$1^\circ = \frac{\pi}{180}$

$40^\circ = 40 \times \frac{\pi}{180} = \frac{2\pi}{9}$

Ans: $\frac{2\pi}{9}$

---

3. Express $17^\circ 12'$ in radians. [2]

$12' = \frac{12}{60}^\circ = \frac{1}{5}^\circ$

$17^\circ 12' = \frac{86}{5}^\circ$

$= \frac{86}{5} \times \frac{\pi}{180} = \frac{43\pi}{450}$

Ans: $\frac{43\pi}{450}$

---

4. A wire of length 22 cm is bent into an arc of a circle of radius 14 cm. Find the angle subtended at the centre in degrees (take $\pi = \frac{22}{7}$). [2]

$\theta = \frac{l}{r} = \frac{22}{14} = \frac{11}{7}$ radians

$= \frac{11}{7} \times \frac{180}{\pi} = \frac{11}{7} \times 180 \times \frac{7}{22} = 90^\circ$

Ans: 90°

---

5. The terminal side of an angle θ passes through $(16, 63)$. Find $\sin\theta$, $\cos\theta$ and $\tan\theta$. [2]

$r = \sqrt{(16)^2 + (63)^2} = 65$

$\sin\theta = \frac{63}{65}$

$\cos\theta = \frac{16}{65}$

$\tan\theta = \frac{63}{16}$

Ans: $\sin\theta = \frac{63}{65}$, $\cos\theta = \frac{16}{65}$, $\tan\theta = \frac{63}{16}$

---

6. If $\cos\theta = -\frac{55}{73}$ and θ lies in the third quadrant, find the other five trigonometric ratios. [3]

Take $x = -55$, $y = -48$, $r = 73$, with the signs of x and y fixed by the third quadrant.

$\sin\theta = -\frac{48}{73}$, $\tan\theta = \frac{48}{55}$, $\cot\theta = \frac{55}{48}$, $\sec\theta = -\frac{73}{55}$, $\operatorname{cosec}\theta = -\frac{73}{48}$

Ans: $\sin\theta = -\frac{48}{73}$, $\tan\theta = \frac{48}{55}$, $\cot\theta = \frac{55}{48}$, $\sec\theta = -\frac{73}{55}$, $\operatorname{cosec}\theta = -\frac{73}{48}$

---

7. If $\sec\theta + \tan\theta = \frac{7}{4}$, find $\sec\theta$ and $\tan\theta$. [3]

$\sec^2\theta - \tan^2\theta = 1 \Rightarrow \sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta} = \frac{4}{7}$

Adding: $2\sec\theta = \frac{7}{4} + \frac{4}{7} = \frac{65}{28}$, so $\sec\theta = \frac{65}{56}$

Subtracting: $2\tan\theta = \frac{33}{28}$, so $\tan\theta = \frac{33}{56}$

Ans: $\sec\theta = \frac{65}{56}$, $\tan\theta = \frac{33}{56}$

---

8. Prove that $\frac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}=\frac{1+\cos\theta}{\sin\theta}$. [5]

Write $1=\operatorname{cosec}^2\theta-\cot^2\theta$ in the numerator:

Numerator $=(\operatorname{cosec}\theta+\cot\theta)(1-\operatorname{cosec}\theta+\cot\theta)$

So LHS $=\operatorname{cosec}\theta+\cot\theta=\frac{1+\cos\theta}{\sin\theta}=$ RHS

Ans: Proved

---

9. In which quadrant does θ lie if $\operatorname{cosec}\theta>0$ and $\cot\theta>0$? [1]

$\operatorname{cosec}\theta>0$ in quadrants 1 and 2.

$\cot\theta>0$ in quadrants 1 and 3.

Both hold only in the first quadrant.

Ans: The first quadrant

---

10. Evaluate $5\sin^2 30^\circ+3\sin^2 45^\circ-\sin^2 60^\circ$. [2]

$\sin 30^\circ=\frac{1}{2}$, $\sin 45^\circ=\frac{\sqrt{2}}{2}$, $\sin 60^\circ=\frac{\sqrt{3}}{2}$

$5\sin^2 30^\circ+3\sin^2 45^\circ-\sin^2 60^\circ=5\left(\frac{1}{4}\right)+3\left(\frac{1}{2}\right)-\left(\frac{3}{4}\right)$

$=2$

Ans: 2

---

11. Find the value of $\sec 330^\circ$. [1]

$\sec 330^\circ=\sec(360^\circ-30^\circ)=\sec 30^\circ=\frac{2\sqrt{3}}{3}$

Ans: $\frac{2\sqrt{3}}{3}$

---

12. Evaluate $\sec(495^\circ)+\sec(570^\circ)-\sec(210^\circ)$. [3]

$\sec(495^\circ)=-\sqrt{2}$

$\sec(570^\circ)=-\frac{2\sqrt{3}}{3}$

$\sec(210^\circ)=-\frac{2\sqrt{3}}{3}$

Value $=-\sqrt{2}$

Ans: $-\sqrt{2}$

---

13. Simplify $\dfrac{\cos(180^\circ+\theta)\sin(270^\circ+\theta)}{\cos(180^\circ-\theta)\cos(-\theta)\sin(-\theta)}$. [5]

Reduce each ratio (the ratio changes for $90^\circ\pm\theta$ and $270^\circ\pm\theta$; the sign is that of the quadrant):

$\cos(180^\circ+\theta)=-\cos\theta$

$\sin(270^\circ+\theta)=-\cos\theta$

$\cos(180^\circ-\theta)=-\cos\theta$

$\cos(-\theta)=\cos\theta$

$\sin(-\theta)=-\sin\theta$

$\text{Expression}=\dfrac{(-\cos\theta)\cdot(-\cos\theta)}{(-\cos\theta)\cdot(\cos\theta)\cdot(-\sin\theta)}$

$=\operatorname{cosec}\theta$

Ans: $\operatorname{cosec}\theta$

---

14. Find the range of $6\sin x - 9$. [1]

$-1 \le \sin x \le 1$

$-15 \le 6\sin x - 9 \le -3$

Ans: $[-15, -3]$

---

15. Arrange $\sin(\frac{4}{5})$, $\frac{4}{5}$ and $\tan(\frac{4}{5})$ in increasing order, giving the reason. [2]

$\frac{4}{5}$ lies in $\left(0, \frac{\pi}{2}\right)$ since $\frac{\pi}{2} \approx 1.57$.

For such x, $\sin x < x < \tan x$.

So $\sin\frac{4}{5} < \frac{4}{5} < \tan\frac{4}{5}$.

Ans: $\sin\frac{4}{5} < \frac{4}{5} < \tan\frac{4}{5}$

---

16. If $A + B = 90^\circ$ and $\sin A = \frac{11}{61}$, find $\cos B$. [1]

$B = 90^\circ - A$

$\cos B = \cos(90^\circ - A) = \sin A = \frac{11}{61}$

Ans: $\frac{11}{61}$

---

17. If $\sin\theta + \cos\theta = \frac{8}{7}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5]

$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta = \frac{64}{49}$

$\sin\theta\cos\theta = \frac{15}{98}$

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta = 1 - 2\left(\frac{15}{98}\right)^2 = \frac{4577}{4802}$

$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta = 1 - 3\left(\frac{15}{98}\right)^2 = \frac{8929}{9604}$

Ans: $\frac{15}{98}$, $\frac{4577}{4802}$, $\frac{8929}{9604}$

---

18. If $\tan\theta = \frac{4}{3}$, find the value of $\dfrac{-2\sin\theta + 4\cos\theta}{4\sin\theta - 5\cos\theta}$. [3]

Divide numerator and denominator by $\cos\theta$:

$\frac{-2\sin\theta + 4\cos\theta}{4\sin\theta - 5\cos\theta} = \frac{-2\tan\theta + 4}{4\tan\theta - 5}$

$= \frac{\frac{4}{3}}{\frac{1}{3}} = 4$

Ans: 4

---

19. If $x = 10\cos\theta + 9\sin\theta$ and $y = 10\sin\theta - 9\cos\theta$, prove that $x^2 + y^2 = 181$. Hence find y^2 when $x = 8$. [5]

$x^2 = 100\cos^2\theta + 180\sin\theta\cos\theta + 81\sin^2\theta$

$y^2 = 100\sin^2\theta - 180\sin\theta\cos\theta + 81\cos^2\theta$

Adding, the middle terms cancel: $x^2 + y^2 = 100(\cos^2\theta + \sin^2\theta) + 81(\sin^2\theta + \cos^2\theta) = 181$

$x = 8 \Rightarrow y^2 = 181 - 64 = 117$

Ans: $x^2 + y^2 = 181$; $y^2 = 117$

---

20. Solve $-3x - 4y - 2 = 0$ and $-2x + 2y + 8 = 0$ by cross multiplication. [2]

$a_1 = -3, b_1 = -4, c_1 = -2;\ a_2 = -2, b_2 = 2, c_2 = 8$

$b_1c_2 - b_2c_1 = -28,\ c_1a_2 - c_2a_1 = 28,\ a_1b_2 - a_2b_1 = -14$

$x = \frac{-28}{-14} = 2, \quad y = \frac{28}{-14} = -2$

Ans: $x = 2, y = -2$
