EXPRESSWALA ACADEMY

Maths 1A: Trigonometric Ratios

Question Paper, Set 05

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. In which quadrant does the point $(3, 12)$ lie? Give the reason. [1 mark]

2. Express $\frac{9\pi}{4}$ radians in degrees. [1 mark]

3. Express $98^\circ 20'$ in radians. [2 marks]

4. An arc of a circle of radius 35 cm subtends 36° at the centre. Find the length of the arc (take $\pi = \frac{22}{7}$). [2 marks]

5. The terminal side of an angle θ passes through $(-48, -14)$. Find $\sin\theta$, $\cos\theta$ and $\tan\theta$. [2 marks]

6. If $\sec\theta = \frac{73}{48}$ and θ lies in the fourth quadrant, find the other five trigonometric ratios. [3 marks]

7. If $\sec\theta + \tan\theta = \frac{9}{5}$, find $\sec\theta$ and $\tan\theta$. [3 marks]

8. Prove that $\frac{\tan\theta}{\sec\theta - 1} + \frac{\tan\theta}{\sec\theta + 1} = 2\,\mathrm{cosec}\,\theta$. [5 marks]

9. In which quadrant does θ lie if $\cot\theta < 0$ and $\sin\theta > 0$? [1 mark]

10. Evaluate $5\sin^2 45^\circ + \mathrm{cosec}^2\, 60^\circ + 5\sec^2 45^\circ$. [2 marks]

11. Find the value of $\mathrm{cosec}\, 210^\circ$. [1 mark]

12. Evaluate $\cos(405^\circ) + \sec(315^\circ) - \sec(-300^\circ)$. [3 marks]

13. Simplify $\dfrac{\sec(270^\circ + \theta)\cot(360^\circ - \theta)\sin(90^\circ + \theta)}{\sec(180^\circ - \theta)\tan(270^\circ + \theta)\,\mathrm{cosec}(180^\circ + \theta)}$. [5 marks]

14. Find the range of $-4\cos x + 9$. [1 mark]

15. Arrange $\sin(-\frac{11}{10})$, $-\frac{11}{10}$ and $\tan(-\frac{11}{10})$ in increasing order, giving the reason. [2 marks]

16. If $A + B = 180^\circ$ and $\cos A = \frac{5}{8}$, find $\cos B$. [1 mark]

17. If $\sin\theta + \cos\theta = \frac{7}{6}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5 marks]

18. If $\tan\theta = \frac{3}{4}$, find the value of $\dfrac{\sin\theta - 4\cos\theta}{6\sin\theta + 6\cos\theta}$. [3 marks]

19. If $x = 3\sec\theta + 2\tan\theta$ and $y = 3\tan\theta + 2\sec\theta$, prove that $x^2 - y^2 = 5$. Hence find y^2 when $x = 12$. [5 marks]

20. Solve $-x + 5y - 19 = 0$ and $2x + 6y - 42 = 0$ by cross multiplication. [2 marks]

Key sheet

Q	Answer
1	The first quadrant
2	405°
3	$\frac{59\pi}{108}$
4	22 cm
5	$\sin\theta = -\frac{7}{25}$, $\cos\theta = -\frac{24}{25}$, $\tan\theta = \frac{7}{24}$
6	$\sin\theta = -\frac{55}{73}$, $\cos\theta = \frac{48}{73}$, $\tan\theta = -\frac{55}{48}$, $\cot\theta = -\frac{48}{55}$, $\operatorname{cosec}\theta = -\frac{73}{55}$
7	$\sec\theta = \frac{53}{45}$, $\tan\theta = \frac{28}{45}$
8	Proved
9	The second quadrant
10	$\frac{83}{6}$
11	-2
12	$\frac{3\sqrt{2}}{2} - 2$
13	$\cos^2\theta$
14	$[5, 13]$
15	$\tan(-\frac{11}{10}) < -\frac{11}{10} < \sin(-\frac{11}{10})$
16	$-\frac{5}{8}$
17	$\frac{13}{72}$, $\frac{2423}{2592}$, $\frac{1559}{1728}$
18	$-\frac{13}{42}$
19	$x^2 - y^2 = 5; y^2 = 139$
20	$x = 6, y = 5$

Solutions

1. In which quadrant does the point $(3, 12)$ lie? Give the reason. [1]

Here $x > 0$ and $y > 0$.

So the point is in the first quadrant.

Ans: The first quadrant

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2. Express $\frac{9\pi}{4}$ radians in degrees. [1]

$\pi^c = 180^\circ$

$\frac{9\pi}{4} = \frac{9}{4} \times 180^\circ = 405^\circ$

Ans: 405°

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3. Express $98^\circ 20'$ in radians. [2]

$20' = \frac{20}{60}^\circ = \frac{1}{3}^\circ$

$98^\circ 20' = \frac{295}{3}^\circ$

$= \frac{295}{3} \times \frac{\pi}{180} = \frac{59\pi}{108}$

Ans: $\frac{59\pi}{108}$

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4. An arc of a circle of radius 35 cm subtends 36° at the centre. Find the length of the arc (take $\pi = \frac{22}{7}$). [2]

$\theta = 36^\circ = 36 \times \frac{\pi}{180}$

$l = r\theta = 35 \times \frac{1}{5} \times \frac{22}{7} = 22$ cm

Ans: 22 cm

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5. The terminal side of an angle θ passes through $(-48, -14)$. Find $\sin\theta$, $\cos\theta$ and $\tan\theta$. [2]

$r = \sqrt{(-48)^2 + (-14)^2} = 50$

$\sin\theta = -\frac{7}{25}$

$\cos\theta = -\frac{24}{25}$

$\tan\theta = \frac{7}{24}$

Ans: $\sin\theta = -\frac{7}{25}$, $\cos\theta = -\frac{24}{25}$, $\tan\theta = \frac{7}{24}$

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6. If $\sec\theta = \frac{73}{48}$ and θ lies in the fourth quadrant, find the other five trigonometric ratios. [3]

Take $x = 48$, $y = -55$, $r = 73$, with the signs of x and y fixed by the fourth quadrant.

$\sin\theta = -\frac{55}{73}$, $\cos\theta = \frac{48}{73}$, $\tan\theta = -\frac{55}{48}$, $\cot\theta = -\frac{48}{55}$, $\operatorname{cosec}\theta = -\frac{73}{55}$

Ans: $\sin\theta = -\frac{55}{73}$, $\cos\theta = \frac{48}{73}$, $\tan\theta = -\frac{55}{48}$, $\cot\theta = -\frac{48}{55}$, $\operatorname{cosec}\theta = -\frac{73}{55}$

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7. If $\sec\theta + \tan\theta = \frac{9}{5}$, find $\sec\theta$ and $\tan\theta$. [3]

$\sec^2\theta - \tan^2\theta = 1 \Rightarrow \sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta} = \frac{5}{9}$

Adding: $2\sec\theta = \frac{9}{5} + \frac{5}{9} = \frac{106}{45}$, so $\sec\theta = \frac{53}{45}$

Subtracting: $2\tan\theta = \frac{56}{45}$, so $\tan\theta = \frac{28}{45}$

Ans: $\sec\theta = \frac{53}{45}$, $\tan\theta = \frac{28}{45}$

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8. Prove that $\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}=2\operatorname{cosec}\theta$. [5]

LHS $=\frac{\tan\theta(\sec\theta+1+\sec\theta-1)}{\sec^2\theta-1}=\frac{2\tan\theta\sec\theta}{\tan^2\theta}$

$=\frac{2\sec\theta}{\tan\theta}=\frac{2}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta=$ RHS

Ans: Proved

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9. In which quadrant does θ lie if $\cot\theta<0$ and $\sin\theta>0$? [1]

$\cot\theta<0$ in quadrants 2 and 4.

$\sin\theta>0$ in quadrants 1 and 2.

Both hold only in the second quadrant.

Ans: The second quadrant

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10. Evaluate $5\sin^2 45^\circ+\operatorname{cosec}^2 60^\circ+5\sec^2 45^\circ$. [2]

$\sin 45^\circ=\frac{\sqrt{2}}{2}$, $\operatorname{cosec}60^\circ=\frac{2\sqrt{3}}{3}$, $\sec 45^\circ=\sqrt{2}$

$5\sin^2 45^\circ+\operatorname{cosec}^2 60^\circ+5\sec^2 45^\circ=5\left(\frac{1}{2}\right)+\left(\frac{4}{3}\right)+5\,(2)$

$=\frac{83}{6}$

Ans: $\frac{83}{6}$

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11. Find the value of $\operatorname{cosec}210^\circ$. [1]

$\operatorname{cosec}210^\circ=\operatorname{cosec}(180^\circ+30^\circ)=-\operatorname{cosec}30^\circ=-2$

Ans: -2

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12. Evaluate $\cos(405^\circ)+\sec(315^\circ)-\sec(-300^\circ)$. [3]

$\cos(405^\circ)=\frac{\sqrt{2}}{2}$

$\sec(315^\circ)=\sqrt{2}$

$\sec(-300^\circ)=2$

Value $=\frac{3\sqrt{2}}{2}-2$

Ans: $\frac{3\sqrt{2}}{2}-2$

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13. Simplify $\dfrac{\sec(270^\circ+\theta)\cot(360^\circ-\theta)\sin(90^\circ+\theta)}{\sec(180^\circ-\theta)\tan(270^\circ+\theta)\operatorname{cosec}(180^\circ+\theta)}$. [5]

Reduce each ratio (the ratio changes for $90^\circ\pm\theta$ and $270^\circ\pm\theta$; the sign is that of the quadrant):

$\sec(270^\circ+\theta)=\operatorname{cosec}\theta$

$\cot(360^\circ-\theta)=-\cot\theta$

$\sin(90^\circ+\theta)=\cos\theta$

$\sec(180^\circ-\theta)=-\sec\theta$

$\tan(270^\circ+\theta)=-\cot\theta$

$\operatorname{cosec}(180^\circ+\theta)=-\operatorname{cosec}\theta$

$\text{Expression}=\dfrac{(\operatorname{cosec}\theta)\cdot(-\cot\theta)\cdot(\cos\theta)}{(-\sec\theta)\cdot(-\cot\theta)\cdot(-\operatorname{cosec}\theta)}$

$=\cos^2\theta$

Ans: $\cos^2\theta$

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14. Find the range of $-4\cos x + 9$. [1]

$-1 \le \cos x \le 1$

$5 \le -4\cos x + 9 \le 13$

Ans: $[5, 13]$

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15. Arrange $\sin(-\frac{11}{10})$, $-\frac{11}{10}$ and $\tan(-\frac{11}{10})$ in increasing order, giving the reason. [2]

$-\frac{11}{10}$ lies in $\left(-\frac{\pi}{2}, 0\right)$ since $\frac{\pi}{2} \approx 1.57$.

For such x, $\tan x < x < \sin x$.

So $\tan(-\frac{11}{10}) < -\frac{11}{10} < \sin(-\frac{11}{10})$.

Ans: $\tan(-\frac{11}{10}) < -\frac{11}{10} < \sin(-\frac{11}{10})$

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16. If $A + B = 180^\circ$ and $\cos A = \frac{5}{8}$, find $\cos B$. [1]

$B = 180^\circ - A$

$\cos B = \cos(180^\circ - A) = -\cos A = -\frac{5}{8}$

Ans: $-\frac{5}{8}$

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17. If $\sin\theta + \cos\theta = \frac{7}{6}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5]

$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta = \frac{49}{36}$

$\sin\theta\cos\theta = \frac{13}{72}$

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta = 1 - 2\left(\frac{13}{72}\right)^2 = \frac{2423}{2592}$

$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta = 1 - 3\left(\frac{13}{72}\right)^2 = \frac{1559}{1728}$

Ans: $\frac{13}{72}$, $\frac{2423}{2592}$, $\frac{1559}{1728}$

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18. If $\tan\theta = \frac{3}{4}$, find the value of $\dfrac{\sin\theta - 4\cos\theta}{6\sin\theta + 6\cos\theta}$. [3]

Divide numerator and denominator by $\cos\theta$:

$\frac{\sin\theta - 4\cos\theta}{6\sin\theta + 6\cos\theta} = \frac{\tan\theta - 4}{6\tan\theta + 6}$

$= \frac{-\frac{13}{4}}{\frac{21}{2}} = -\frac{13}{42}$

Ans: $-\frac{13}{42}$

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19. If $x = 3\sec\theta + 2\tan\theta$ and $y = 3\tan\theta + 2\sec\theta$, prove that $x^2 - y^2 = 5$. Hence find y^2 when $x = 12$. [5]

$x^2 = 9\sec^2\theta + 12\sec\theta\tan\theta + 4\tan^2\theta$

$y^2 = 9\tan^2\theta + 12\sec\theta\tan\theta + 4\sec^2\theta$

Subtracting, the middle terms cancel: $x^2 - y^2 = 9(\sec^2\theta - \tan^2\theta) - 4(\sec^2\theta - \tan^2\theta) = 9 - 4 = 5$

$x = 12 \Rightarrow y^2 = 144 - (5) = 139$

Ans: $x^2 - y^2 = 5$; $y^2 = 139$

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20. Solve $-x + 5y - 19 = 0$ and $2x + 6y - 42 = 0$ by cross multiplication. [2]

$a_1 = -1, b_1 = 5, c_1 = -19;\ a_2 = 2, b_2 = 6, c_2 = -42$

$b_1c_2 - b_2c_1 = -96,\ c_1a_2 - c_2a_1 = -80,\ a_1b_2 - a_2b_1 = -16$

$x = \frac{-96}{-16} = 6, \quad y = \frac{-80}{-16} = 5$

Ans: $x = 6$, $y = 5$
