EXPRESSWALA ACADEMY

Maths 1A: Trigonometric Ratios

Question Paper, Set 04

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the distances of the point $(1, 7)$ from the x-axis and from the y-axis. [1 mark]

2. Express 305° in radians. [1 mark]

3. The angles of a triangle are in the ratio $3 : 8 : 4$. Find them in degrees and in radians. [2 marks]

4. A wheel makes 75 revolutions per minute. Through how many radians does it turn in one second? [2 marks]

5. If the point $(70, 24)$ is on the terminal side of θ, find $\operatorname{cosec}\theta$, $\sec\theta$ and $\cot\theta$. [2 marks]

6. If $\sec\theta = -\frac{29}{21}$ and θ lies in the third quadrant, find the other five trigonometric ratios. [3 marks]

7. If $\operatorname{cosec}\theta - \cot\theta = \frac{4}{9}$, find $\operatorname{cosec}\theta$ and $\cot\theta$. [3 marks]

8. Prove that $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$. [5 marks]

9. In which quadrant does θ lie if $\operatorname{cosec}\theta > 0$ and $\tan\theta < 0$? [1 mark]

10. Evaluate $4\sec^2 45^\circ + 3\cos^2 45^\circ + \cos^2 30^\circ$. [2 marks]

11. Find the value of $\operatorname{cosec} 765^\circ$. [1 mark]

12. Evaluate $\operatorname{cosec}(240^\circ)\cdot\cos(330^\circ) + \sin(-150^\circ)$. [3 marks]

13. Simplify $\dfrac{\tan(90^\circ-\theta)\cos(270^\circ+\theta)}{\sin(270^\circ+\theta)\cot(-\theta)\sec(-\theta)}$. [5 marks]

14. Find the range of $6 - 3\sin x$. [1 mark]

15. Is $\tan x$ increasing or decreasing on $\left(\frac{\pi}{2}, \pi\right)$? Give the reason. [2 marks]

16. If $A + B = 90^\circ$ and $\cos A = \frac{40}{41}$, find $\sin B$. [1 mark]

17. If $\sin\theta - \cos\theta = \frac{\sqrt{14}}{5}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5 marks]

18. If $\dfrac{1+\sin\theta}{1-\sin\theta} = 4$, find $\sin\theta$ using componendo and dividendo. [3 marks]

19. If $x = 2\cos\theta + 8\sin\theta$ and $y = 2\sin\theta - 8\cos\theta$, prove that $x^2 + y^2 = 68$. Hence find y^2 when $x = 1$. [5 marks]

20. Find $\sin\theta + \sin(\pi + \theta) + \sin(2\pi + \theta) + \cdots + \sin(6\pi + \theta)$. [2 marks]

Key sheet

Q	Answer
1	7 and 1
2	$\frac{61\pi}{36}$
3	$36^\circ, 96^\circ, 48^\circ$; that is $\frac{\pi}{5}, \frac{8\pi}{15}, \frac{4\pi}{15}$
4	$\frac{5\pi}{2}$
5	$\operatorname{cosec}\theta = \frac{37}{12}$, $\sec\theta = \frac{37}{35}$, $\cot\theta = \frac{35}{12}$
6	$\sin\theta = -\frac{20}{29}$, $\cos\theta = -\frac{21}{29}$, $\tan\theta = \frac{20}{21}$, $\cot\theta = \frac{21}{20}$, $\operatorname{cosec}\theta = -\frac{29}{20}$
7	$\operatorname{cosec}\theta = \frac{97}{72}$, $\cot\theta = \frac{65}{72}$
8	Proved
9	The second quadrant
10	$\frac{41}{4}$
11	$\sqrt{2}$
12	$-\frac{3}{2}$
13	$\sin\theta$
14	$[3, 9]$
15	Increasing
16	$\frac{40}{41}$
17	$\frac{11}{50}$, $\frac{1129}{1250}$, $\frac{2137}{2500}$
18	$\sin\theta = \frac{3}{5}$
19	$x^2 + y^2 = 68$; $y^2 = 67$
20	$\sin\theta$

Solutions

1. Find the distances of the point $(1, 7)$ from the x-axis and from the y-axis. [1]

Distance from the x-axis $= |y| = |7| = 7$.

Distance from the y-axis $= |x| = |1| = 1$.

Ans: 7 **and** 1

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2. Express 305° in radians. [1]

$1^\circ = \frac{\pi}{180}$

$305^\circ = 305 \times \frac{\pi}{180} = \frac{61\pi}{36}$

Ans: $\frac{61\pi}{36}$

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3. The angles of a triangle are in the ratio $3 : 8 : 4$. Find them in degrees and in radians. [2]

Let the angles be $3k, 8k, 4k$. Then $15k = 180^\circ$, so $k = 12^\circ$.

Angles: $36^\circ, 96^\circ, 48^\circ$.

In radians: $\frac{\pi}{5}, \frac{8\pi}{15}, \frac{4\pi}{15}$.

Ans: $36^\circ, 96^\circ, 48^\circ$**; that is** $\frac{\pi}{5}, \frac{8\pi}{15}, \frac{4\pi}{15}$

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4. A wheel makes 75 revolutions per minute. Through how many radians does it turn in one second? [2]

In one second it makes $\frac{75}{60} = \frac{5}{4}$ revolutions.

One revolution is 2π radians.

$\frac{5}{4} \times 2\pi = \frac{5\pi}{2}$

Ans: $\frac{5\pi}{2}$

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5. If the point $(70, 24)$ is on the terminal side of θ, find $\operatorname{cosec}\theta$, $\sec\theta$ and $\cot\theta$. [2]

$r = \sqrt{(70)^2 + (24)^2} = 74$

$\operatorname{cosec}\theta = \frac{37}{12}$

$\sec\theta = \frac{37}{35}$

$\cot\theta = \frac{35}{12}$

Ans: $\operatorname{cosec}\theta = \frac{37}{12}$, $\sec\theta = \frac{37}{35}$, $\cot\theta = \frac{35}{12}$

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6. If $\sec\theta = -\frac{29}{21}$ and θ lies in the third quadrant, find the other five trigonometric ratios. [3]

Take $x = -21$, $y = -20$, $r = 29$, with the signs of x and y fixed by the third quadrant.

$\sin\theta = -\frac{20}{29}$, $\cos\theta = -\frac{21}{29}$, $\tan\theta = \frac{20}{21}$, $\cot\theta = \frac{21}{20}$, $\operatorname{cosec}\theta = -\frac{29}{20}$

Ans: $\sin\theta = -\frac{20}{29}$, $\cos\theta = -\frac{21}{29}$, $\tan\theta = \frac{20}{21}$, $\cot\theta = \frac{21}{20}$, $\operatorname{cosec}\theta = -\frac{29}{20}$

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7. If $\operatorname{cosec}\theta - \cot\theta = \frac{4}{9}$, find $\operatorname{cosec}\theta$ and $\cot\theta$. [3]

$\operatorname{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow \operatorname{cosec}\theta + \cot\theta = \frac{9}{4}$

Adding: $2\operatorname{cosec}\theta = \frac{97}{36}$, so $\operatorname{cosec}\theta = \frac{97}{72}$

Subtracting: $2\cot\theta = \frac{65}{36}$, so $\cot\theta = \frac{65}{72}$

Ans: $\operatorname{cosec}\theta = \frac{97}{72}$, $\cot\theta = \frac{65}{72}$

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8. Prove that $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$. [5]

LHS $= \frac{\cos^2\theta}{\cos\theta-\sin\theta} + \frac{\sin^2\theta}{\sin\theta-\cos\theta}$

$= \frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta} = \frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\cos\theta-\sin\theta}$

$= \sin\theta + \cos\theta =$ RHS

Ans: Proved

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9. In which quadrant does θ lie if $\operatorname{cosec}\theta > 0$ and $\tan\theta < 0$? [1]

$\operatorname{cosec}\theta > 0$ in quadrants 1 and 2.

$\tan\theta < 0$ in quadrants 2 and 4.

Both hold only in the second quadrant.

Ans: The second quadrant

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10. Evaluate $4\sec^2 45^\circ + 3\cos^2 45^\circ + \cos^2 30^\circ$. [2]

$\sec 45^\circ = \sqrt{2}$, $\cos 45^\circ = \frac{\sqrt{2}}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$4\sec^2 45^\circ + 3\cos^2 45^\circ + \cos^2 30^\circ = 4\,(2) + 3\left(\frac{1}{2}\right) + \left(\frac{3}{4}\right)$

$= \frac{41}{4}$

Ans: $\frac{41}{4}$

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11. Find the value of $\operatorname{cosec} 765^\circ$. [1]

$765^\circ = 2 \times 360^\circ + 45^\circ$, so $\operatorname{cosec} 765^\circ = \operatorname{cosec} 45^\circ$.

$\operatorname{cosec} 45^\circ = \sqrt{2}$

Ans: $\sqrt{2}$

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12. Evaluate $\operatorname{cosec}(240^\circ) \cdot \cos(330^\circ) + \sin(-150^\circ)$. [3]

$\operatorname{cosec}(240^\circ) = -\frac{2\sqrt{3}}{3}$

$\cos(330^\circ) = \frac{\sqrt{3}}{2}$

$\sin(-150^\circ) = -\frac{1}{2}$

Value $= -\frac{3}{2}$

Ans: $-\frac{3}{2}$

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13. Simplify $\dfrac{\tan(90^\circ - \theta)\cos(270^\circ + \theta)}{\sin(270^\circ + \theta)\cot(-\theta)\sec(-\theta)}$. [5]

Reduce each ratio (the ratio changes for $90^\circ \pm \theta$ and $270^\circ \pm \theta$; the sign is that of the quadrant):

$\tan(90^\circ - \theta) = \cot\theta$

$\cos(270^\circ + \theta) = \sin\theta$

$\sin(270^\circ + \theta) = -\cos\theta$

$\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$

$\text{Expression} = \dfrac{(\cot\theta)\cdot(\sin\theta)}{(-\cos\theta)\cdot(-\cot\theta)\cdot(\sec\theta)}$

$= \sin\theta$

Ans: $\sin\theta$

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14. Find the range of $6 - 3\sin x$. [1]

$-1 \le \sin x \le 1$

$3 \le 6 - 3\sin x \le 9$

Ans: $[3, 9]$

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15. Is $\tan x$ increasing or decreasing on $\left(\frac{\pi}{2}, \pi\right)$? Give the reason. [2]

$\left(\frac{\pi}{2}, \pi\right)$ is the second quadrant.

As x moves across this quadrant, $\tan x$ rises, so it is increasing there.

Ans: Increasing

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16. If $A + B = 90^\circ$ and $\cos A = \frac{40}{41}$, find $\sin B$. [1]

$B = 90^\circ - A$

$\sin B = \sin(90^\circ - A) = \cos A = \frac{40}{41}$

Ans: $\frac{40}{41}$

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17. If $\sin\theta - \cos\theta = \frac{\sqrt{14}}{5}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5]

$(\sin\theta - \cos\theta)^2 = 1 - 2\sin\theta\cos\theta = \frac{14}{25}$

$\sin\theta\cos\theta = \frac{11}{50}$

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta = 1 - 2\left(\frac{11}{50}\right)^2 = \frac{1129}{1250}$

$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta = 1 - 3\left(\frac{11}{50}\right)^2 = \frac{2137}{2500}$

Ans: $\frac{11}{50}$, $\frac{1129}{1250}$, $\frac{2137}{2500}$

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18. If $\dfrac{1+\sin\theta}{1-\sin\theta} = 4$, find $\sin\theta$ using componendo and dividendo. [3]

$\frac{(1+\sin\theta)+(1-\sin\theta)}{(1+\sin\theta)-(1-\sin\theta)} = \frac{4+1}{4-1}$

$\frac{2}{2\sin\theta} = \frac{5}{3}$

$\sin\theta = \frac{3}{5}$

Ans: $\sin\theta = \frac{3}{5}$

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19. If $x = 2\cos\theta + 8\sin\theta$ and $y = 2\sin\theta - 8\cos\theta$, prove that $x^2 + y^2 = 68$. Hence find y^2 when $x = 1$. [5]

$x^2 = 4\cos^2\theta + 32\sin\theta\cos\theta + 64\sin^2\theta$

$y^2 = 4\sin^2\theta - 32\sin\theta\cos\theta + 64\cos^2\theta$

Adding, the middle terms cancel: $x^2 + y^2 = 4(\cos^2\theta + \sin^2\theta) + 64(\sin^2\theta + \cos^2\theta) = 68$

$x = 1 \Rightarrow y^2 = 68 - 1 = 67$

Ans: $x^2 + y^2 = 68$; $y^2 = 67$

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20. Find $\sin\theta + \sin(\pi + \theta) + \sin(2\pi + \theta) + \cdots + \sin(6\pi + \theta)$. [2]

$\sin(k\pi + \theta) = (-1)^k \sin\theta$

There are 7 terms, which cancel in pairs, leaving the first.

Sum $= \sin\theta$

Ans: $\sin\theta$
