EXPRESSWALA ACADEMY

Maths 1A: Trigonometric Ratios

Question Paper, Set 03

Time: 1 hour 30 minutes Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. In which quadrant does the point $(-10, 13)$ lie? Give the reason. [1 mark]

2. Express 288° in grades. [1 mark]

3. Express $67^\circ 40'$ in radians. [2 marks]

4. A wire of length $\frac{88}{3}$ cm is bent into an arc of a circle of radius 14 cm. Find the angle subtended at the centre in degrees (take $\pi = \frac{22}{7}$). [2 marks]

5. The terminal side of an angle θ passes through $(55, -48)$. Find $\sin\theta$, $\cos\theta$ and $\tan\theta$. [2 marks]

6. If $\operatorname{cosec}\theta = -\frac{73}{48}$ and θ lies in the fourth quadrant, find the other five trigonometric ratios. [3 marks]

7. If $\sec\theta + \tan\theta = \frac{9}{4}$, find $\sec\theta$ and $\tan\theta$. [3 marks]

8. Prove that $\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$. [5 marks]

9. In which quadrant does θ lie if $\cos\theta > 0$ and $\operatorname{cosec}\theta < 0$? [1 mark]

10. Evaluate $-\cos^2 60^\circ + 3\cot^2 30^\circ + 5\tan^2 45^\circ$. [2 marks]

11. Find the value of $\operatorname{cosec} 120^\circ$. [1 mark]

12. Evaluate $\sin(750^\circ) + \cos(240^\circ) - \operatorname{cosec}(-120^\circ)$. [3 marks]

13. Simplify $\dfrac{\operatorname{cosec}(180^\circ+\theta)\tan(180^\circ-\theta)\sin(180^\circ+\theta)}{\cos(180^\circ+\theta)\cot(270^\circ-\theta)}$. [5 marks]

14. Find the range of $-\sin x + 10$. [1 mark]

15. Arrange $\sin(-\frac{1}{5})$, $-\frac{1}{5}$ and $\tan(-\frac{1}{5})$ in increasing order, giving the reason. [2 marks]

16. Three angles of a quadrilateral are 114°, 93° and 69°. Find the fourth angle in radians. [1 mark]

17. If $\sin\theta + \cos\theta = \frac{4}{3}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5 marks]

18. If $\tan\theta = \frac{33}{56}$, find the value of $\dfrac{-2\sin\theta + 5\cos\theta}{4\sin\theta + 3\cos\theta}$. [3 marks]

19. If $x = 2\,\text{cosec}\,\theta + 11\cot\theta$ and $y = 2\cot\theta + 11\,\text{cosec}\,\theta$, prove that $x^2 - y^2 = -117$. Hence find y^2 when $x = 2$. [5 marks]

20. Solve $5x + y + 19 = 0$ and $x - 3y + 23 = 0$ by cross multiplication. [2 marks]

Key sheet

Q	Answer
1	The second quadrant
2	320^g
3	$\frac{203\pi}{540}$
4	120°
5	$\sin\theta = -\frac{48}{73}$, $\cos\theta = \frac{55}{73}$, $\tan\theta = -\frac{48}{55}$
6	$\sin\theta = -\frac{48}{73}$, $\cos\theta = \frac{55}{73}$, $\tan\theta = -\frac{48}{55}$, $\cot\theta = -\frac{55}{48}$, $\sec\theta = \frac{73}{55}$
7	$\sec\theta = \frac{97}{72}$, $\tan\theta = \frac{65}{72}$
8	Proved
9	The fourth quadrant
10	$\frac{55}{4}$
11	$\frac{2\sqrt{3}}{3}$
12	$\frac{2\sqrt{3}}{3}$
13	$\sec\theta$
14	$[9, 11]$
15	$\tan(-\frac{1}{5}) < -\frac{1}{5} < \sin(-\frac{1}{5})$
16	$\frac{7\pi}{15}$
17	$\frac{7}{18}$, $\frac{113}{162}$, $\frac{59}{108}$
18	$\frac{107}{150}$
19	$x^2 - y^2 = -117$; $y^2 = 121$
20	$x = -5$, $y = 6$

Solutions

1. In which quadrant does the point $(-10, 13)$ lie? Give the reason. [1]

Here $x < 0$ and $y > 0$.

So the point is in the second quadrant.

Ans: The second quadrant

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2. Express 288° in grades. [1]

$90^\circ = 100^g \Rightarrow 1^\circ = \frac{10}{9}\,^g$

$288^\circ = 288 \times \frac{10}{9} = 320^g$

Ans: 320^g

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3. Express $67^\circ 40'$ in radians. [2]

$40' = \frac{40}{60}\,^\circ = \frac{2}{3}^\circ$

$67^\circ 40' = \frac{203}{3}^\circ$

$= \frac{203}{3} \times \frac{\pi}{180} = \frac{203\pi}{540}$

Ans: $\frac{203\pi}{540}$

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4. A wire of length $\frac{88}{3}$ cm is bent into an arc of a circle of radius 14 cm. Find the angle subtended at the centre in degrees (take $\pi = \frac{22}{7}$). [2]

$\theta = \frac{l}{r} = \frac{\frac{88}{3}}{14} = \frac{44}{21}$ radians

$= \frac{44}{21} \times \frac{180}{\pi} = \frac{44}{21} \times 180 \times \frac{7}{22} = 120^\circ$

Ans: 120°

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5. The terminal side of an angle θ passes through $(55, -48)$. Find $\sin\theta$, $\cos\theta$ and $\tan\theta$. [2]

$r = \sqrt{(55)^2 + (-48)^2} = 73$

$\sin\theta = -\frac{48}{73}$

$\cos\theta = \frac{55}{73}$

$\tan\theta = -\frac{48}{55}$

Ans: $\sin\theta = -\frac{48}{73}$, $\cos\theta = \frac{55}{73}$, $\tan\theta = -\frac{48}{55}$

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6. If $\operatorname{cosec}\theta = -\frac{73}{48}$ and θ lies in the fourth quadrant, find the other five trigonometric ratios. [3]

Take $x = 165$, $y = -144$, $r = 219$, with the signs of x and y fixed by the fourth quadrant.

$\sin\theta = -\frac{48}{73}$, $\cos\theta = \frac{55}{73}$, $\tan\theta = -\frac{48}{55}$, $\cot\theta = -\frac{55}{48}$, $\sec\theta = \frac{73}{55}$

Ans: $\sin\theta = -\frac{48}{73}$, $\cos\theta = \frac{55}{73}$, $\tan\theta = -\frac{48}{55}$, $\cot\theta = -\frac{55}{48}$, $\sec\theta = \frac{73}{55}$

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7. If $\sec\theta + \tan\theta = \frac{9}{4}$, find $\sec\theta$ and $\tan\theta$. [3]

$\sec^2\theta - \tan^2\theta = 1 \Rightarrow \sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta} = \frac{4}{9}$

Adding: $2\sec\theta = \frac{9}{4} + \frac{4}{9} = \frac{97}{36}$, so $\sec\theta = \frac{97}{72}$

Subtracting: $2\tan\theta = \frac{65}{36}$, so $\tan\theta = \frac{65}{72}$

Ans: $\sec\theta = \frac{97}{72}$, $\tan\theta = \frac{65}{72}$

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8. Prove that $\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$. [5]

LHS $= \frac{\sin\theta(1+\cos\theta)+\sin\theta(1-\cos\theta)}{1-\cos^2\theta}$

$= \frac{2\sin\theta}{\sin^2\theta} = \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta =$ RHS

Ans: Proved

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9. In which quadrant does θ lie if $\cos\theta > 0$ and $\operatorname{cosec}\theta < 0$? [1]

$\cos\theta > 0$ in quadrants 1 and 4.

$\operatorname{cosec}\theta < 0$ in quadrants 3 and 4.

Both hold only in the fourth quadrant.

Ans: The fourth quadrant

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10. Evaluate $-\cos^2 60^\circ + 3\cot^2 30^\circ + 5\tan^2 45^\circ$. [2]

$\cos 60^\circ = \frac{1}{2}$, $\cot 30^\circ = \sqrt{3}$, $\tan 45^\circ = 1$

$-\cos^2 60^\circ + 3\cot^2 30^\circ + 5\tan^2 45^\circ = -\left(\frac{1}{4}\right) + 3\,(3) + 5\,(1)$

$= \frac{55}{4}$

Ans: $\frac{55}{4}$

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11. Find the value of $\operatorname{cosec} 120^\circ$. [1]

$\operatorname{cosec} 120^\circ = \operatorname{cosec}(180^\circ - 60^\circ) = \operatorname{cosec} 60^\circ = \frac{2\sqrt{3}}{3}$

Ans: $\frac{2\sqrt{3}}{3}$

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12. Evaluate $\sin(750^\circ) + \cos(240^\circ) - \operatorname{cosec}(-120^\circ)$. [3]

$\sin(750^\circ) = \frac{1}{2}$

$\cos(240^\circ) = -\frac{1}{2}$

$\operatorname{cosec}(-120^\circ) = -\frac{2\sqrt{3}}{3}$

Value $= \frac{2\sqrt{3}}{3}$

Ans: $\frac{2\sqrt{3}}{3}$

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13. Simplify $\dfrac{\operatorname{cosec}(180^\circ+\theta)\tan(180^\circ-\theta)\sin(180^\circ+\theta)}{\cos(180^\circ+\theta)\cot(270^\circ-\theta)}$. [5]

Reduce each ratio (the ratio changes for $90^\circ \pm \theta$ and $270^\circ \pm \theta$; the sign is that of the quadrant):

$\operatorname{cosec}(180^\circ+\theta) = -\operatorname{cosec}\theta$

$\tan(180^\circ-\theta) = -\tan\theta$

$\sin(180^\circ+\theta) = -\sin\theta$

$\cos(180^\circ+\theta) = -\cos\theta$

$\cot(270^\circ-\theta) = \tan\theta$

$\text{Expression} = \dfrac{(-\operatorname{cosec}\theta)\cdot(-\tan\theta)\cdot(-\sin\theta)}{(-\cos\theta)\cdot(\tan\theta)}$

$= \sec\theta$

Ans: $\sec\theta$

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14. Find the range of $-\sin x + 10$. [1]

$-1 \le \sin x \le 1$

$9 \le -\sin x + 10 \le 11$

Ans: $[9, 11]$

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15. Arrange $\sin(-\frac{1}{5})$, $-\frac{1}{5}$ and $\tan(-\frac{1}{5})$ in increasing order, giving the reason. [2]

$-\frac{1}{5}$ lies in $\left(-\frac{\pi}{2}, 0\right)$ since $\frac{\pi}{2} \approx 1.57$.

For such x, $\tan x < x < \sin x$.

So $\tan(-\frac{1}{5}) < -\frac{1}{5} < \sin(-\frac{1}{5})$.

Ans: $\tan(-\frac{1}{5}) < -\frac{1}{5} < \sin(-\frac{1}{5})$

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16. Three angles of a quadrilateral are 114°, 93° and 69°. Find the fourth angle in radians. [1]

The angles add up to 360°: fourth $= 360^\circ - 276^\circ = 84^\circ$.

$84^\circ = 84 \times \frac{\pi}{180} = \frac{7\pi}{15}$

Ans: $\frac{7\pi}{15}$

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17. If $\sin\theta + \cos\theta = \frac{4}{3}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5]

$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta = \frac{16}{9}$

$\sin\theta\cos\theta = \frac{7}{18}$

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta = 1 - 2\left(\frac{7}{18}\right)^2 = \frac{113}{162}$

$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta = 1 - 3\left(\frac{7}{18}\right)^2 = \frac{59}{108}$

Ans: $\frac{7}{18}$, $\frac{113}{162}$, $\frac{59}{108}$

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18. If $\tan\theta = \frac{33}{56}$, find the value of $\dfrac{-2\sin\theta + 5\cos\theta}{4\sin\theta + 3\cos\theta}$. [3]

Divide numerator and denominator by $\cos\theta$:

$\frac{-2\sin\theta+5\cos\theta}{4\sin\theta+3\cos\theta} = \frac{-2\tan\theta+5}{4\tan\theta+3}$

$= \frac{\frac{107}{28}}{\frac{75}{14}} = \frac{107}{150}$

Ans: $\frac{107}{150}$

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19. If $x = 2\,\mathrm{cosec}\,\theta + 11\cot\theta$ and $y = 2\cot\theta + 11\,\mathrm{cosec}\,\theta$, prove that $x^2 - y^2 = -117$. Hence find y^2 when $x = 2$. [5]

$x^2 = 4\,\mathrm{cosec}^2\theta + 44\,\mathrm{cosec}\,\theta\cot\theta + 121\cot^2\theta$

$y^2 = 4\cot^2\theta + 44\,\mathrm{cosec}\,\theta\cot\theta + 121\,\mathrm{cosec}^2\theta$

Subtracting, the middle terms cancel: $x^2 - y^2 = 4(\mathrm{cosec}^2\theta - \cot^2\theta) - 121(\mathrm{cosec}^2\theta - \cot^2\theta) = 4 - 121 = -117$

$x = 2 \Rightarrow y^2 = 4 - (-117) = 121$

Ans: $x^2 - y^2 = -117$; $y^2 = 121$

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20. Solve $5x + y + 19 = 0$ and $x - 3y + 23 = 0$ by cross multiplication. [2]

$a_1 = 5, b_1 = 1, c_1 = 19;\ a_2 = 1, b_2 = -3, c_2 = 23$

$b_1c_2 - b_2c_1 = 80,\ c_1a_2 - c_2a_1 = -96,\ a_1b_2 - a_2b_1 = -16$

$x = \frac{80}{-16} = -5, \quad y = \frac{-96}{-16} = 6$

Ans: $x = -5$, $y = 6$
