EXPRESSWALA ACADEMY

Maths 1A: Trigonometric Ratios

Question Paper, Set 02

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the distances of the point $(-11, -4)$ from the x-axis and from the y-axis. [1 mark]

2. Express $\frac{4\pi}{3}$ radians in degrees. [1 mark]

3. The angles of a triangle are in the ratio $9 : 5 : 6$. Find them in degrees and in radians. [2 marks]

4. An arc of length 46 cm subtends an angle of $\frac{1}{4}$ radians at the centre of a circle. Find the radius. [2 marks]

5. If the point $(9, 40)$ is on the terminal side of θ, find $\operatorname{cosec}\theta$, $\sec\theta$ and $\cot\theta$. [2 marks]

6. If $\cos\theta = -\frac{24}{25}$ and θ lies in the third quadrant, find the other five trigonometric ratios. [3 marks]

7. If $\operatorname{cosec}\theta - \cot\theta = \frac{5}{7}$, find $\operatorname{cosec}\theta$ and $\cot\theta$. [3 marks]

8. Prove that $\frac{1-\cos\theta}{1+\cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$. [5 marks]

9. In which quadrant does θ lie if $\sec\theta < 0$ and $\cot\theta < 0$? [1 mark]

10. Evaluate $3\tan^2 45^\circ + 3\operatorname{cosec}^2 90^\circ - 4\tan^2 30^\circ$. [2 marks]

11. Find the value of $\cot 120^\circ$. [1 mark]

12. Evaluate $\cos(-315^\circ)\cdot\cos(390^\circ) + \sec(1125^\circ)$. [3 marks]

13. Simplify $\dfrac{\cos(360^\circ - \theta)\sec(-\theta)}{\operatorname{cosec}(90^\circ + \theta)\sec(360^\circ - \theta)}$. [5 marks]

14. Find the range of $10 + 6\cos x$. [1 mark]

15. Is $\sec x$ increasing or decreasing on $\left(\frac{3\pi}{2}, 2\pi\right)$? Give the reason. [2 marks]

16. If $A + B = 180^\circ$ and $\tan A = \frac{25}{7}$, find $\tan B$. [1 mark]

17. If $\sin\theta - \cos\theta = \frac{\sqrt{2}}{3}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5 marks]

18. If $\dfrac{1+\cos\theta}{1-\cos\theta} = \frac{8}{7}$, find $\cos\theta$ using componendo and dividendo. [3 marks]

19. If $x = 6\sec\theta + 7\tan\theta$ and $y = 6\tan\theta + 7\sec\theta$, prove that $x^2 - y^2 = -13$. Hence find y^2 when $x = 8$. [5 marks]

20. Find $\sin\theta + \sin(\pi + \theta) + \sin(2\pi + \theta) + \cdots + \sin(13\pi + \theta)$. [2 marks]

Key sheet

Q	Answer
1	4 and 11
2	240°
3	$81^\circ, 45^\circ, 54^\circ$; that is $\frac{9\pi}{20}, \frac{\pi}{4}, \frac{3\pi}{10}$
4	184 cm
5	$\operatorname{cosec}\theta = \frac{41}{40}$, $\sec\theta = \frac{41}{9}$, $\cot\theta = \frac{9}{40}$
6	$\sin\theta = -\frac{7}{25}$, $\tan\theta = \frac{7}{24}$, $\cot\theta = \frac{24}{7}$, $\sec\theta = -\frac{25}{24}$, $\operatorname{cosec}\theta = -\frac{25}{7}$
7	$\operatorname{cosec}\theta = \frac{37}{35}$, $\cot\theta = \frac{12}{35}$
8	Proved
9	The second quadrant
10	$\frac{14}{3}$
11	$-\frac{\sqrt{3}}{3}$
12	$\frac{\sqrt{2}\sqrt{8\sqrt{3}+19}}{4}$
13	$\cos^2\theta$
14	$[4, 16]$
15	Decreasing
16	$-\frac{25}{7}$
17	$\frac{7}{18}$, $\frac{113}{162}$, $\frac{59}{108}$
18	$\cos\theta = \frac{1}{15}$
19	$x^2 - y^2 = -13$; $y^2 = 77$
20	0

Solutions

1. Find the distances of the point $(-11, -4)$ from the x-axis and from the y-axis. [1]

Distance from the x-axis $= |y| = |-4| = 4$.

Distance from the y-axis $= |x| = |-11| = 11$.

Ans: 4 **and** 11

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2. Express $\frac{4\pi}{3}$ radians in degrees. [1]

$\pi^c = 180^\circ$

$\frac{4\pi}{3} = \frac{4}{3} \times 180^\circ = 240^\circ$

Ans: 240°

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3. The angles of a triangle are in the ratio $9 : 5 : 6$. Find them in degrees and in radians. [2]

Let the angles be $9k, 5k, 6k$. Then $20k = 180^\circ$, so $k = 9^\circ$.

Angles: $81^\circ, 45^\circ, 54^\circ$.

In radians: $\frac{9\pi}{20}, \frac{\pi}{4}, \frac{3\pi}{10}$.

Ans: $81^\circ, 45^\circ, 54^\circ$; **that is** $\frac{9\pi}{20}, \frac{\pi}{4}, \frac{3\pi}{10}$

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4. An arc of length 46 cm subtends an angle of $\frac{1}{4}$ radians at the centre of a circle. Find the radius. [2]

$l = r\theta \Rightarrow r = \frac{l}{\theta}$

$r = \frac{46}{\frac{1}{4}} = 184$ cm

Ans: 184 **cm**

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5. If the point $(9, 40)$ is on the terminal side of θ, find $\operatorname{cosec}\theta$, $\sec\theta$ and $\cot\theta$. [2]

$r = \sqrt{(9)^2 + (40)^2} = 41$

$\operatorname{cosec}\theta = \frac{41}{40}$

$\sec\theta = \frac{41}{9}$

$\cot\theta = \frac{9}{40}$

Ans: $\operatorname{cosec}\theta = \frac{41}{40}$, $\sec\theta = \frac{41}{9}$, $\cot\theta = \frac{9}{40}$

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6. If $\cos\theta = -\frac{24}{25}$ and θ lies in the third quadrant, find the other five trigonometric ratios. [3]

Take $x = -48$, $y = -14$, $r = 50$, with the signs of x and y fixed by the third quadrant.

$\sin\theta = -\frac{7}{25}$, $\tan\theta = \frac{7}{24}$, $\cot\theta = \frac{24}{7}$, $\sec\theta = -\frac{25}{24}$, $\operatorname{cosec}\theta = -\frac{25}{7}$

Ans: $\sin\theta = -\frac{7}{25}$, $\tan\theta = \frac{7}{24}$, $\cot\theta = \frac{24}{7}$, $\sec\theta = -\frac{25}{24}$, $\operatorname{cosec}\theta = -\frac{25}{7}$

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7. If $\operatorname{cosec}\theta - \cot\theta = \frac{5}{7}$, find $\operatorname{cosec}\theta$ and $\cot\theta$. [3]

$\operatorname{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow \operatorname{cosec}\theta + \cot\theta = \frac{7}{5}$

Adding: $2\operatorname{cosec}\theta = \frac{74}{35}$, so $\operatorname{cosec}\theta = \frac{37}{35}$

Subtracting: $2\cot\theta = \frac{24}{35}$, so $\cot\theta = \frac{12}{35}$

Ans: $\operatorname{cosec}\theta = \frac{37}{35}$, $\cot\theta = \frac{12}{35}$

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8. Prove that $\frac{1-\cos\theta}{1+\cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$. [5]

RHS $= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} =$ LHS

Ans: Proved

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9. In which quadrant does θ lie if $\sec\theta < 0$ and $\cot\theta < 0$? [1]

$\sec\theta < 0$ in quadrants 2 and 3.

$\cot\theta < 0$ in quadrants 2 and 4.

Both hold only in the second quadrant.

Ans: The second quadrant

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10. Evaluate $3\tan^2 45^\circ + 3\operatorname{cosec}^2 90^\circ - 4\tan^2 30^\circ$. [2]

$\tan 45^\circ = 1$, $\operatorname{cosec} 90^\circ = 1$, $\tan 30^\circ = \frac{\sqrt{3}}{3}$

$3\tan^2 45^\circ + 3\operatorname{cosec}^2 90^\circ - 4\tan^2 30^\circ = 3(1) + 3(1) - 4\left(\frac{1}{3}\right)$

$= \frac{14}{3}$

Ans: $\frac{14}{3}$

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11. Find the value of $\cot 120^\circ$. [1]

$\cot 120^\circ = \cot(180^\circ - 60^\circ) = -\cot 60^\circ = -\frac{\sqrt{3}}{3}$

Ans: $-\frac{\sqrt{3}}{3}$

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12. Evaluate $\cos(-315^\circ)\cdot\cos(390^\circ) + \sec(1125^\circ)$. [3]

$\cos(-315^\circ) = \frac{\sqrt{2}}{2}$

$\cos(390^\circ) = \frac{\sqrt{3}}{2}$

$\sec(1125^\circ) = \sqrt{2}$

Value $= \frac{\sqrt{2}\sqrt{8\sqrt{3}+19}}{4}$

Ans: $\frac{\sqrt{2}\sqrt{8\sqrt{3}+19}}{4}$

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13. Simplify $\dfrac{\cos(360^\circ - \theta)\sec(-\theta)}{\operatorname{cosec}(90^\circ + \theta)\sec(360^\circ - \theta)}$. [5]

Reduce each ratio (the ratio changes for $90^\circ \pm \theta$ and $270^\circ \pm \theta$; the sign is that of the quadrant):

$\cos(360^\circ - \theta) = \cos\theta$

$\sec(-\theta) = \sec\theta$

$\operatorname{cosec}(90^\circ + \theta) = \sec\theta$

$\sec(360^\circ - \theta) = \sec\theta$

$\text{Expression} = \dfrac{(\cos\theta)\cdot(\sec\theta)}{(\sec\theta)\cdot(\sec\theta)}$

$= \cos^2\theta$

Ans: $\cos^2\theta$

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14. Find the range of $10 + 6\cos x$. [1]

$-1 \le \cos x \le 1$

$4 \le 10 + 6\cos x \le 16$

Ans: $[4, 16]$

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15. Is $\sec x$ increasing or decreasing on $\left(\frac{3\pi}{2}, 2\pi\right)$? Give the reason. [2]

$\left(\frac{3\pi}{2}, 2\pi\right)$ is the fourth quadrant.

As x moves across this quadrant, $\sec x$ falls, so it is decreasing there.

Ans: Decreasing

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16. If $A + B = 180^\circ$ and $\tan A = \frac{25}{7}$, find $\tan B$. [1]

$B = 180^\circ - A$

$\tan B = \tan(180^\circ - A) = -\tan A = -\frac{25}{7}$

Ans: $-\frac{25}{7}$

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17. If $\sin\theta - \cos\theta = \frac{\sqrt{2}}{3}$, find $\sin\theta\cos\theta$, $\sin^4\theta + \cos^4\theta$ and $\sin^6\theta + \cos^6\theta$. [5]

$(\sin\theta - \cos\theta)^2 = 1 - 2\sin\theta\cos\theta = \frac{2}{9}$

$\sin\theta\cos\theta = \frac{7}{18}$

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta = 1 - 2\left(\frac{7}{18}\right)^2 = \frac{113}{162}$

$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta = 1 - 3\left(\frac{7}{18}\right)^2 = \frac{59}{108}$

Ans: $\frac{7}{18}$, $\frac{113}{162}$, $\frac{59}{108}$

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18. If $\dfrac{1+\cos\theta}{1-\cos\theta} = \frac{8}{7}$, find $\cos\theta$ using componendo and dividendo. [3]

$\frac{(1+\cos\theta)+(1-\cos\theta)}{(1+\cos\theta)-(1-\cos\theta)} = \frac{\frac{8}{7}+1}{\frac{8}{7}-1}$

$\frac{2}{2\cos\theta} = \frac{\frac{15}{7}}{\frac{1}{7}}$

$\cos\theta = \frac{1}{15}$

Ans: $\cos\theta = \frac{1}{15}$

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19. If $x = 6\sec\theta + 7\tan\theta$ and $y = 6\tan\theta + 7\sec\theta$, prove that $x^2 - y^2 = -13$. Hence find y^2 when $x = 8$. [5]

$x^2 = 36\sec^2\theta + 84\sec\theta\tan\theta + 49\tan^2\theta$

$y^2 = 36\tan^2\theta + 84\sec\theta\tan\theta + 49\sec^2\theta$

Subtracting, the middle terms cancel: $x^2 - y^2 = 36(\sec^2\theta - \tan^2\theta) - 49(\sec^2\theta - \tan^2\theta) = 36 - 49 = -13$

$x = 8 \Rightarrow y^2 = 64 - (-13) = 77$

Ans: $x^2 - y^2 = -13$; $y^2 = 77$

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20. Find $\sin\theta + \sin(\pi + \theta) + \sin(2\pi + \theta) + \cdots + \sin(13\pi + \theta)$. [2]

$\sin(k\pi + \theta) = (-1)^k \sin\theta$

There are 14 terms, which cancel in pairs.

Sum $= 0$

Ans: 0
