EXPRESSWALA ACADEMY

Maths 1A: Transformations

Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write $2\sin 9\theta \sin 4\theta$ as a sum or difference of two trigonometric ratios. [1 mark]

2. Evaluate $4\cos 165^\circ \cos 15^\circ$ by changing the product into a sum. [2 marks]

3. Express $\sin 5\theta + \sin\theta$ as a product. [1 mark]

4. Simplify $\cos 137^\circ + \cos 103^\circ$. [1 mark]

5. Evaluate $\sin 165^\circ + \sin 105^\circ$ using a sum to product formula. [2 marks]

6. Show that $\sin 52^\circ + \sin 8^\circ - \sin 112^\circ = 0$. [3 marks]

7. Evaluate $\frac{\cos 99^\circ + \cos 21^\circ}{\sin 99^\circ + \sin 21^\circ}$. [1 mark]

8. Simplify $\frac{\cos 8\theta + \cos 2\theta}{\sin 8\theta - \sin 2\theta}$. [2 marks]

9. Simplify $\frac{\sin 8\theta + \sin 6\theta + \sin 7\theta}{\cos 8\theta + \cos 6\theta + \cos 7\theta}$. [3 marks]

10. Show that $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$. [5 marks]

11. In a triangle ABC, express $\cos(B + C)$ in terms of A. [1 mark]

12. Verify $-\sin^2 A + \sin^2 B + \sin^2 C = 2\cos A \sin B \sin C$ for the triangle with $A = 30^\circ$, $B = 30^\circ$, $C = 120^\circ$. [2 marks]

13. If $A + B + C = 180^\circ$, prove that $\sin 2A - \sin 2B + \sin 2C = 4\cos A \sin B \cos C$. [5 marks]

14. In a triangle ABC, prove that $\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2\sin A \sin B \cos C$. [5 marks]

15. If $\cos x - \cos y = \frac{2}{3}$ and $\sin x - \sin y = \frac{3}{7}$, find $\tan\frac{x+y}{2}$. [2 marks]

16. If $\cos x + \cos y = \frac{3}{4}$ and $\sin x - \sin y = \frac{1}{5}$, find $\sin(x - y)$ and $\cos(x - y)$. [3 marks]

17. If $\cos x - \cos y = \frac{2}{5}$ and $\sin x + \sin y = \frac{1}{2}$, find $\tan\frac{x-y}{2}$, $\sin(x - y)$, $\cos(x - y)$ and $\tan(x - y)$. [5 marks]

18. If $\cos x - \cos y = \frac{5}{6}$ and $\sin x - \sin y = \frac{3}{7}$, find $\cos(x - y)$. [2 marks]

19. If $\cos x + \cos y = \frac{7}{5}$ and $\sin x + \sin y = \frac{5}{6}$, find $\cos(x - y)$ and $\cos\frac{x-y}{2}$. [3 marks]

20. In a triangle ABC, $A = 30^\circ$ and $B = 75^\circ$. Find $\cos A + \cos B + \cos C$ and check that it does not exceed $\frac{3}{2}$. [1 mark]

Key sheet

Q	Answer
1	$\cos 5\theta - \cos 13\theta$
2	$-2 - \sqrt{3}$
3	$2\sin 3\theta \cos 2\theta$
4	$-\cos 17^\circ$
5	$\frac{\sqrt{6}}{2}$
6	Shown: the expression is 0
7	$\frac{\sqrt{3}}{3}$
8	$\cot 3\theta$
9	$\tan 7\theta$
10	Shown: $\frac{3}{16}$
11	$-\cos A$
12	Both sides equal $\frac{3}{4}$
13	Proved
14	Proved
15	$\tan\frac{x+y}{2} = -\frac{14}{9}$
16	$\sin(x-y) = \frac{120}{241},\ \cos(x-y) = \frac{209}{241}$
17	$\tan\frac{x-y}{2} = -\frac{4}{5},\ \sin(x-y) = -\frac{40}{41},\ \cos(x-y) = \frac{9}{41},\ \tan(x-y) = -\frac{40}{9}$
18	$\cos(x-y) = \frac{1979}{3528}$
19	$\cos(x-y) = \frac{589}{1800},\ \cos\frac{x-y}{2} = \pm\frac{\sqrt{2389}}{60}$
20	$\frac{-\sqrt{2}+\sqrt{3}+\sqrt{6}}{2}$

Solutions

1. Write $2\sin 9\theta \sin 4\theta$ as a sum or difference of two trigonometric ratios. [1]

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$ with $A = 9\theta,\ B = 4\theta$.

$2\sin 9\theta \sin 4\theta = \cos 5\theta - \cos 13\theta$

Ans: $\cos 5\theta - \cos 13\theta$

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2. Evaluate $4\cos 165^\circ \cos 15^\circ$ by changing the product into a sum. [2]

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$4\cos 165^\circ \cos 15^\circ = 2\left(\cos 180^\circ + \cos 150^\circ\right)$

$= 2\left(-1 - \frac{\sqrt{3}}{2}\right) = -2 - \sqrt{3}$

Ans: $-2 - \sqrt{3}$

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3. Express $\sin 5\theta + \sin\theta$ as a product. [1]

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin 5\theta + \sin\theta = 2\sin 3\theta\cos 2\theta$

Ans: $2\sin 3\theta\cos 2\theta$

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4. Simplify $\cos 137^\circ + \cos 103^\circ$. [1]

The half sum of the angles is 120°, a standard angle.

$\cos 137^\circ + \cos 103^\circ = 2\cos 120^\circ \cos 17^\circ = -\cos 17^\circ$

Ans: $-\cos 17^\circ$

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5. Evaluate $\sin 165^\circ + \sin 105^\circ$ using a sum to product formula. [2]

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin 165^\circ + \sin 105^\circ = 2\sin 135^\circ \cos 30^\circ$

$= \frac{\sqrt{6}}{2}$

Ans: $\frac{\sqrt{6}}{2}$

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6. Show that $\sin 52^\circ + \sin 8^\circ - \sin 112^\circ = 0$. [3]

$\sin 8^\circ + \sin 52^\circ = 2\sin\frac{8^\circ+52^\circ}{2}\cos\frac{8^\circ-52^\circ}{2} = 2\sin 30^\circ\cos -22^\circ = \cos -22^\circ$

$\sin 112^\circ = \sin(90^\circ - -22^\circ) = \cos -22^\circ$

$\therefore \sin 52^\circ + \sin 8^\circ - \sin 112^\circ = \cos -22^\circ - \cos -22^\circ = 0$

Ans: Shown: the expression is 0

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7. Evaluate $\frac{\cos 99^\circ + \cos 21^\circ}{\sin 99^\circ + \sin 21^\circ}$. [1]

$\frac{\cos 99^\circ + \cos 21^\circ}{\sin 99^\circ + \sin 21^\circ} = \frac{2\cos 60^\circ \cos 39^\circ}{2\sin 60^\circ \cos 39^\circ}$

$= \cot 60^\circ = \frac{\sqrt{3}}{3}$

Ans: $\frac{\sqrt{3}}{3}$

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8. Simplify $\frac{\cos 8\theta+\cos 2\theta}{\sin 8\theta-\sin 2\theta}$. [2]

$\frac{\cos 8\theta+\cos 2\theta}{\sin 8\theta-\sin 2\theta} = \frac{2\cos 5\theta\cos 3\theta}{2\cos 5\theta\sin 3\theta}$

$= \cot 3\theta$

Ans: $\cot 3\theta$

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9. Simplify $\frac{\sin 8\theta+\sin 6\theta+\sin 7\theta}{\cos 8\theta+\cos 6\theta+\cos 7\theta}$. [3]

$\sin 8\theta + \sin 6\theta = 2\sin 7\theta\cos\theta$, $\cos 8\theta + \cos 6\theta = 2\cos 7\theta\cos\theta$

$\text{Numerator} = \sin 7\theta(2\cos\theta + 1)$, $\text{Denominator} = \cos 7\theta(2\cos\theta + 1)$

Cancel the common factor $2\cos\theta + 1$: the fraction is $\tan 7\theta$.

Ans: $\tan 7\theta$

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10. Show that $\cos 10^\circ\cos 30^\circ\cos 50^\circ\cos 70^\circ = \frac{3}{16}$. [5]

$\cos 30^\circ = \frac{\sqrt{3}}{2}$, so it is a constant factor.

$\cos 50^\circ\cos 70^\circ = \frac{1}{2}\cos 20^\circ - \frac{1}{4}$

Multiplying by $\cos 10^\circ$ and using product to sum again: $\cos 10^\circ\cos 50^\circ\cos 70^\circ = \frac{1}{4}\cos 30^\circ = \frac{\sqrt{3}}{8}$

$\therefore \cos 10^\circ\cos 30^\circ\cos 50^\circ\cos 70^\circ = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{8} = \frac{3}{16}$

Ans: Shown: $\frac{3}{16}$

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11. In a triangle ABC, express $\cos(B + C)$ in terms of A. [1]

In a triangle, $B + C = 180^\circ - A$.

$\cos(B + C) = -\cos A$

Ans: $-\cos A$

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12. Verify $-\sin^2 A + \sin^2 B + \sin^2 C = 2\cos A\sin B\sin C$ for the triangle with $A = 30^\circ$, $B = 30^\circ$, $C = 120^\circ$. [2]

LHS $= \frac{3}{4}$

RHS $= \frac{3}{4}$

Both sides equal $\frac{3}{4}$, so the identity holds.

Ans: Both sides equal $\frac{3}{4}$

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13. If $A + B + C = 180^\circ$, prove that $\sin 2A - \sin 2B + \sin 2C = 4\cos A\sin B\cos C$. [5]

$A + B + C = 180^\circ$, so $A + B = 180^\circ - C$ and similar.

$\sin 2A + \sin 2C = 2\sin(A + C)\cos(A - C) = 2\sin B\cos(A - C)$

$\sin 2B = 2\sin B\cos B = -2\sin B\cos(A + C)$

$\text{LHS} = 2\sin B[\cos(A - C) + \cos(A + C)] = 2\sin B\cdot 2\cos A\cos C$

$= 4\cos A\sin B\cos C = \text{RHS}$

Ans: Proved

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14. In a triangle ABC, prove that $\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2\sin A \sin B \cos C$. [5]

$A + B + C = 180°$, so $A + B = 180° - C$ and similar.

$\cos^2 A + \cos^2 B = 1 + \frac{1}{2}(\cos 2A + \cos 2B) = 1 + \cos(A+B)\cos(A-B) = 1 - \cos C \cos(A-B)$

$-\cos^2 C = -\cos C \cos C = \cos C \cos(A+B)$

$\text{LHS} = 1 - \cos C[\cos(A-B) - \cos(A+B)] = 1 - \cos C \cdot 2\sin A \sin B$

$= 1 - 2\sin A \sin B \cos C = \text{RHS}$

Ans: Proved

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15. If $\cos x - \cos y = \frac{2}{3}$ and $\sin x - \sin y = \frac{3}{7}$, find $\tan\frac{x+y}{2}$. [2]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{2}{3}$ and $2\cos\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{3}{7}$.

Dividing: $\tan\frac{x+y}{2} = -\frac{14}{9}$

Ans: $\tan\frac{x+y}{2} = -\frac{14}{9}$

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16. If $\cos x + \cos y = \frac{3}{4}$ and $\sin x - \sin y = \frac{1}{5}$, find $\sin(x-y)$ and $\cos(x-y)$. [3]

Transforming: $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{3}{4}$ and $2\cos\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{1}{5}$.

Dividing: $\tan\frac{x-y}{2} = \frac{4}{15}$

With $t = \frac{4}{15}$: $\sin(x-y) = \frac{2t}{1+t^2} = \frac{120}{241}$, $\cos(x-y) = \frac{1-t^2}{1+t^2} = \frac{209}{241}$

Ans: $\sin(x-y) = \frac{120}{241}$, $\cos(x-y) = \frac{209}{241}$

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17. If $\cos x - \cos y = \frac{2}{5}$ and $\sin x + \sin y = \frac{1}{2}$, find $\tan\frac{x-y}{2}$, $\sin(x-y)$, $\cos(x-y)$ and $\tan(x-y)$. [5]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{2}{5}$ and $2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{1}{2}$.

Dividing: $\tan\frac{x-y}{2} = -\frac{4}{5}$

With $t = -\frac{4}{5}$: $\sin(x-y) = \frac{2t}{1+t^2} = -\frac{40}{41}$, $\cos(x-y) = \frac{1-t^2}{1+t^2} = \frac{9}{41}$

$\tan(x-y) = \frac{\sin(x-y)}{\cos(x-y)} = -\frac{40}{9}$

Ans: $\tan\frac{x-y}{2} = -\frac{4}{5}$, $\sin(x-y) = -\frac{40}{41}$, $\cos(x-y) = \frac{9}{41}$, $\tan(x-y) = -\frac{40}{9}$

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18. If $\cos x - \cos y = \frac{5}{6}$ and $\sin x - \sin y = \frac{3}{7}$, find $\cos(x-y)$. [2]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 - 2(\cos x \cos y + \sin x \sin y) = 2 - 2\cos(x-y)$

$\frac{25}{36} + \frac{9}{49} = 2 - 2\cos(x-y)$

$\cos(x-y) = \frac{1979}{3528}$

Ans: $\cos(x-y) = \frac{1979}{3528}$

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19. If $\cos x + \cos y = \frac{7}{5}$ and $\sin x + \sin y = \frac{5}{6}$, find $\cos(x-y)$ and $\cos\frac{x-y}{2}$. [3]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 + 2(\cos x \cos y + \sin x \sin y) = 2 + 2\cos(x-y)$

$\frac{49}{25} + \frac{25}{36} = 2 + 2\cos(x-y)$

$\cos(x-y) = \frac{589}{1800}$

$\cos^2\frac{x-y}{2} = \frac{1+\cos(x-y)}{2} = \frac{2389}{3600}$

$\cos\frac{x-y}{2} = \pm\frac{\sqrt{2389}}{60}$

Ans: $\cos(x-y) = \frac{589}{1800}$, $\cos\frac{x-y}{2} = \pm\frac{\sqrt{2389}}{60}$

20. In a triangle ABC, $A = 30^\circ$ and $B = 75^\circ$. Find $\cos A + \cos B + \cos C$ and check that it does not exceed $\frac{3}{2}$. [1]

$C = 180^\circ - 30^\circ - 75^\circ = 75^\circ$

$\cos 30^\circ + \cos 75^\circ + \cos 75^\circ = \frac{\sqrt{3}}{2} + \frac{-\sqrt{2}+\sqrt{6}}{4} + \frac{-\sqrt{2}+\sqrt{6}}{4} = \frac{-\sqrt{2}+\sqrt{3}+\sqrt{6}}{2}$

$\frac{-\sqrt{2}+\sqrt{3}+\sqrt{6}}{2} \approx 1.384 \leq 1.5$

Ans: $\frac{-\sqrt{2}+\sqrt{3}+\sqrt{6}}{2}$