EXPRESSWALA ACADEMY

Maths 1A: Transformations

Question Paper, Set 07

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write $2\sin 9\theta\cos 4\theta$ as a sum or difference of two trigonometric ratios. [1 mark]

2. Evaluate $\cos 165^\circ \sin 15^\circ$ by changing the product into a sum. [2 marks]

3. Write $\sin 10\theta - \sin 6\theta$ in the form of a product. [1 mark]

4. Simplify $\sin 59^\circ + \sin 1^\circ$. [1 mark]

5. Evaluate $\cos 75^\circ - \cos 15^\circ$ using a sum to product formula. [2 marks]

6. Show that $\sin 85^\circ - \sin 25^\circ + \cos 125^\circ = 0$. [3 marks]

7. Find the value of $\frac{\cos 82^\circ + \cos 38^\circ}{\sin 82^\circ + \sin 38^\circ}$. [1 mark]

8. Simplify $\frac{\sin 11\theta + \sin\theta}{\cos\theta - \cos 11\theta}$. [2 marks]

9. Prove that $\frac{\sin 11\theta + \sin 7\theta + \sin 3\theta}{\cos 11\theta + \cos 7\theta + \cos 3\theta} = \tan 7\theta$. [3 marks]

10. Show that $\tan 20^\circ \tan 40^\circ \tan 80^\circ = \sqrt{3}$. [5 marks]

11. In a triangle ABC, express $\cos(2A + 2C)$ in terms of B. [1 mark]

12. Verify $\cos 2A + \cos 2B - \cos 2C = 1 - 4\sin A \sin B \cos C$ for the triangle with $A = 30^\circ$, $B = 120^\circ$, $C = 30^\circ$. [2 marks]

13. In a triangle ABC, prove that $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$. [5 marks]

14. In a triangle ABC, prove that $-\cos A + \cos B + \cos C = -1 + 4\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$. [5 marks]

15. If $\cos x - \cos y = \frac{3}{5}$ and $\sin x + \sin y = \frac{1}{3}$, find $\tan\frac{x-y}{2}$. [2 marks]

16. If $\cos x + \cos y = \frac{1}{5}$ and $\sin x + \sin y = \frac{5}{3}$, find $\sin(x+y)$ and $\cos(x+y)$. [3 marks]

17. If $\cos x - \cos y = \frac{2}{5}$ and $\sin x - \sin y = 1$, find $\tan\frac{x+y}{2}$, $\sin(x+y)$, $\cos(x+y)$ and $\tan(x+y)$. [5 marks]

18. If $\cos x - \cos y = \frac{3}{2}$ and $\sin x - \sin y = \frac{1}{6}$, find $\cos(x-y)$. [2 marks]

19. If $\cos x + \cos y = \frac{1}{2}$ and $\sin x + \sin y = \frac{3}{4}$, find $\cos(x-y)$ and $\cos\frac{x-y}{2}$. [3 marks]

20. What is the least value of $\tan A + \tan B + \tan C$ in an acute angled triangle ABC, and when is it reached? [1 mark]

Key sheet

Q	Answer
1	$\sin 13\theta + \sin 5\theta$
2	$-\frac{1}{4}$
3	$2\cos 8\theta \sin 2\theta$
4	$\cos 29^\circ$
5	$-\frac{\sqrt{2}}{2}$
6	Shown: the expression is 0
7	$\frac{\sqrt{3}}{3}$
8	$\cot 5\theta$
9	$\tan 7\theta$
10	Shown: $\sqrt{3}$
11	$\cos 2B$
12	Both sides equal $-\frac{1}{2}$
13	Proved
14	Proved
15	$\tan\frac{x-y}{2} = -\frac{9}{5}$
16	$\sin(x+y) = \frac{75}{317},\ \cos(x+y) = -\frac{308}{317}$
17	$\tan\frac{x+y}{2} = -\frac{2}{5},\ \sin(x+y) = -\frac{20}{29},\ \cos(x+y) = \frac{21}{29},\ \tan(x+y) = -\frac{20}{21}$
18	$\cos(x-y) = -\frac{5}{36}$
19	$\cos(x-y) = -\frac{19}{32},\ \cos\frac{x-y}{2} = \pm\frac{\sqrt{13}}{8}$
20	$3\sqrt{3}$, for the equilateral triangle

Solutions

1. Write $2\sin 9\theta\cos 4\theta$ as a sum or difference of two trigonometric ratios. [1]

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$ with $A = 9\theta,\ B = 4\theta$.

$2\sin 9\theta\cos 4\theta = \sin 13\theta + \sin 5\theta$

Ans: $\sin 13\theta + \sin 5\theta$

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2. Evaluate $\cos 165^\circ \sin 15^\circ$ by changing the product into a sum. [2]

$2\cos A\sin B = \sin(A+B)-\sin(A-B)$

$\cos 165^\circ \sin 15^\circ = \frac{1}{2}(\sin 180^\circ - \sin 150^\circ)$

$= \frac{1}{2}\left(0 - \frac{1}{2}\right) = -\frac{1}{4}$

Ans: $-\frac{1}{4}$

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3. Write $\sin 10\theta - \sin 6\theta$ in the form of a product. [1]

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\sin 10\theta - \sin 6\theta = 2\cos 8\theta \sin 2\theta$

Ans: $2\cos 8\theta \sin 2\theta$

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4. Simplify $\sin 59^\circ + \sin 1^\circ$. [1]

The half sum of the angles is 30°, a standard angle.

$\sin 59^\circ + \sin 1^\circ = 2\sin 30^\circ \cos 29^\circ = \cos 29^\circ$

Ans: $\cos 29^\circ$

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5. Evaluate $\cos 75^\circ - \cos 15^\circ$ using a sum to product formula. [2]

$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos 75^\circ - \cos 15^\circ = -2\sin 45^\circ \sin 30^\circ$

$= -\frac{\sqrt{2}}{2}$

Ans: $-\frac{\sqrt{2}}{2}$

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6. Show that $\sin 85^\circ - \sin 25^\circ + \cos 125^\circ = 0$. [3]

$\sin 85^\circ - \sin 25^\circ = 2\cos\frac{85^\circ+25^\circ}{2}\sin\frac{85^\circ-25^\circ}{2} = 2\cos 55^\circ \sin 30^\circ = \cos 55^\circ$

$\cos 125^\circ = \cos(180^\circ - 55^\circ) = -\cos 55^\circ$

$\therefore \sin 85^\circ - \sin 25^\circ + \cos 125^\circ = \cos 55^\circ - \cos 55^\circ = 0$

Ans: Shown: the expression is 0

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7. Find the value of $\frac{\cos 82^\circ + \cos 38^\circ}{\sin 82^\circ + \sin 38^\circ}$. [1]

$\frac{\cos 82^\circ+\cos 38^\circ}{\sin 82^\circ+\sin 38^\circ} = \frac{2\cos 60^\circ \cos 22^\circ}{2\sin 60^\circ \cos 22^\circ}$

$= \cot 60^\circ = \frac{\sqrt{3}}{3}$

Ans: $\frac{\sqrt{3}}{3}$

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8. Simplify $\frac{\sin 11\theta+\sin\theta}{\cos\theta-\cos 11\theta}$. [2]

$\frac{\sin 11\theta+\sin\theta}{\cos\theta-\cos 11\theta} = \frac{2\sin 6\theta\cos 5\theta}{2\sin 6\theta\sin 5\theta}$

$= \cot 5\theta$

Ans: $\cot 5\theta$

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9. Prove that $\frac{\sin 11\theta+\sin 7\theta+\sin 3\theta}{\cos 11\theta+\cos 7\theta+\cos 3\theta} = \tan 7\theta$. [3]

$\sin 11\theta + \sin 3\theta = 2\sin 7\theta\cos 4\theta$, $\cos 11\theta + \cos 3\theta = 2\cos 7\theta\cos 4\theta$

$\text{Numerator} = \sin 7\theta(2\cos 4\theta + 1)$, $\text{Denominator} = \cos 7\theta(2\cos 4\theta + 1)$

Cancel the common factor $2\cos 4\theta + 1$: the fraction is $\tan 7\theta$.

Ans: $\tan 7\theta$

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10. Show that $\tan 20^\circ \tan 40^\circ \tan 80^\circ = \sqrt{3}$. [5]

By the product to sum formulas, $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{1}{4}\sin 60^\circ$.

($\sin 40^\circ \sin 80^\circ = \frac{1}{2}\cos 40^\circ + \frac{1}{4}$, then multiply by $\sin 20^\circ$.)

In the same way $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{4}\cos 60^\circ$.

$\therefore \tan 20^\circ \tan 40^\circ \tan 80^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \tan 60^\circ = \sqrt{3}$

Ans: Shown: $\sqrt{3}$

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11. In a triangle ABC, express $\cos(2A + 2C)$ in terms of B. [1]

In a triangle, $A + C = 180^\circ - B$.

$\cos(2A + 2C) = \cos 2B$

Ans: $\cos 2B$

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12. Verify $\cos 2A + \cos 2B - \cos 2C = 1 - 4\sin A\sin B\cos C$ for the triangle with $A = 30^\circ$, $B = 120^\circ$, $C = 30^\circ$. [2]

LHS $= -\frac{1}{2}$

RHS $= -\frac{1}{2}$

Both sides equal $-\frac{1}{2}$, so the identity holds.

Ans: Both sides equal $-\frac{1}{2}$

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13. In a triangle ABC, prove that $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$. [5]

$A + B + C = 180^\circ$, so $A + B = 180^\circ - C$ and similar.

$\cos 2A + \cos 2B = 2\cos(A + B)\cos(A - B) = -2\cos C\cos(A - B)$

$\cos 2C = 2\cos^2 C - 1 = -1 - 2\cos C\cos(A + B)$

$\text{LHS} = -1 - 2\cos C[\cos(A - B) + \cos(A + B)] = -1 - 2\cos C \cdot 2\cos A\cos B$

$= -1 - 4\cos A\cos B\cos C = \text{RHS}$

Ans: Proved

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14. In a triangle ABC, prove that $-\cos A + \cos B + \cos C = -1 + 4\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$. [5]

$A + B + C = 180°$, so $A + B = 180° - C$ and similar.

$\cos B + \cos C = 2\cos\frac{B+C}{2}\cos\frac{B-C}{2} = 2\sin\frac{A}{2}\cos\frac{B-C}{2}$

$-\cos A = -1 + 2\sin^2\frac{A}{2} = -1 + 2\sin\frac{A}{2}\cos\frac{B+C}{2}$

$\text{LHS} = -1 + 2\sin\frac{A}{2}\left[\cos\frac{B-C}{2} + \cos\frac{B+C}{2}\right] = -1 + 2\sin\frac{A}{2}\cdot 2\cos\frac{B}{2}\cos\frac{C}{2}$

$= -1 + 4\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} = \text{RHS}$

Ans: Proved

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15. If $\cos x - \cos y = \frac{3}{5}$ and $\sin x + \sin y = \frac{1}{3}$, find $\tan\frac{x-y}{2}$. [2]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{3}{5}$ and $2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{1}{3}$.

Dividing: $\tan\frac{x-y}{2} = -\frac{9}{5}$

Ans: $\tan\frac{x-y}{2} = -\frac{9}{5}$

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16. If $\cos x + \cos y = \frac{1}{5}$ and $\sin x + \sin y = \frac{5}{3}$, find $\sin(x+y)$ and $\cos(x+y)$. [3]

Transforming: $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{1}{5}$ and $2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{5}{3}$.

Dividing: $\tan\frac{x+y}{2} = \frac{25}{3}$

With $t = \frac{25}{3}$: $\sin(x+y) = \frac{2t}{1+t^2} = \frac{75}{317}$, $\cos(x+y) = \frac{1-t^2}{1+t^2} = -\frac{308}{317}$

Ans: $\sin(x+y) = \frac{75}{317}$, $\cos(x+y) = -\frac{308}{317}$

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17. If $\cos x - \cos y = \frac{2}{5}$ and $\sin x - \sin y = 1$, find $\tan\frac{x+y}{2}$, $\sin(x+y)$, $\cos(x+y)$ and $\tan(x+y)$. [5]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{2}{5}$ and $2\cos\frac{x+y}{2}\sin\frac{x-y}{2} = 1$.

Dividing: $\tan\frac{x+y}{2} = -\frac{2}{5}$

With $t = -\frac{2}{5}$: $\sin(x+y) = \frac{2t}{1+t^2} = -\frac{20}{29}$, $\cos(x+y) = \frac{1-t^2}{1+t^2} = \frac{21}{29}$

$\tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = -\frac{20}{21}$

Ans: $\tan\frac{x+y}{2} = -\frac{2}{5}$, $\sin(x+y) = -\frac{20}{29}$, $\cos(x+y) = \frac{21}{29}$, $\tan(x+y) = -\frac{20}{21}$

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18. If $\cos x - \cos y = \frac{3}{2}$ and $\sin x - \sin y = \frac{1}{6}$, find $\cos(x-y)$. [2]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 - 2(\cos x\cos y + \sin x\sin y) = 2 - 2\cos(x-y)$

$\frac{9}{4} + \frac{1}{36} = 2 - 2\cos(x-y)$

$\cos(x-y) = -\frac{5}{36}$

Ans: $\cos(x-y) = -\frac{5}{36}$

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19. If $\cos x + \cos y = \frac{1}{2}$ and $\sin x + \sin y = \frac{3}{4}$, find $\cos(x-y)$ and $\cos\frac{x-y}{2}$. [3]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 + 2(\cos x\cos y + \sin x\sin y) = 2 + 2\cos(x-y)$

$\frac{1}{4} + \frac{9}{16} = 2 + 2\cos(x-y)$

$\cos(x-y) = -\frac{19}{32}$

$\cos^2\frac{x-y}{2} = \frac{1+\cos(x-y)}{2} = \frac{13}{64}$

$\cos\frac{x-y}{2} = \pm\frac{\sqrt{13}}{8}$

Ans: $\cos(x-y) = -\frac{19}{32}$, $\cos\frac{x-y}{2} = \pm\frac{\sqrt{13}}{8}$

20. What is the least value of $\tan A + \tan B + \tan C$ in an acute angled triangle ABC, and when is it reached? [1]

Equality holds for the equilateral triangle, where every angle is 60°.

The least value is $3\sqrt{3}$.

Ans: $3\sqrt{3}$, for the equilateral triangle