EXPRESSWALA ACADEMY

Maths 1A: Transformations

Question Paper, Set 06

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write $2\sin 7\theta\cos 4\theta$ as a sum or difference of two trigonometric ratios. [1 mark]

2. Evaluate $2\sin 75^\circ\cos 45^\circ$ by changing the product into a sum. [2 marks]

3. Write $\sin 3\theta - \sin\theta$ in the form of a product. [1 mark]

4. Simplify $\sin 102^\circ - \sin 18^\circ$. [1 mark]

5. Find the value of $\cos 15^\circ - \cos 45^\circ$. [2 marks]

6. Show that $\sin 51^\circ + \sin 9^\circ - \sin 111^\circ = 0$. [3 marks]

7. Find the value of $\frac{\sin 42^\circ - \sin 18^\circ}{\cos 18^\circ - \cos 42^\circ}$. [1 mark]

8. Simplify $\frac{\sin 11\theta + \sin 5\theta}{\cos 11\theta + \cos 5\theta}$. [2 marks]

9. Prove that $\frac{\sin 8\theta + \sin 6\theta + \sin 7\theta}{\cos 8\theta + \cos 6\theta + \cos 7\theta} = \tan 7\theta$. [3 marks]

10. Show that $\cos 15^\circ\cos 45^\circ\cos 75^\circ = \frac{\sqrt{2}}{8}$. [5 marks]

11. In a triangle ABC, express $\tan(A+B)$ in terms of C. [1 mark]

12. Verify $-\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A\sin B\sin C$ for the triangle with $A = 30^\circ$, $B = 30^\circ$, $C = 120^\circ$. [2 marks]

13. If $A + B + C = 180^\circ$, prove that $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A\cos B\cos C$. [5 marks]

14. In a triangle ABC, prove that $\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$. [5 marks]

15. If $\cos x + \cos y = \frac{3}{2}$ and $\sin x - \sin y = \frac{2}{5}$, find $\tan\frac{x-y}{2}$. [2 marks]

16. If $\cos x + \cos y = \frac{1}{5}$ and $\sin x + \sin y = \frac{3}{4}$, find $\sin(x+y)$ and $\cos(x+y)$. [3 marks]

17. If $\cos x - \cos y = \frac{1}{5}$ and $\sin x + \sin y = \frac{3}{2}$, find $\tan\frac{x-y}{2}$, $\sin(x-y)$, $\cos(x-y)$ and $\tan(x-y)$. [5 marks]

18. If $\cos x - \cos y = \frac{3}{2}$ and $\sin x + \sin y = \frac{4}{5}$, find $\cos(x+y)$. [2 marks]

19. If $\cos x + \cos y = \frac{1}{2}$ and $\sin x + \sin y = \frac{4}{5}$, find $\cos(x-y)$ and $\cos\frac{x-y}{2}$. [3 marks]

20. What is the greatest value of $\cos A + \cos B + \cos C$ in a triangle ABC, and when is it reached? [1 mark]

Key sheet

Q	Answer
1	$\sin 11\theta + \sin 3\theta$
2	$\frac{1}{2} + \frac{\sqrt{3}}{2}$
3	$2\cos 2\theta \sin \theta$
4	$\sin 42^\circ$
5	$\frac{-\sqrt{2}+\sqrt{6}}{4}$
6	Shown: the expression is 0
7	$\sqrt{3}$
8	$\tan 8\theta$
9	$\tan 7\theta$
10	Shown: $\frac{\sqrt{2}}{8}$
11	$-\tan C$
12	Both sides equal $\frac{1}{4}$
13	Proved
14	Proved
15	$\tan\frac{x-y}{2} = \frac{4}{15}$
16	$\sin(x+y) = \frac{120}{241},\ \cos(x+y) = -\frac{209}{241}$
17	$\tan\frac{x-y}{2} = -\frac{2}{15},\ \sin(x-y) = -\frac{60}{229},\ \cos(x-y) = \frac{221}{229},\ \tan(x-y) = -\frac{60}{221}$
18	$\cos(x+y) = -\frac{89}{200}$
19	$\cos(x-y) = -\frac{111}{200},\ \cos\frac{x-y}{2} = \pm\frac{\sqrt{89}}{20}$
20	$\frac{3}{2}$, for the equilateral triangle

Solutions

1. Write $2\sin 7\theta\cos 4\theta$ as a sum or difference of two trigonometric ratios. [1]

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$ with $A = 7\theta,\ B = 4\theta$.

$2\sin 7\theta\cos 4\theta = \sin 11\theta + \sin 3\theta$

Ans: $\sin 11\theta + \sin 3\theta$

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2. Evaluate $2\sin 75^\circ\cos 45^\circ$ by changing the product into a sum. [2]

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$2\sin 75^\circ\cos 45^\circ = \sin 120^\circ + \sin 30^\circ$

$= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1}{2} + \frac{\sqrt{3}}{2}$

Ans: $\frac{1}{2} + \frac{\sqrt{3}}{2}$

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3. Write $\sin 3\theta - \sin\theta$ in the form of a product. [1]

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\sin 3\theta - \sin\theta = 2\cos 2\theta\sin\theta$

Ans: $2\cos 2\theta\sin\theta$

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4. Simplify $\sin 102^\circ - \sin 18^\circ$. [1]

The half sum of the angles is 60°, a standard angle.

$\sin 102^\circ - \sin 18^\circ = 2\cos 60^\circ\sin 42^\circ = \sin 42^\circ$

Ans: $\sin 42^\circ$

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5. Find the value of $\cos 15^\circ - \cos 45^\circ$. [2]

$\cos D - \cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos 15^\circ - \cos 45^\circ = 2\sin 30^\circ\sin 15^\circ$

$= \frac{-\sqrt{2}+\sqrt{6}}{4}$

Ans: $\frac{-\sqrt{2}+\sqrt{6}}{4}$

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6. Show that $\sin 51^\circ + \sin 9^\circ - \sin 111^\circ = 0$. [3]

$\sin 9^\circ + \sin 51^\circ = 2\sin\frac{9^\circ+51^\circ}{2}\cos\frac{9^\circ-51^\circ}{2} = 2\sin 30^\circ\cos -21^\circ = \cos -21^\circ$

$\sin 111^\circ = \sin(90^\circ - -21^\circ) = \cos -21^\circ$

$\therefore \sin 51^\circ + \sin 9^\circ - \sin 111^\circ = \cos -21^\circ - \cos -21^\circ = 0$

Ans: Shown: the expression is 0

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7. Find the value of $\frac{\sin 42^\circ - \sin 18^\circ}{\cos 18^\circ - \cos 42^\circ}$. [1]

$\frac{\sin 42^\circ - \sin 18^\circ}{\cos 18^\circ - \cos 42^\circ} = \frac{2\cos 30^\circ\sin 12^\circ}{2\sin 30^\circ\sin 12^\circ}$

$= \cot 30^\circ = \sqrt{3}$

Ans: $\sqrt{3}$

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8. Simplify $\frac{\sin 11\theta+\sin 5\theta}{\cos 11\theta+\cos 5\theta}$. [2]

$\frac{\sin 11\theta+\sin 5\theta}{\cos 11\theta+\cos 5\theta} = \frac{2\sin 8\theta\cos 3\theta}{2\cos 8\theta\cos 3\theta}$

$= \tan 8\theta$

Ans: $\tan 8\theta$

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9. Prove that $\frac{\sin 8\theta+\sin 6\theta+\sin 7\theta}{\cos 8\theta+\cos 6\theta+\cos 7\theta} = \tan 7\theta$. [3]

$\sin 8\theta + \sin 6\theta = 2\sin 7\theta\cos\theta$, $\cos 8\theta + \cos 6\theta = 2\cos 7\theta\cos\theta$

$\text{Numerator} = \sin 7\theta(2\cos\theta + 1)$, $\text{Denominator} = \cos 7\theta(2\cos\theta + 1)$

Cancel the common factor $2\cos\theta + 1$: the fraction is $\tan 7\theta$.

Ans: $\tan 7\theta$

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10. Show that $\cos 15^\circ \cos 45^\circ \cos 75^\circ = \frac{\sqrt{2}}{8}$. [5]

$\cos 45^\circ \cos 75^\circ = \frac{1}{2}[\cos 120^\circ + \cos 30^\circ] = \frac{1}{2}\cos 30^\circ - \frac{1}{4}$

$\cos 15^\circ\left(\frac{1}{2}\cos 30^\circ - \frac{1}{4}\right) = \frac{1}{4}(2\cos 30^\circ\cos 15^\circ) - \frac{1}{4}\cos 15^\circ$

$= \frac{1}{4}[\cos 45^\circ + \cos 15^\circ] - \frac{1}{4}\cos 15^\circ = \frac{1}{4}\cos 45^\circ$

$= \frac{1}{4}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{8}$

Ans: Shown: $\frac{\sqrt{2}}{8}$

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11. In a triangle ABC, express $\tan(A+B)$ in terms of C. [1]

In a triangle, $A + B = 180^\circ - C$.

$\tan(A+B) = -\tan C$

Ans: $-\tan C$

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12. Verify $-\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A\sin B\sin C$ for the triangle with $A = 30^\circ$, $B = 30^\circ$, $C = 120^\circ$. [2]

LHS $= \frac{1}{4}$

RHS $= \frac{1}{4}$

Both sides equal $\frac{1}{4}$, so the identity holds.

Ans: Both sides equal $\frac{1}{4}$

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13. If $A + B + C = 180^\circ$, prove that $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A\cos B\cos C$. [5]

$A + B + C = 180^\circ$, so $A + B = 180^\circ - C$ and similar.

$\sin^2 A + \sin^2 B = 1 - \frac{1}{2}(\cos 2A + \cos 2B) = 1 - \cos(A+B)\cos(A-B) = 1 + \cos C\cos(A-B)$

$\sin^2 C = 1 - \cos^2 C = 1 + \cos C\cos(A+B)$

$\text{LHS} = 2 + \cos C[\cos(A-B) + \cos(A+B)] = 2 + \cos C\cdot 2\cos A\cos B$

$= 2 + 2\cos A\cos B\cos C = \text{RHS}$

Ans: Proved

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14. In a triangle ABC, prove that $\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$. [5]

$A + B + C = 180°$, so $A + B = 180° - C$ and similar.

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} = 2\sin\frac{C}{2}\cos\frac{A-B}{2}$

$-\cos C = -1 + 2\sin^2\frac{C}{2} = -1 + 2\sin\frac{C}{2}\cos\frac{A+B}{2}$

$\text{LHS} = -1 + 2\sin\frac{C}{2}\left[\cos\frac{A-B}{2} + \cos\frac{A+B}{2}\right] = -1 + 2\sin\frac{C}{2}\cdot 2\cos\frac{A}{2}\cos\frac{B}{2}$

$= -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} = \text{RHS}$

Ans: Proved

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15. If $\cos x + \cos y = \frac{3}{2}$ and $\sin x - \sin y = \frac{2}{5}$, find $\tan\frac{x-y}{2}$. [2]

Transforming: $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{3}{2}$ and $2\cos\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{2}{5}$.

Dividing: $\tan\frac{x-y}{2} = \frac{4}{15}$

Ans: $\tan\frac{x-y}{2} = \frac{4}{15}$

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16. If $\cos x + \cos y = \frac{1}{5}$ and $\sin x + \sin y = \frac{3}{4}$, find $\sin(x+y)$ and $\cos(x+y)$. [3]

Transforming: $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{1}{5}$ and $2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{3}{4}$.

Dividing: $\tan\frac{x+y}{2} = \frac{15}{4}$

With $t = \frac{15}{4}$: $\sin(x+y) = \frac{2t}{1+t^2} = \frac{120}{241}$, $\cos(x+y) = \frac{1-t^2}{1+t^2} = -\frac{209}{241}$

Ans: $\sin(x+y) = \frac{120}{241}$, $\cos(x+y) = -\frac{209}{241}$

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17. If $\cos x - \cos y = \frac{1}{5}$ and $\sin x + \sin y = \frac{3}{2}$, find $\tan\frac{x-y}{2}$, $\sin(x-y)$, $\cos(x-y)$ and $\tan(x-y)$. [5]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{1}{5}$ and $2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{3}{2}$.

Dividing: $\tan\frac{x-y}{2} = -\frac{2}{15}$

With $t = -\frac{2}{15}$: $\sin(x-y) = \frac{2t}{1+t^2} = -\frac{60}{229}$, $\cos(x-y) = \frac{1-t^2}{1+t^2} = \frac{221}{229}$

$\tan(x-y) = \frac{\sin(x-y)}{\cos(x-y)} = -\frac{60}{221}$

Ans: $\tan\frac{x-y}{2} = -\frac{2}{15}$, $\sin(x-y) = -\frac{60}{229}$, $\cos(x-y) = \frac{221}{229}$, $\tan(x-y) = -\frac{60}{221}$

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18. If $\cos x - \cos y = \frac{3}{2}$ and $\sin x + \sin y = \frac{4}{5}$, find $\cos(x+y)$. [2]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 - 2(\cos x\cos y - \sin x\sin y) = 2 - 2\cos(x+y)$

$\frac{9}{4} + \frac{16}{25} = 2 - 2\cos(x+y)$

$\cos(x+y) = -\frac{89}{200}$

Ans: $\cos(x+y) = -\frac{89}{200}$

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19. If $\cos x + \cos y = \frac{1}{2}$ and $\sin x + \sin y = \frac{4}{5}$, find $\cos(x-y)$ and $\cos\frac{x-y}{2}$. [3]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 + 2(\cos x\cos y + \sin x\sin y) = 2 + 2\cos(x-y)$

$\frac{1}{4} + \frac{16}{25} = 2 + 2\cos(x-y)$

$\cos(x-y) = -\frac{111}{200}$

$\cos^2\frac{x-y}{2} = \frac{1+\cos(x-y)}{2} = \frac{89}{400}$

$\cos\frac{x-y}{2} = \pm\frac{\sqrt{89}}{20}$

Ans: $\cos(x-y) = -\frac{111}{200}$, $\cos\frac{x-y}{2} = \pm\frac{\sqrt{89}}{20}$

20. What is the greatest value of $\cos A + \cos B + \cos C$ in a triangle ABC, and when is it reached? [1]

Equality holds for the equilateral triangle, where every angle is 60°.

The greatest value is $\frac{3}{2}$.

Ans: $\frac{3}{2}$, for the equilateral triangle
