EXPRESSWALA ACADEMY

Maths 1A: Transformations

Question Paper, Set 04

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Express $2\sin 5\theta \sin 4\theta$ as a sum or difference. [1 mark]

2. Evaluate $\sin 45^\circ \cos 15^\circ$ by changing the product into a sum. [2 marks]

3. Write $\cos 11\theta - \cos 7\theta$ in the form of a product. [1 mark]

4. Simplify $\sin 73^\circ - \sin 47^\circ$. [1 mark]

5. Find the value of $\sin 45^\circ + \sin 15^\circ$. [2 marks]

6. Show that $\cos 12^\circ + \cos 108^\circ + \cos 132^\circ = 0$. [3 marks]

7. Find the value of $\frac{\sin 51^\circ + \sin 39^\circ}{\cos 51^\circ + \cos 39^\circ}$. [1 mark]

8. Simplify $\frac{\sin 8\theta - \sin 2\theta}{\cos 8\theta + \cos 2\theta}$. [2 marks]

9. Prove that $\frac{\sin 7\theta + \sin 11\theta + \sin 3\theta}{\cos 7\theta + \cos 11\theta + \cos 3\theta} = \tan 7\theta$. [3 marks]

10. Show that $\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$. [5 marks]

11. In a triangle ABC, express $\cos(2B + 2A)$ in terms of C. [1 mark]

12. Verify $-\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \sin B \sin C$ for the triangle with $A = 60^\circ$, $B = 90^\circ$, $C = 30^\circ$. [2 marks]

13. If $A + B + C = 180^\circ$, prove that $-\sin 2A + \sin 2B + \sin 2C = 4\sin A \cos B \cos C$. [5 marks]

14. If $A + B + C = 180^\circ$, prove that $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cos B \cos C$. [5 marks]

15. If $\cos x - \cos y = \frac{4}{5}$ and $\sin x + \sin y = 1$, find $\tan \frac{x-y}{2}$. [2 marks]

16. If $\cos x - \cos y = \frac{5}{6}$ and $\sin x - \sin y = \frac{1}{2}$, find $\sin(x + y)$ and $\cos(x + y)$. [3 marks]

17. If $\cos x + \cos y = \frac{5}{4}$ and $\sin x - \sin y = \frac{1}{4}$, find $\tan \frac{x-y}{2}$, $\sin(x - y)$, $\cos(x - y)$ and $\tan(x - y)$. [5 marks]

18. If $\cos x - \cos y = \frac{4}{5}$ and $\sin x + \sin y = \frac{1}{4}$, find $\cos(x + y)$. [2 marks]

19. If $\cos x + \cos y = \frac{1}{2}$ and $\sin x + \sin y = \frac{5}{6}$, find $\cos(x - y)$ and $\cos \frac{x-y}{2}$. [3 marks]

20. In an acute angled triangle ABC, write the least value of $\tan A + \tan B + \tan C$. [1 mark]

Key sheet

Q	Answer
1	$\cos\theta - \cos 9\theta$
2	$\frac{1}{4} + \frac{\sqrt{3}}{4}$
3	$-2\sin 9\theta \sin 2\theta$
4	$\sin 13^\circ$
5	$\frac{\sqrt{2}+\sqrt{6}}{4}$
6	Shown: the expression is 0
7	1
8	$\tan 3\theta$
9	$\tan 7\theta$
10	Shown: $\frac{1}{8}$
11	$\cos 2C$
12	Both sides equal $\frac{1}{2}$
13	Proved
14	Proved
15	$\tan\frac{x-y}{2} = -\frac{4}{5}$
16	$\sin(x+y) = -\frac{15}{17},\ \cos(x+y) = -\frac{8}{17}$
17	$\tan\frac{x-y}{2} = \frac{1}{5},\ \sin(x-y) = \frac{5}{13},\ \cos(x-y) = \frac{12}{13},\ \tan(x-y) = \frac{5}{12}$
18	$\cos(x+y) = \frac{519}{800}$
19	$\cos(x-y) = -\frac{19}{36},\ \cos\frac{x-y}{2} = \pm\frac{\sqrt{34}}{12}$
20	$3\sqrt{3}$, for the equilateral triangle

Solutions

1. Express $2\sin 5\theta \sin 4\theta$ as a sum or difference. [1]

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$ with $A = 5\theta,\ B = 4\theta$.

$2\sin 5\theta \sin 4\theta = \cos\theta - \cos 9\theta$

Ans: $\cos\theta - \cos 9\theta$

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2. Evaluate $\sin 45^\circ \cos 15^\circ$ by changing the product into a sum. [2]

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$\sin 45^\circ \cos 15^\circ = \frac{1}{2}(\sin 60^\circ + \sin 30^\circ)$

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \frac{1}{4} + \frac{\sqrt{3}}{4}$

Ans: $\frac{1}{4} + \frac{\sqrt{3}}{4}$

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3. Write $\cos 11\theta - \cos 7\theta$ in the form of a product. [1]

$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos 11\theta - \cos 7\theta = -2\sin 9\theta \sin 2\theta$

Ans: $-2\sin 9\theta \sin 2\theta$

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4. Simplify $\sin 73^\circ - \sin 47^\circ$. [1]

The half sum of the angles is 60°, a standard angle.

$\sin 73^\circ - \sin 47^\circ = 2\cos 60^\circ \sin 13^\circ = \sin 13^\circ$

Ans: $\sin 13^\circ$

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5. Find the value of $\sin 45^\circ + \sin 15^\circ$. [2]

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin 45^\circ + \sin 15^\circ = 2\sin 30^\circ \cos 15^\circ$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

Ans: $\frac{\sqrt{2}+\sqrt{6}}{4}$

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6. Show that $\cos 12^\circ + \cos 108^\circ + \cos 132^\circ = 0$. [3]

$\cos 132^\circ + \cos 108^\circ = 2\cos\frac{132^\circ+108^\circ}{2}\cos\frac{132^\circ-108^\circ}{2} = 2\cos 120^\circ \cos 12^\circ$

$= 2\left(-\frac{1}{2}\right)\cos 12^\circ = -\cos 12^\circ$

$\therefore \cos 12^\circ + \cos 108^\circ + \cos 132^\circ = \cos 12^\circ - \cos 12^\circ = 0$

Ans: Shown: the expression is 0

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7. Find the value of $\frac{\sin 51^\circ + \sin 39^\circ}{\cos 51^\circ + \cos 39^\circ}$. [1]

$\frac{\sin 51^\circ+\sin 39^\circ}{\cos 51^\circ+\cos 39^\circ} = \frac{2\sin 45^\circ \cos 6^\circ}{2\cos 45^\circ \cos 6^\circ}$

$= \tan 45^\circ = 1$

Ans: 1

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8. Simplify $\frac{\sin 8\theta - \sin 2\theta}{\cos 8\theta + \cos 2\theta}$. [2]

$\frac{\sin 8\theta - \sin 2\theta}{\cos 8\theta + \cos 2\theta} = \frac{2\cos 5\theta \sin 3\theta}{2\cos 5\theta \cos 3\theta}$

$= \tan 3\theta$

Ans: $\tan 3\theta$

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9. Prove that $\frac{\sin 7\theta + \sin 11\theta + \sin 3\theta}{\cos 7\theta + \cos 11\theta + \cos 3\theta} = \tan 7\theta$. [3]

$\sin 11\theta + \sin 3\theta = 2\sin 7\theta \cos 4\theta$, $\cos 11\theta + \cos 3\theta = 2\cos 7\theta \cos 4\theta$

$\text{Numerator} = \sin 7\theta(2\cos 4\theta + 1)$, $\text{Denominator} = \cos 7\theta(2\cos 4\theta + 1)$

Cancel the common factor $2\cos 4\theta + 1$: the fraction is $\tan 7\theta$.

Ans: $\tan 7\theta$

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10. Show that $\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$. [5]

$\sin 50^\circ \sin 70^\circ = \frac{1}{2}[\cos 20^\circ - \cos 120^\circ] = \frac{1}{2}\cos 20^\circ + \frac{1}{4}$

$\sin 10^\circ \left(\frac{1}{2}\cos 20^\circ + \frac{1}{4}\right) = \frac{1}{4}(2\sin 10^\circ \cos 20^\circ) + \frac{1}{4}\sin 10^\circ$

$= \frac{1}{4}[\sin 30^\circ - \sin 10^\circ] + \frac{1}{4}\sin 10^\circ = \frac{1}{4}\sin 30^\circ$

$= \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$

Ans: Shown: $\frac{1}{8}$

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11. In a triangle ABC, express $\cos(2B + 2A)$ in terms of C. [1]

In a triangle, $B + A = 180^\circ - C$.

$\cos(2B + 2A) = \cos 2C$

Ans: $\cos 2C$

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12. Verify $-\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \sin B \sin C$ for the triangle with $A = 60^\circ$, $B = 90^\circ$, $C = 30^\circ$. [2]

LHS $= \frac{1}{2}$

RHS $= \frac{1}{2}$

Both sides equal $\frac{1}{2}$, so the identity holds.

Ans: Both sides equal $\frac{1}{2}$

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13. If $A + B + C = 180^\circ$, prove that $-\sin 2A + \sin 2B + \sin 2C = 4\sin A \cos B \cos C$. [5]

$A + B + C = 180^\circ$, so $A + B = 180^\circ - C$ and similar.

$\sin 2B + \sin 2C = 2\sin(B + C)\cos(B - C) = 2\sin A \cos(B - C)$

$\sin 2A = 2\sin A \cos A = -2\sin A \cos(B + C)$

$\text{LHS} = 2\sin A[\cos(B - C) + \cos(B + C)] = 2\sin A \cdot 2\cos B \cos C$

$= 4\sin A \cos B \cos C = \text{RHS}$

Ans: Proved

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14. If $A + B + C = 180°$, prove that $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cos B \cos C$. [5]

$A + B + C = 180°$, so $A + B = 180° - C$ and similar.

$\cos^2 A + \cos^2 B = 1 + \frac{1}{2}(\cos 2A + \cos 2B) = 1 + \cos(A+B)\cos(A-B) = 1 - \cos C \cos(A-B)$

$\cos^2 C = -\cos C \cos(A+B)$

$\text{LHS} = 1 - \cos C[\cos(A-B) + \cos(A+B)] = 1 - \cos C \cdot 2\cos A \cos B$

$= 1 - 2\cos A \cos B \cos C = \text{RHS}$

Ans: Proved

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15. If $\cos x - \cos y = \frac{4}{5}$ and $\sin x + \sin y = 1$, find $\tan\frac{x-y}{2}$. [2]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{4}{5}$ and $2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = 1$.

Dividing: $\tan\frac{x-y}{2} = -\frac{4}{5}$

Ans: $\tan\frac{x-y}{2} = -\frac{4}{5}$

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16. If $\cos x - \cos y = \frac{5}{6}$ and $\sin x - \sin y = \frac{1}{2}$, find $\sin(x+y)$ and $\cos(x+y)$. [3]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{5}{6}$ and $2\cos\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{1}{2}$.

Dividing: $\tan\frac{x+y}{2} = -\frac{5}{3}$

With $t = -\frac{5}{3}$: $\sin(x+y) = \frac{2t}{1+t^2} = -\frac{15}{17}$, $\cos(x+y) = \frac{1-t^2}{1+t^2} = -\frac{8}{17}$

Ans: $\sin(x+y) = -\frac{15}{17}$, $\cos(x+y) = -\frac{8}{17}$

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17. If $\cos x + \cos y = \frac{5}{4}$ and $\sin x - \sin y = \frac{1}{4}$, find $\tan\frac{x-y}{2}$, $\sin(x-y)$, $\cos(x-y)$ and $\tan(x-y)$. [5]

Transforming: $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{5}{4}$ and $2\cos\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{1}{4}$.

Dividing: $\tan\frac{x-y}{2} = \frac{1}{5}$

With $t = \frac{1}{5}$: $\sin(x-y) = \frac{2t}{1+t^2} = \frac{5}{13}$, $\cos(x-y) = \frac{1-t^2}{1+t^2} = \frac{12}{13}$

$\tan(x-y) = \frac{\sin(x-y)}{\cos(x-y)} = \frac{5}{12}$

Ans: $\tan\frac{x-y}{2} = \frac{1}{5}$, $\sin(x-y) = \frac{5}{13}$, $\cos(x-y) = \frac{12}{13}$, $\tan(x-y) = \frac{5}{12}$

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18. If $\cos x - \cos y = \frac{4}{5}$ and $\sin x + \sin y = \frac{1}{4}$, find $\cos(x+y)$. [2]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 - 2(\cos x \cos y - \sin x \sin y) = 2 - 2\cos(x+y)$

$\frac{16}{25} + \frac{1}{16} = 2 - 2\cos(x+y)$

$\cos(x+y) = \frac{519}{800}$

Ans: $\cos(x+y) = \frac{519}{800}$

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19. If $\cos x + \cos y = \frac{1}{2}$ and $\sin x + \sin y = \frac{5}{6}$, find $\cos(x-y)$ and $\cos\frac{x-y}{2}$. [3]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 + 2(\cos x \cos y + \sin x \sin y) = 2 + 2\cos(x-y)$

$\frac{1}{4} + \frac{25}{36} = 2 + 2\cos(x-y)$

$\cos(x-y) = -\frac{19}{36}$

$\cos^2\frac{x-y}{2} = \frac{1+\cos(x-y)}{2} = \frac{17}{72}$

$\cos\frac{x-y}{2} = \pm\frac{\sqrt{34}}{12}$

Ans: $\cos(x-y) = -\frac{19}{36}$, $\cos\frac{x-y}{2} = \pm\frac{\sqrt{34}}{12}$

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20. In an acute angled triangle ABC, write the least value of $\tan A + \tan B + \tan C$. [1]

Equality holds for the equilateral triangle, where every angle is 60°.

The least value is $3\sqrt{3}$.

Ans: $3\sqrt{3}$, for the equilateral triangle
