EXPRESSWALA ACADEMY

Maths 1A: Transformations

Question Paper, Set 03

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Express $2\sin 9\theta\cos 7\theta$ as a sum or difference. [1 mark]

2. Evaluate $4\sin 105^\circ \sin 75^\circ$ by changing the product into a sum. [2 marks]

3. Write $\sin 5\theta + \sin\theta$ in the form of a product. [1 mark]

4. Simplify $\sin 39^\circ + \sin 21^\circ$. [1 mark]

5. Evaluate $\sin 105^\circ + \sin 15^\circ$ using a sum to product formula. [2 marks]

6. Show that $\sin 5^\circ + \sin 55^\circ - \sin 65^\circ = 0$. [3 marks]

7. Evaluate $\frac{\sin 35^\circ - \sin 25^\circ}{\cos 25^\circ - \cos 35^\circ}$. [1 mark]

8. Show that $\frac{\sin 10\theta - \sin 8\theta}{\cos 10\theta + \cos 8\theta} = \tan\theta$. [2 marks]

9. Simplify $\frac{\sin 4\theta + \sin\theta + \sin 7\theta}{\cos 4\theta + \cos\theta + \cos 7\theta}$. [3 marks]

10. Show that $\sin 15^\circ \sin 45^\circ \sin 75^\circ = \frac{\sqrt{2}}{8}$. [5 marks]

11. In a triangle ABC, express $\sin(B + C)$ in terms of A. [1 mark]

12. Verify $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ for the triangle with $A = 60^\circ$, $B = 60^\circ$, $C = 60^\circ$. [2 marks]

13. In a triangle ABC, prove that $\sin 2A + \sin 2B - \sin 2C = 4\cos A\cos B\sin C$. [5 marks]

14. In a triangle ABC, prove that $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$. [5 marks]

15. If $\cos x + \cos y = \frac{3}{2}$ and $\sin x + \sin y = \frac{2}{7}$, find $\tan\frac{x+y}{2}$. [2 marks]

16. If $\cos x - \cos y = \frac{3}{4}$ and $\sin x - \sin y = \frac{2}{3}$, find $\sin(x + y)$ and $\cos(x + y)$. [3 marks]

17. If $\cos x + \cos y = \frac{5}{6}$ and $\sin x - \sin y = \frac{5}{3}$, find $\tan\frac{x-y}{2}$, $\sin(x - y)$, $\cos(x - y)$ and $\tan(x - y)$. [5 marks]

18. If $\cos x + \cos y = \frac{1}{3}$ and $\sin x + \sin y = \frac{3}{4}$, find $\cos(x - y)$. [2 marks]

19. If $\cos x + \cos y = \frac{1}{5}$ and $\sin x + \sin y = \frac{4}{5}$, find $\cos(x - y)$ and $\cos\frac{x-y}{2}$. [3 marks]

20. In a triangle ABC, $A = 60^\circ$ and $B = 60^\circ$. Find $\cos A + \cos B + \cos C$ and check that it does not exceed $\frac{3}{2}$. [1 mark]

Key sheet

Q	Answer
1	$\sin 16\theta + \sin 2\theta$
2	$\sqrt{3} + 2$
3	$2\sin 3\theta \cos 2\theta$
4	$\cos 9^\circ$
5	$\frac{\sqrt{6}}{2}$
6	Shown: the expression is 0
7	$\sqrt{3}$
8	$\tan\theta$
9	$\tan 4\theta$
10	Shown: $\frac{\sqrt{2}}{8}$
11	$\sin A$
12	Both sides equal $\frac{3\sqrt{3}}{2}$
13	Proved
14	Proved
15	$\tan\frac{x+y}{2} = \frac{4}{21}$
16	$\sin(x+y) = -\frac{144}{145},\ \cos(x+y) = -\frac{17}{145}$
17	$\tan\frac{x-y}{2} = 2,\ \sin(x-y) = \frac{4}{5},\ \cos(x-y) = -\frac{3}{5},\ \tan(x-y) = -\frac{4}{3}$
18	$\cos(x-y) = -\frac{191}{288}$
19	$\cos(x-y) = -\frac{33}{50},\ \cos\frac{x-y}{2} = \pm\frac{\sqrt{17}}{10}$
20	$\frac{3}{2}$

Solutions

1. Express $2\sin 9\theta\cos 7\theta$ as a sum or difference. [1]

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$ with $A = 9\theta,\ B = 7\theta$.

$2\sin 9\theta\cos 7\theta = \sin 16\theta + \sin 2\theta$

Ans: $\sin 16\theta + \sin 2\theta$

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2. Evaluate $4\sin 105^\circ \sin 75^\circ$ by changing the product into a sum. [2]

$2\sin A\sin B = \cos(A-B)-\cos(A+B)$

$4\sin 105^\circ \sin 75^\circ = 2\left(\cos 30^\circ - \cos 180^\circ\right)$

$= 2\left(\frac{\sqrt{3}}{2}+1\right) = \sqrt{3}+2$

Ans: $\sqrt{3}+2$

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3. Write $\sin 5\theta + \sin\theta$ in the form of a product. [1]

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin 5\theta + \sin\theta = 2\sin 3\theta\cos 2\theta$

Ans: $2\sin 3\theta\cos 2\theta$

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4. Simplify $\sin 39^\circ + \sin 21^\circ$. [1]

The half sum of the angles is 30°, a standard angle.

$\sin 39^\circ + \sin 21^\circ = 2\sin 30^\circ\cos 9^\circ = \cos 9^\circ$

Ans: $\cos 9^\circ$

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5. Evaluate $\sin 105^\circ + \sin 15^\circ$ using a sum to product formula. [2]

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin 105^\circ + \sin 15^\circ = 2\sin 60^\circ\cos 45^\circ$

$= \frac{\sqrt{6}}{2}$

Ans: $\frac{\sqrt{6}}{2}$

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6. Show that $\sin 5^\circ + \sin 55^\circ - \sin 65^\circ = 0$. [3]

$\sin 55^\circ + \sin 5^\circ = 2\sin\frac{55^\circ+5^\circ}{2}\cos\frac{55^\circ-5^\circ}{2} = 2\sin 30^\circ\cos 25^\circ = \cos 25^\circ$

$\sin 65^\circ = \sin(90^\circ - 25^\circ) = \cos 25^\circ$

$\therefore \sin 5^\circ + \sin 55^\circ - \sin 65^\circ = \cos 25^\circ - \cos 25^\circ = 0$

Ans: Shown: the expression is 0

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7. Evaluate $\frac{\sin 35^\circ - \sin 25^\circ}{\cos 25^\circ - \cos 35^\circ}$. [1]

$\frac{\sin 35^\circ - \sin 25^\circ}{\cos 25^\circ - \cos 35^\circ} = \frac{2\cos 30^\circ\sin 5^\circ}{2\sin 30^\circ\sin 5^\circ}$

$= \cot 30^\circ = \sqrt{3}$

Ans: $\sqrt{3}$

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8. Show that $\frac{\sin 10\theta - \sin 8\theta}{\cos 10\theta + \cos 8\theta} = \tan\theta$. [2]

$\frac{\sin 10\theta - \sin 8\theta}{\cos 10\theta + \cos 8\theta} = \frac{2\cos 9\theta \sin\theta}{2\cos 9\theta \cos\theta}$

$= \tan\theta$

Ans: $\tan\theta$

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9. Simplify $\frac{\sin 4\theta + \sin\theta + \sin 7\theta}{\cos 4\theta + \cos\theta + \cos 7\theta}$. [3]

$\sin 7\theta + \sin\theta = 2\sin 4\theta\cos 3\theta$, $\cos 7\theta + \cos\theta = 2\cos 4\theta\cos 3\theta$

$\text{Numerator} = \sin 4\theta(2\cos 3\theta + 1)$, $\text{Denominator} = \cos 4\theta(2\cos 3\theta + 1)$

Cancel the common factor $2\cos 3\theta + 1$: the fraction is $\tan 4\theta$.

Ans: $\tan 4\theta$

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10. Show that $\sin 15^\circ \sin 45^\circ \sin 75^\circ = \frac{\sqrt{2}}{8}$. [5]

$\sin 45^\circ \sin 75^\circ = \frac{1}{2}[\cos 30^\circ - \cos 120^\circ] = \frac{1}{2}\cos 30^\circ + \frac{1}{4}$

$\sin 15^\circ\left(\frac{1}{2}\cos 30^\circ + \frac{1}{4}\right) = \frac{1}{4}(2\sin 15^\circ \cos 30^\circ) + \frac{1}{4}\sin 15^\circ$

$= \frac{1}{4}[\sin 45^\circ - \sin 15^\circ] + \frac{1}{4}\sin 15^\circ = \frac{1}{4}\sin 45^\circ$

$= \frac{1}{4}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{8}$

Ans: Shown: $\frac{\sqrt{2}}{8}$

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11. In a triangle ABC, express $\sin(B + C)$ in terms of A. [1]

In a triangle, $B + C = 180^\circ - A$.

$\sin(B + C) = \sin A$

Ans: $\sin A$

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12. Verify $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ for the triangle with $A = 60^\circ$, $B = 60^\circ$, $C = 60^\circ$. [2]

LHS $= \frac{3\sqrt{3}}{2}$

RHS $= \frac{3\sqrt{3}}{2}$

Both sides equal $\frac{3\sqrt{3}}{2}$, so the identity holds.

Ans: Both sides equal $\frac{3\sqrt{3}}{2}$

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13. In a triangle ABC, prove that $\sin 2A + \sin 2B - \sin 2C = 4\cos A\cos B\sin C$. [5]

$A + B + C = 180^\circ$, so $A + B = 180^\circ - C$ and similar.

$\sin 2A + \sin 2B = 2\sin(A + B)\cos(A - B) = 2\sin C\cos(A - B)$

$\sin 2C = 2\sin C\cos C = -2\sin C\cos(A + B)$

$\text{LHS} = 2\sin C[\cos(A - B) + \cos(A + B)] = 2\sin C \cdot 2\cos A\cos B$

$= 4\cos A\cos B\sin C = \text{RHS}$

Ans: Proved

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14. In a triangle ABC, prove that $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$. [5]

$A + B + C = 180°$, so $A + B = 180° - C$ and similar.

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} = 2\cos\frac{C}{2}\cos\frac{A-B}{2}$

$\sin C = 2\sin\frac{C}{2}\cos\frac{C}{2} = 2\cos\frac{C}{2}\cos\frac{A+B}{2}$

$\text{LHS} = 2\cos\frac{C}{2}\left[\cos\frac{A-B}{2} + \cos\frac{A+B}{2}\right] = 2\cos\frac{C}{2}\cdot 2\cos\frac{A}{2}\cos\frac{B}{2}$

$= 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} = \text{RHS}$

Ans: Proved

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15. If $\cos x + \cos y = \frac{3}{2}$ and $\sin x + \sin y = \frac{2}{7}$, find $\tan\frac{x+y}{2}$. [2]

Transforming: $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{3}{2}$ and $2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{2}{7}$.

Dividing: $\tan\frac{x+y}{2} = \frac{4}{21}$

Ans: $\tan\frac{x+y}{2} = \frac{4}{21}$

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16. If $\cos x - \cos y = \frac{3}{4}$ and $\sin x - \sin y = \frac{2}{3}$, find $\sin(x+y)$ and $\cos(x+y)$. [3]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{3}{4}$ and $2\cos\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{2}{3}$.

Dividing: $\tan\frac{x+y}{2} = -\frac{9}{8}$

With $t = -\frac{9}{8}$: $\sin(x+y) = \frac{2t}{1+t^2} = -\frac{144}{145}$, $\cos(x+y) = \frac{1-t^2}{1+t^2} = -\frac{17}{145}$

Ans: $\sin(x+y) = -\frac{144}{145}$, $\cos(x+y) = -\frac{17}{145}$

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17. If $\cos x + \cos y = \frac{5}{6}$ and $\sin x - \sin y = \frac{5}{3}$, find $\tan\frac{x-y}{2}$, $\sin(x-y)$, $\cos(x-y)$ and $\tan(x-y)$. [5]

Transforming: $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{5}{6}$ and $2\cos\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{5}{3}$.

Dividing: $\tan\frac{x-y}{2} = 2$

With $t = 2$: $\sin(x-y) = \frac{2t}{1+t^2} = \frac{4}{5}$, $\cos(x-y) = \frac{1-t^2}{1+t^2} = -\frac{3}{5}$

$\tan(x-y) = \frac{\sin(x-y)}{\cos(x-y)} = -\frac{4}{3}$

Ans: $\tan\frac{x-y}{2} = 2$, $\sin(x-y) = \frac{4}{5}$, $\cos(x-y) = -\frac{3}{5}$, $\tan(x-y) = -\frac{4}{3}$

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18. If $\cos x + \cos y = \frac{1}{3}$ and $\sin x + \sin y = \frac{3}{4}$, find $\cos(x-y)$. [2]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 + 2(\cos x\cos y + \sin x\sin y) = 2 + 2\cos(x-y)$

$\frac{1}{9} + \frac{9}{16} = 2 + 2\cos(x-y)$

$\cos(x-y) = -\frac{191}{288}$

Ans: $\cos(x-y) = -\frac{191}{288}$

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19. If $\cos x + \cos y = \frac{1}{5}$ and $\sin x + \sin y = \frac{4}{5}$, find $\cos(x-y)$ and $\cos\frac{x-y}{2}$. [3]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 + 2(\cos x\cos y + \sin x\sin y) = 2 + 2\cos(x-y)$

$\frac{1}{25} + \frac{16}{25} = 2 + 2\cos(x-y)$

$\cos(x-y) = -\frac{33}{50}$

$\cos^2\frac{x-y}{2} = \frac{1+\cos(x-y)}{2} = \frac{17}{100}$

$\cos\frac{x-y}{2} = \pm\frac{\sqrt{17}}{10}$

Ans: $\cos(x-y) = -\frac{33}{50}$, $\cos\frac{x-y}{2} = \pm\frac{\sqrt{17}}{10}$

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20. In a triangle ABC, $A = 60^\circ$ and $B = 60^\circ$. Find $\cos A + \cos B + \cos C$ and check that it does not exceed $\frac{3}{2}$. [1]

$C = 180^\circ - 60^\circ - 60^\circ = 60^\circ$

$\cos 60^\circ + \cos 60^\circ + \cos 60^\circ = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$

$\frac{3}{2} \approx 1.500 \leq 1.5$

Ans: $\frac{3}{2}$
