EXPRESSWALA ACADEMY

Maths 1A: Transformations

Question Paper, Set 02

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write $2\sin 4\theta \sin 2\theta$ as a sum or difference of two trigonometric ratios. [1 mark]

2. Evaluate $2\cos 105^\circ \cos 15^\circ$ by changing the product into a sum. [2 marks]

3. Express $\cos 3\theta - \cos 11\theta$ as a product. [1 mark]

4. Simplify $\sin 44^\circ + \sin 16^\circ$. [1 mark]

5. Evaluate $\cos 165^\circ + \cos 75^\circ$ using a sum to product formula. [2 marks]

6. Show that $\cos 34^\circ + \cos 86^\circ + \cos 154^\circ = 0$. [3 marks]

7. Find the value of $\frac{\sin 44^\circ + \sin 16^\circ}{\cos 44^\circ + \cos 16^\circ}$. [1 mark]

8. Show that $\frac{\cos\theta - \cos 11\theta}{\sin 11\theta + \sin\theta} = \tan 5\theta$. [2 marks]

9. Prove that $\frac{\sin 6\theta + \sin 8\theta + \sin 7\theta}{\cos 6\theta + \cos 8\theta + \cos 7\theta} = \tan 7\theta$. [3 marks]

10. Show that $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$. [5 marks]

11. In a triangle ABC, express $\cos(2C + 2B)$ in terms of A. [1 mark]

12. Verify $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2 + 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ for the triangle with $A = 45^\circ$, $B = 90^\circ$, $C = 45^\circ$. [2 marks]

13. If $A + B + C = 180^\circ$, prove that $-\sin^2 A + \sin^2 B + \sin^2 C = 2\cos A \sin B \sin C$. [5 marks]

14. If $A + B + C = 180^\circ$, prove that $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$. [5 marks]

15. If $\cos x - \cos y = \frac{3}{4}$ and $\sin x + \sin y = \frac{5}{4}$, find $\tan\frac{x-y}{2}$. [2 marks]

16. If $\cos x - \cos y = \frac{3}{4}$ and $\sin x - \sin y = \frac{4}{5}$, find $\sin(x + y)$ and $\cos(x + y)$. [3 marks]

17. If $\cos x - \cos y = \frac{5}{6}$ and $\sin x - \sin y = \frac{2}{3}$, find $\tan\frac{x+y}{2}$, $\sin(x + y)$, $\cos(x + y)$ and $\tan(x + y)$. [5 marks]

18. If $\cos x + \cos y = \frac{2}{5}$ and $\sin x + \sin y = \frac{4}{5}$, find $\cos(x - y)$. [2 marks]

19. If $\cos x + \cos y = \frac{5}{3}$ and $\sin x + \sin y = \frac{1}{2}$, find $\cos(x - y)$ and $\cos\frac{x-y}{2}$. [3 marks]

20. In a triangle ABC, $A = 15^\circ$ and $B = 75^\circ$. Find $\cos A + \cos B + \cos C$ and check that it does not exceed $\frac{3}{2}$. [1 mark]

Key sheet

Q	Answer
1	$\cos 2\theta - \cos 6\theta$
2	$-\frac{1}{2}$
3	$2\sin 7\theta \sin 4\theta$
4	$\cos 14^\circ$
5	$-\frac{\sqrt{2}}{2}$
6	Shown: the expression is 0
7	$\frac{\sqrt{3}}{3}$
8	$\tan 5\theta$
9	$\tan 7\theta$
10	Shown: $\frac{3}{16}$
11	$\cos 2A$
12	Both sides equal $\frac{\sqrt{2}}{2} + \frac{3}{2}$
13	Proved
14	Proved
15	$\tan\frac{x-y}{2} = -\frac{3}{5}$
16	$\sin(x+y) = -\frac{480}{481},\ \cos(x+y) = \frac{31}{481}$
17	$\tan\frac{x+y}{2} = -\frac{5}{4},\ \sin(x+y) = -\frac{40}{41},\ \cos(x+y) = -\frac{9}{41},\ \tan(x+y) = \frac{40}{9}$
18	$\cos(x-y) = -\frac{3}{5}$
19	$\cos(x-y) = \frac{37}{72},\ \cos\frac{x-y}{2} = \pm\frac{\sqrt{109}}{12}$
20	$\frac{\sqrt{6}}{2}$

Solutions

1. Write $2\sin 4\theta \sin 2\theta$ as a sum or difference of two trigonometric ratios. [1]

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$ with $A = 4\theta,\ B = 2\theta$.

$2\sin 4\theta \sin 2\theta = \cos 2\theta - \cos 6\theta$

Ans: $\cos 2\theta - \cos 6\theta$

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2. Evaluate $2\cos 105^\circ \cos 15^\circ$ by changing the product into a sum. [2]

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\cos 105^\circ \cos 15^\circ = \cos 120^\circ + \cos 90^\circ$

$= -\frac{1}{2} + 0 = -\frac{1}{2}$

Ans: $-\frac{1}{2}$

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3. Express $\cos 3\theta - \cos 11\theta$ as a product. [1]

$\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

$\cos 3\theta - \cos 11\theta = 2\sin 7\theta \sin 4\theta$

Ans: $2\sin 7\theta \sin 4\theta$

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4. Simplify $\sin 44^\circ + \sin 16^\circ$. [1]

The half sum of the angles is 30°, a standard angle.

$\sin 44^\circ + \sin 16^\circ = 2\sin 30^\circ \cos 14^\circ = \cos 14^\circ$

Ans: $\cos 14^\circ$

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5. Evaluate $\cos 165^\circ + \cos 75^\circ$ using a sum to product formula. [2]

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos 165^\circ + \cos 75^\circ = 2\cos 120^\circ \cos 45^\circ$

$= -\frac{\sqrt{2}}{2}$

Ans: $-\frac{\sqrt{2}}{2}$

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6. Show that $\cos 34^\circ + \cos 86^\circ + \cos 154^\circ = 0$. [3]

$\cos 154^\circ + \cos 86^\circ = 2\cos\frac{154^\circ+86^\circ}{2}\cos\frac{154^\circ-86^\circ}{2} = 2\cos 120^\circ \cos 34^\circ$

$= 2\left(-\frac{1}{2}\right)\cos 34^\circ = -\cos 34^\circ$

$\therefore \cos 34^\circ + \cos 86^\circ + \cos 154^\circ = \cos 34^\circ - \cos 34^\circ = 0$

Ans: Shown: the expression is 0

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7. Find the value of $\frac{\sin 44^\circ + \sin 16^\circ}{\cos 44^\circ + \cos 16^\circ}$. [1]

$\frac{\sin 44^\circ + \sin 16^\circ}{\cos 44^\circ + \cos 16^\circ} = \frac{2\sin 30^\circ \cos 14^\circ}{2\cos 30^\circ \cos 14^\circ}$

$= \tan 30^\circ = \frac{\sqrt{3}}{3}$

Ans: $\frac{\sqrt{3}}{3}$

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8. Show that $\frac{\cos\theta-\cos 11\theta}{\sin 11\theta+\sin\theta}=\tan 5\theta$. [2]

$\frac{\cos\theta-\cos 11\theta}{\sin 11\theta+\sin\theta}=\frac{2\sin 6\theta\sin 5\theta}{2\sin 6\theta\cos 5\theta}$

$=\tan 5\theta$

Ans: $\tan 5\theta$

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9. Prove that $\frac{\sin 6\theta+\sin 8\theta+\sin 7\theta}{\cos 6\theta+\cos 8\theta+\cos 7\theta}=\tan 7\theta$. [3]

$\sin 8\theta+\sin 6\theta=2\sin 7\theta\cos\theta$, $\cos 8\theta+\cos 6\theta=2\cos 7\theta\cos\theta$

$\text{Numerator}=\sin 7\theta(2\cos\theta+1)$, $\text{Denominator}=\cos 7\theta(2\cos\theta+1)$

Cancel the common factor $2\cos\theta+1$: the fraction is $\tan 7\theta$.

Ans: $\tan 7\theta$

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10. Show that $\sin 20^\circ\sin 40^\circ\sin 60^\circ\sin 80^\circ=\frac{3}{16}$. [5]

$\sin 60^\circ=\frac{\sqrt{3}}{2}$, so it is a constant factor.

$\sin 40^\circ\sin 80^\circ=\frac{1}{2}\cos 40^\circ+\frac{1}{4}$

Multiplying by $\sin 20^\circ$ and using product to sum again: $\sin 20^\circ\sin 40^\circ\sin 80^\circ=\frac{1}{4}\sin 60^\circ=\frac{\sqrt{3}}{8}$

$\therefore\ \sin 20^\circ\sin 40^\circ\sin 60^\circ\sin 80^\circ=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{8}=\frac{3}{16}$

Ans: Shown: $\frac{3}{16}$

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11. In a triangle ABC, express $\cos(2C+2B)$ in terms of A. [1]

In a triangle, $C+B=180^\circ-A$.

$\cos(2C+2B)=\cos 2A$

Ans: $\cos 2A$

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12. Verify $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=2+2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ for the triangle with $A=45^\circ$, $B=90^\circ$, $C=45^\circ$. [2]

LHS $=\frac{\sqrt{2}}{2}+\frac{3}{2}$

RHS $=\frac{\sqrt{2}}{2}+\frac{3}{2}$

Both sides equal $\frac{\sqrt{2}}{2}+\frac{3}{2}$, so the identity holds.

Ans: Both sides equal $\frac{\sqrt{2}}{2}+\frac{3}{2}$

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13. If $A+B+C=180^\circ$, prove that $-\sin^2 A+\sin^2 B+\sin^2 C=2\cos A\sin B\sin C$. [5]

$A+B+C=180^\circ$, so $A+B=180^\circ-C$ and similar.

$\sin^2 B+\sin^2 C=1-\frac{1}{2}(\cos 2B+\cos 2C)=1-\cos(B+C)\cos(B-C)=1+\cos A\cos(B-C)$

$-\sin^2 A=-1+\cos^2 A=-1-\cos A\cos(B+C)$

$\text{LHS}=\cos A[\cos(B-C)-\cos(B+C)]=\cos A\cdot 2\sin B\sin C$

$=2\cos A\sin B\sin C=\text{RHS}$

Ans: Proved

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14. If $A+B+C=180^\circ$, prove that $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$. [5]

$A+B+C=180^\circ$, so $A+B=180^\circ-C$ and similar.

$\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}=2\sin\frac{C}{2}\cos\frac{A-B}{2}$

$\cos C=1-2\sin^2\frac{C}{2}=1-2\sin\frac{C}{2}\cos\frac{A+B}{2}$

$\text{LHS}=1+2\sin\frac{C}{2}\left[\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right]=1+2\sin\frac{C}{2}\cdot 2\sin\frac{A}{2}\sin\frac{B}{2}$

$=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\text{RHS}$

Ans: Proved

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15. If $\cos x-\cos y=\frac{3}{4}$ and $\sin x+\sin y=\frac{5}{4}$, find $\tan\frac{x-y}{2}$. [2]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}=\frac{3}{4}$ and $2\sin\frac{x+y}{2}\cos\frac{x-y}{2}=\frac{5}{4}$.

Dividing: $\tan\frac{x-y}{2}=-\frac{3}{5}$

Ans: $\tan\frac{x-y}{2}=-\frac{3}{5}$

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16. If $\cos x-\cos y=\frac{3}{4}$ and $\sin x-\sin y=\frac{4}{5}$, find $\sin(x+y)$ and $\cos(x+y)$. [3]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}=\frac{3}{4}$ and $2\cos\frac{x+y}{2}\sin\frac{x-y}{2}=\frac{4}{5}$.

Dividing: $\tan\frac{x+y}{2}=-\frac{15}{16}$

With $t=-\frac{15}{16}$: $\sin(x+y)=\frac{2t}{1+t^2}=-\frac{480}{481}$, $\cos(x+y)=\frac{1-t^2}{1+t^2}=\frac{31}{481}$

Ans: $\sin(x+y)=-\frac{480}{481}$, $\cos(x+y)=\frac{31}{481}$

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17. If $\cos x-\cos y=\frac{5}{6}$ and $\sin x-\sin y=\frac{2}{3}$, find $\tan\frac{x+y}{2}$, $\sin(x+y)$, $\cos(x+y)$ and $\tan(x+y)$. [5]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}=\frac{5}{6}$ and $2\cos\frac{x+y}{2}\sin\frac{x-y}{2}=\frac{2}{3}$.

Dividing: $\tan\frac{x+y}{2}=-\frac{5}{4}$

With $t=-\frac{5}{4}$: $\sin(x+y)=\frac{2t}{1+t^2}=-\frac{40}{41}$, $\cos(x+y)=\frac{1-t^2}{1+t^2}=-\frac{9}{41}$

$\tan(x+y)=\frac{\sin(x+y)}{\cos(x+y)}=\frac{40}{9}$

Ans: $\tan\frac{x+y}{2}=-\frac{5}{4}$, $\sin(x+y)=-\frac{40}{41}$, $\cos(x+y)=-\frac{9}{41}$, $\tan(x+y)=\frac{40}{9}$

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18. If $\cos x+\cos y=\frac{2}{5}$ and $\sin x+\sin y=\frac{4}{5}$, find $\cos(x-y)$. [2]

Square both and add, using $\cos^2+\sin^2=1$:

$a^2+b^2=2+2(\cos x\cos y+\sin x\sin y)=2+2\cos(x-y)$

$\frac{4}{25}+\frac{16}{25}=2+2\cos(x-y)$

$\cos(x-y)=-\frac{3}{5}$

Ans: $\cos(x-y)=-\frac{3}{5}$

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19. If $\cos x+\cos y=\frac{5}{3}$ and $\sin x+\sin y=\frac{1}{2}$, find $\cos(x-y)$ and $\cos\frac{x-y}{2}$. [3]

Square both and add, using $\cos^2+\sin^2=1$:

$a^2+b^2=2+2(\cos x\cos y+\sin x\sin y)=2+2\cos(x-y)$

$\frac{25}{9}+\frac{1}{4}=2+2\cos(x-y)$

$\cos(x-y)=\frac{37}{72}$

$\cos^2\frac{x-y}{2}=\frac{1+\cos(x-y)}{2}=\frac{109}{144}$

$\cos\frac{x-y}{2}=\pm\frac{\sqrt{109}}{12}$

Ans: $\cos(x-y)=\frac{37}{72}$, $\cos\frac{x-y}{2}=\pm\frac{\sqrt{109}}{12}$

20. In a triangle ABC, $A = 15^\circ$ and $B = 75^\circ$. Find $\cos A + \cos B + \cos C$ and check that it does not exceed $\frac{3}{2}$. [1]

$C = 180^\circ - 15^\circ - 75^\circ = 90^\circ$

$\cos 15^\circ + \cos 75^\circ + \cos 90^\circ = \frac{\sqrt{2}+\sqrt{6}}{4} + \frac{-\sqrt{2}+\sqrt{6}}{4} + 0 = \frac{\sqrt{6}}{2}$

$\frac{\sqrt{6}}{2} \approx 1.225 \leq 1.5$

Ans: $\frac{\sqrt{6}}{2}$