EXPRESSWALA ACADEMY

Maths 1A: Transformations

Question Paper, Set 01

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write $2\sin 9\theta\cos 8\theta$ as a sum or difference of two trigonometric ratios. [1 mark]

2. Evaluate $2\sin 75^\circ\cos 15^\circ$ by changing the product into a sum. [2 marks]

3. Write $\sin 10\theta + \sin 6\theta$ in the form of a product. [1 mark]

4. Simplify $\cos 39^\circ + \cos 81^\circ$. [1 mark]

5. Evaluate $\sin 105^\circ - \sin 15^\circ$ using a sum to product formula. [2 marks]

6. Show that $\sin 114^\circ - \sin 54^\circ + \cos 96^\circ = 0$. [3 marks]

7. Find the value of $\frac{\sin 57^\circ - \sin 33^\circ}{\cos 33^\circ - \cos 57^\circ}$. [1 mark]

8. Simplify $\frac{\cos 4\theta - \cos 6\theta}{\sin 6\theta + \sin 4\theta}$. [2 marks]

9. Prove that $\frac{\sin 8\theta + \sin 2\theta + \sin 5\theta}{\cos 8\theta + \cos 2\theta + \cos 5\theta} = \tan 5\theta$. [3 marks]

10. Show that $\cos 20^\circ\cos 40^\circ\cos 60^\circ\cos 80^\circ = \frac{1}{16}$. [5 marks]

11. In a triangle ABC, express $\sin(B + A)$ in terms of C. [1 mark]

12. Verify $\cos 2A + \cos 2B - \cos 2C = 1 - 4\sin A\sin B\cos C$ for the triangle with $A = 60^\circ$, $B = 90^\circ$, $C = 30^\circ$. [2 marks]

13. In a triangle ABC, prove that $\sin^2 A - \sin^2 B + \sin^2 C = 2\sin A\cos B\sin C$. [5 marks]

14. In a triangle ABC, prove that $-\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$. [5 marks]

15. If $\cos x + \cos y = \frac{1}{5}$ and $\sin x - \sin y = \frac{3}{7}$, find $\tan\frac{x-y}{2}$. [2 marks]

16. If $\cos x - \cos y = \frac{5}{6}$ and $\sin x + \sin y = \frac{2}{5}$, find $\sin(x - y)$ and $\cos(x - y)$. [3 marks]

17. If $\cos x + \cos y = \frac{7}{5}$ and $\sin x + \sin y = \frac{3}{7}$, find $\tan\frac{x+y}{2}$, $\sin(x + y)$, $\cos(x + y)$ and $\tan(x + y)$. [5 marks]

18. If $\cos x - \cos y = \frac{1}{2}$ and $\sin x - \sin y = \frac{3}{7}$, find $\cos(x - y)$. [2 marks]

19. If $\cos x + \cos y = \frac{5}{6}$ and $\sin x + \sin y = \frac{4}{3}$, find $\cos(x - y)$ and $\cos\frac{x-y}{2}$. [3 marks]

20. In a triangle ABC, $A = 45^\circ$ and $B = 105^\circ$. Find $\cos A + \cos B + \cos C$ and check that it does not exceed $\frac{3}{2}$. [1 mark]

Key sheet

Q	Answer
1	$\sin 17\theta + \sin\theta$
2	$\frac{\sqrt{3}}{2} + 1$
3	$2\sin 8\theta \cos 2\theta$
4	$\cos 21^\circ$
5	$\frac{\sqrt{2}}{2}$
6	Shown: the expression is 0
7	1
8	$\tan\theta$
9	$\tan 5\theta$
10	Shown: $\frac{1}{16}$
11	$\sin C$
12	Both sides equal -2
13	Proved
14	Proved
15	$\tan\frac{x-y}{2} = \frac{15}{7}$
16	$\sin(x-y) = -\frac{600}{769},\ \cos(x-y) = -\frac{481}{769}$
17	$\tan\frac{x+y}{2} = \frac{15}{49},\ \sin(x+y) = \frac{735}{1313},\ \cos(x+y) = \frac{1088}{1313},\ \tan(x+y) = \frac{735}{1088}$
18	$\cos(x-y) = \frac{307}{392}$
19	$\cos(x-y) = \frac{17}{72},\ \cos\frac{x-y}{2} = \pm\frac{\sqrt{89}}{12}$
20	$\frac{-\sqrt{6}+2\sqrt{3}+3\sqrt{2}}{4}$

Solutions

1. Write $2\sin 9\theta\cos 8\theta$ as a sum or difference of two trigonometric ratios. [1]

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$ with $A = 9\theta,\ B = 8\theta$.

$2\sin 9\theta\cos 8\theta = \sin 17\theta + \sin\theta$

Ans: $\sin 17\theta + \sin\theta$

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2. Evaluate $2\sin 75^\circ\cos 15^\circ$ by changing the product into a sum. [2]

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$2\sin 75^\circ\cos 15^\circ = \sin 90^\circ + \sin 60^\circ$

$= 1 + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} + 1$

Ans: $\frac{\sqrt{3}}{2} + 1$

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3. Write $\sin 10\theta + \sin 6\theta$ in the form of a product. [1]

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin 10\theta + \sin 6\theta = 2\sin 8\theta\cos 2\theta$

Ans: $2\sin 8\theta\cos 2\theta$

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4. Simplify $\cos 39^\circ + \cos 81^\circ$. [1]

The half sum of the angles is 60°, a standard angle.

$\cos 39^\circ + \cos 81^\circ = 2\cos 60^\circ\cos 21^\circ = \cos 21^\circ$

Ans: $\cos 21^\circ$

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5. Evaluate $\sin 105^\circ - \sin 15^\circ$ using a sum to product formula. [2]

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\sin 105^\circ - \sin 15^\circ = 2\cos 60^\circ\sin 45^\circ$

$= \frac{\sqrt{2}}{2}$

Ans: $\frac{\sqrt{2}}{2}$

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6. Show that $\sin 114^\circ - \sin 54^\circ + \cos 96^\circ = 0$. [3]

$\sin 114^\circ - \sin 54^\circ = 2\cos\frac{114^\circ+54^\circ}{2}\sin\frac{114^\circ-54^\circ}{2} = 2\cos 84^\circ\sin 30^\circ = \cos 84^\circ$

$\cos 96^\circ = \cos(180^\circ - 84^\circ) = -\cos 84^\circ$

$\therefore \sin 114^\circ - \sin 54^\circ + \cos 96^\circ = \cos 84^\circ - \cos 84^\circ = 0$

Ans: Shown: the expression is 0

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7. Find the value of $\frac{\sin 57^\circ - \sin 33^\circ}{\cos 33^\circ - \cos 57^\circ}$. [1]

$\frac{\sin 57^\circ - \sin 33^\circ}{\cos 33^\circ - \cos 57^\circ} = \frac{2\cos 45^\circ\sin 12^\circ}{2\sin 45^\circ\sin 12^\circ}$

$= \cot 45^\circ = 1$

Ans: 1

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8. Simplify $\frac{\cos 4\theta - \cos 6\theta}{\sin 6\theta + \sin 4\theta}$. [2]

$\frac{\cos 4\theta - \cos 6\theta}{\sin 6\theta + \sin 4\theta} = \frac{2\sin 5\theta \sin \theta}{2\sin 5\theta \cos \theta}$

$= \tan\theta$

Ans: $\tan\theta$

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9. Prove that $\frac{\sin 8\theta + \sin 2\theta + \sin 5\theta}{\cos 8\theta + \cos 2\theta + \cos 5\theta} = \tan 5\theta$. [3]

$\sin 8\theta + \sin 2\theta = 2\sin 5\theta \cos 3\theta$, $\cos 8\theta + \cos 2\theta = 2\cos 5\theta \cos 3\theta$

$\text{Numerator} = \sin 5\theta(2\cos 3\theta + 1)$, $\text{Denominator} = \cos 5\theta(2\cos 3\theta + 1)$

Cancel the common factor $2\cos 3\theta + 1$: the fraction is $\tan 5\theta$.

Ans: $\tan 5\theta$

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10. Show that $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$. [5]

$\cos 60^\circ = \frac{1}{2}$, so it is a constant factor.

$\cos 40^\circ \cos 80^\circ = \frac{1}{2}\cos 40^\circ - \frac{1}{4}$

Multiplying by $\cos 20^\circ$ and using product to sum again: $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{4}\cos 60^\circ = \frac{1}{8}$

$\therefore \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}$

Ans: Shown: $\frac{1}{16}$

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11. In a triangle ABC, express $\sin(B + A)$ in terms of C. [1]

In a triangle, $B + A = 180^\circ - C$.

$\sin(B + A) = \sin C$

Ans: $\sin C$

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12. Verify $\cos 2A + \cos 2B - \cos 2C = 1 - 4\sin A \sin B \cos C$ for the triangle with $A = 60^\circ$, $B = 90^\circ$, $C = 30^\circ$. [2]

LHS $= -2$

RHS $= -2$

Both sides equal -2, so the identity holds.

Ans: Both sides equal -2

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13. In a triangle ABC, prove that $\sin^2 A - \sin^2 B + \sin^2 C = 2\sin A \cos B \sin C$. [5]

$A + B + C = 180^\circ$, so $A + B = 180^\circ - C$ and similar.

$\sin^2 A + \sin^2 C = 1 - \frac{1}{2}(\cos 2A + \cos 2C) = 1 - \cos(A + C)\cos(A - C) = 1 + \cos B \cos(A - C)$

$-\sin^2 B = -1 + \cos^2 B = -1 - \cos B \cos(A + C)$

$\text{LHS} = \cos B[\cos(A - C) - \cos(A + C)] = \cos B \cdot 2\sin A \sin C$

$= 2\sin A \cos B \sin C = \text{RHS}$

Ans: Proved

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14. In a triangle ABC, prove that $-\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$. [5]

$A+B+C = 180°$, so $A+B = 180° - C$ and similar.

$\sin B + \sin C = 2\sin\frac{B+C}{2}\cos\frac{B-C}{2} = 2\cos\frac{A}{2}\cos\frac{B-C}{2}$

$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\frac{A}{2}\cos\frac{B+C}{2}$

$\text{LHS} = 2\cos\frac{A}{2}\left[\cos\frac{B-C}{2} - \cos\frac{B+C}{2}\right] = 2\cos\frac{A}{2}\cdot 2\sin\frac{B}{2}\sin\frac{C}{2}$

$= 4\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \text{RHS}$

Ans: Proved

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15. If $\cos x + \cos y = \frac{1}{5}$ and $\sin x - \sin y = \frac{3}{7}$, find $\tan\frac{x-y}{2}$. [2]

Transforming: $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{1}{5}$ and $2\cos\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{3}{7}$.

Dividing: $\tan\frac{x-y}{2} = \frac{15}{7}$

Ans: $\tan\frac{x-y}{2} = \frac{15}{7}$

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16. If $\cos x - \cos y = \frac{5}{6}$ and $\sin x + \sin y = \frac{2}{5}$, find $\sin(x-y)$ and $\cos(x-y)$. [3]

Transforming: $-2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = \frac{5}{6}$ and $2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{2}{5}$.

Dividing: $\tan\frac{x-y}{2} = -\frac{25}{12}$

With $t = -\frac{25}{12}$: $\sin(x-y) = \frac{2t}{1+t^2} = -\frac{600}{769}$, $\cos(x-y) = \frac{1-t^2}{1+t^2} = -\frac{481}{769}$

Ans: $\sin(x-y) = -\frac{600}{769}$, $\cos(x-y) = -\frac{481}{769}$

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17. If $\cos x + \cos y = \frac{7}{5}$ and $\sin x + \sin y = \frac{3}{7}$, find $\tan\frac{x+y}{2}$, $\sin(x+y)$, $\cos(x+y)$ and $\tan(x+y)$. [5]

Transforming: $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{7}{5}$ and $2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{3}{7}$.

Dividing: $\tan\frac{x+y}{2} = \frac{15}{49}$

With $t = \frac{15}{49}$: $\sin(x+y) = \frac{2t}{1+t^2} = \frac{735}{1313}$, $\cos(x+y) = \frac{1-t^2}{1+t^2} = \frac{1088}{1313}$

$\tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{735}{1088}$

Ans: $\tan\frac{x+y}{2} = \frac{15}{49}$, $\sin(x+y) = \frac{735}{1313}$, $\cos(x+y) = \frac{1088}{1313}$, $\tan(x+y) = \frac{735}{1088}$

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18. If $\cos x - \cos y = \frac{1}{2}$ and $\sin x - \sin y = \frac{3}{7}$, find $\cos(x-y)$. [2]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 - 2(\cos x\cos y + \sin x\sin y) = 2 - 2\cos(x-y)$

$\frac{1}{4} + \frac{9}{49} = 2 - 2\cos(x-y)$

$\cos(x-y) = \frac{307}{392}$

Ans: $\cos(x-y) = \frac{307}{392}$

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19. If $\cos x + \cos y = \frac{5}{6}$ and $\sin x + \sin y = \frac{4}{3}$, find $\cos(x-y)$ and $\cos\frac{x-y}{2}$. [3]

Square both and add, using $\cos^2 + \sin^2 = 1$:

$a^2 + b^2 = 2 + 2(\cos x\cos y + \sin x\sin y) = 2 + 2\cos(x-y)$

$\frac{25}{36} + \frac{16}{9} = 2 + 2\cos(x-y)$

$\cos(x-y) = \frac{17}{72}$

$\cos^2\frac{x-y}{2} = \frac{1+\cos(x-y)}{2} = \frac{89}{144}$

$\cos\frac{x-y}{2} = \pm\frac{\sqrt{89}}{12}$

Ans: $\cos(x-y) = \frac{17}{72}$, $\cos\frac{x-y}{2} = \pm\frac{\sqrt{89}}{12}$

20. In a triangle ABC, $A = 45^\circ$ and $B = 105^\circ$. Find $\cos A + \cos B + \cos C$ and check that it does not exceed $\frac{3}{2}$. [1]

$C = 180^\circ - 45^\circ - 105^\circ = 30^\circ$

$\cos 45^\circ + \cos 105^\circ + \cos 30^\circ = \frac{\sqrt{2}}{2} + \frac{-\sqrt{6}+\sqrt{2}}{4} + \frac{\sqrt{3}}{2} = \frac{-\sqrt{6}+2\sqrt{3}+3\sqrt{2}}{4}$

$\frac{-\sqrt{6}+2\sqrt{3}+3\sqrt{2}}{4} \approx 1.314 \leq 1.5$

Ans: $\frac{-\sqrt{6}+2\sqrt{3}+3\sqrt{2}}{4}$