EXPRESSWALA ACADEMY

Maths 1B: Straight Lines

Question Paper, Set 09

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the slope of a line perpendicular to the line through $(-3, 3)$ and $(-2, -6)$. [1 mark]

2. Find a if the distance between $(a, -2)$ and $(2, 3)$ is 13. [2 marks]

3. Find the point dividing the join of $A(2, 2)$ and $B(-3, 3)$ internally in the ratio $1 : 1$. [2 marks]

4. The midpoint of AB is $(-4, -3)$ and $A = (5, 1)$. Find B. [1 mark]

5. Show that the points $A(3, -3)$, $B(8, 9)$, $C(21, 9)$, $D(16, -3)$ taken in order form a rhombus, and find its area. [5 marks]

6. Find a if the points $(-5, 3)$, $(1, 3)$ and $(-2, a)$ are collinear. [3 marks]

7. Find the locus of a point P at which the segment joining $A(3, -2)$ and $B(0, 4)$ subtends a right angle. [3 marks]

8. Find the equation of the line through $(-1, -3)$ parallel to $4x + 4y - 1 = 0$. [2 marks]

9. Find the area of the triangle formed by $2x + y - 10 = 0$ with the coordinate axes. [1 mark]

10. Write $x - \sqrt{3}\,y - 16 = 0$ in the normal form and find p and α. [2 marks]

11. Find the points on the line through $(0, 5)$ with inclination 45°, at a distance $6\sqrt{2}$ from $(0, 5)$. [3 marks]

12. Find the perpendicular distance from $(1, 1)$ to $4x - 3y + 4 = 0$. [1 mark]

13. Do $(-1, 6)$ and $(6, 1)$ lie on the same side of $2x + y + 9 = 0$ or on opposite sides? [1 mark]

14. Find the equation of the line through the point of intersection of $x - y - 6 = 0$ and $3x - y - 14 = 0$ and the point $(-1, 7)$. [2 marks]

15. Find the orthocentre of the triangle with vertices $A(3, 4)$, $B(6, 5)$, $C(5, -2)$. [5 marks]

16. Find a if $x + ay - 4 = 0$ is parallel to $2x + 2y + 1 = 0$. [1 mark]

17. Find the bisectors of the angles between $12x + 5y + 3 = 0$ and $15x + 8y + 3 = 0$, and say which one bisects the acute angle. [5 marks]

18. Show that the lines $2y - 2 = 0$, $2y + 1 = 0$, $x + y - 8 = 0$, $x + y + 3 = 0$ form a parallelogram and find its area. [5 marks]

19. Find the image of $(-5, -3)$ in the line $3x + 4y - 3 = 0$. [2 marks]

20. A line through $(1, 25)$ meets the positive x- and y-axes at A and B. Find the least value of $OA + OB$. [3 marks]

Key sheet

Q	Answer
1	$\frac{1}{9}$
2	$a = 14$ or $a = -10$
3	$(-\frac{1}{2}, \frac{5}{2})$
4	$B = (-13, -7)$
5	A rhombus of area 156
6	$a = 3$
7	$x^2 + y^2 - 3x - 2y - 8 = 0$
8	$x + y + 4 = 0$
9	25
10	$x\cos 300^\circ + y\sin 300^\circ = 8,\ p = 8,\ \alpha = 300^\circ$
11	$(6, 11),\ (-6, -1)$
12	1
13	The same side
14	$9x + 5y - 26 = 0$
15	$(3, 4)$
16	$a = 1$
17	$9x - 19y + 12 = 0,\ 133x + 63y + 30 = 0$; the acute angle bisector is $133x + 63y + 30 = 0$
18	A parallelogram of area $\frac{33}{2}$
19	$(\frac{11}{5}, \frac{33}{5})$
20	36

Solutions

1. Find the slope of a line perpendicular to the line through $(-3, 3)$ and $(-2, -6)$. [1]

$m = \frac{-6-3}{-2-(-3)} = -9$

A perpendicular line has slope $-\frac{1}{m} = \frac{1}{9}$

Ans: $\frac{1}{9}$

---

2. Find a if the distance between $(a, -2)$ and $(2, 3)$ is 13. [2]

$(a-2)^2 + (-2-3)^2 = 13^2$

$(a-2)^2 = 169 - 25 = 144$

$a - 2 = \pm 12$

$a = 14 \text{ or } -10$

Ans: $a = 14$ or $a = -10$

---

3. Find the point dividing the join of $A(2, 2)$ and $B(-3, 3)$ internally in the ratio $1 : 1$. [2]

$P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$= \left(\frac{1(-3)+1(2)}{2}, \frac{1(3)+1(2)}{2}\right)$

$= (-\frac{1}{2}, \frac{5}{2})$

Ans: $(-\frac{1}{2}, \frac{5}{2})$

---

4. The midpoint of AB is $(-4, -3)$ and $A = (5, 1)$. Find B. [1]

$B = 2M - A = (-8-5, -6-1) = (-13, -7)$

Ans: $B = (-13, -7)$

---

5. Show that the points $A(3, -3)$, $B(8, 9)$, $C(21, 9)$, $D(16, -3)$ taken in order form a rhombus, and find its area. [5]

Midpoint of $AC = (12, 3)$ = midpoint of BD, so the diagonals bisect each other: $ABCD$ is a parallelogram.

$AB^2 = 169,\ BC^2 = 169,\ AC^2 = 468,\ BD^2 = 208$

$AB = BC$ but $AC \neq BD$: equal sides, unequal diagonals, so it is a rhombus.

$\text{Area} = \frac{1}{2}\,|(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)| = 156$

Ans: A rhombus of area 156

---

6. Find a if the points $(-5, 3)$, $(1, 3)$ and $(-2, a)$ are collinear. [3]

Area $= 0$: $-5(3-a) + 1(a-3) + (-2)(3-3) = 0$

$a = 3$

Ans: $a = 3$

---

7. Find the locus of a point P at which the segment joining $A(3,-2)$ and $B(0,4)$ subtends a right angle. [3]

Let $P(x,y)$ be on the locus: $PA \perp PB:\ (x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$x^2+y^2-3x-2y-8=0$

This is a circle.

Ans: $x^2+y^2-3x-2y-8=0$

---

8. Find the equation of the line through $(-1,-3)$ parallel to $4x+4y-1=0$. [2]

A parallel line is $4x+4y=k$

$k=-16$

$x+y+4=0$

Ans: $x+y+4=0$

---

9. Find the area of the triangle formed by $2x+y-10=0$ with the coordinate axes. [1]

Intercepts 5 and 10.

$\text{Area}=\frac{c^2}{2|ab|}=\frac{100}{4}=25$

Ans: 25

---

10. Write $x-\sqrt{3}\,y-16=0$ in the normal form and find p and α. [2]

Move the constant to the right so that it is positive, then divide by $\sqrt{a^2+b^2}=2$

$x\cos 300^\circ+y\sin 300^\circ=8$

Ans: $x\cos 300^\circ+y\sin 300^\circ=8,\ p=8,\ \alpha=300^\circ$

---

11. Find the points on the line through $(0,5)$ with inclination 45°, at a distance $6\sqrt{2}$ from $(0,5)$. [3]

$\cos\theta=\frac{\sqrt{2}}{2},\ \sin\theta=\frac{\sqrt{2}}{2}$

$(x_1,y_1)\pm r(\cos\theta,\sin\theta)=(0,5)\pm 6\sqrt{2}\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

$(6,11)$ and $(-6,-1)$

Ans: $(6,11),\ (-6,-1)$

---

12. Find the perpendicular distance from $(1,1)$ to $4x-3y+4=0$. [1]

$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}=\frac{|5|}{\sqrt{25}}=1$

Ans: 1

---

13. Do $(-1,6)$ and $(6,1)$ lie on the same side of $2x+y+9=0$ or on opposite sides? [1]

$L_{11}=13,\ L_{22}=22$

$L_{11}L_{22}>0$, so they lie on the same side.

Ans: The same side

---

14. Find the equation of the line through the point of intersection of $x - y - 6 = 0$ and $3x - y - 14 = 0$ and the point $(-1, 7)$. [2]

The line is $(x - y - 6) + \lambda(3x - y - 14) = 0$

At $(-1, 7)$: $-14 + \lambda(-24) = 0 \Rightarrow \lambda = -\frac{7}{12}$

$9x + 5y - 26 = 0$

Ans: $9x + 5y - 26 = 0$

---

15. Find the orthocentre of the triangle with vertices $A(3, 4)$, $B(6, 5)$, $C(5, -2)$. [5]

The altitude through A is perpendicular to BC: $x + 7y - 31 = 0$

The altitude through B is perpendicular to CA: $x - 3y + 9 = 0$

Solving the two altitudes: the orthocentre is $(3, 4)$

Ans: $(3, 4)$

---

16. Find a if $x + ay - 4 = 0$ is parallel to $2x + 2y + 1 = 0$. [1]

$\frac{a_1}{a_2} = \frac{b_1}{b_2} : \frac{1}{2} = \frac{a}{2}$

$a = 1$

Ans: $a = 1$

---

17. Find the bisectors of the angles between $12x + 5y + 3 = 0$ and $15x + 8y + 3 = 0$, and say which one bisects the acute angle. [5]

$\frac{12x+5y+3}{13} = \pm\frac{15x+8y+3}{17}$

$+$ sign: $9x - 19y + 12 = 0$

$-$ sign: $133x + 63y + 30 = 0$

Both constants are positive and $a_1a_2 + b_1b_2 = 220 > 0$, so the $-$ sign gives the acute angle bisector.

Acute angle bisector: $133x + 63y + 30 = 0$

Ans: $9x - 19y + 12 = 0,\ 133x + 63y + 30 = 0$**; the acute angle bisector is** $133x + 63y + 30 = 0$

---

18. Show that the lines $2y - 2 = 0$, $2y + 1 = 0$, $x + y - 8 = 0$, $x + y + 3 = 0$ form a parallelogram and find its area. [5]

The first two lines are parallel, and so are the last two.

$d_1 = \frac{|-3|}{\sqrt{4}} = \frac{3}{2},\ d_2 = \frac{|-11|}{\sqrt{2}} = \frac{11\sqrt{2}}{2}$

$aa_1 + bb_1 = 2$: the pairs are not perpendicular, and the widths are unequal, so it is a parallelogram.

$\text{Area} = \left|\frac{(c_1-c_2)(d_1-d_2)}{a_1b_2-a_2b_1}\right| = \left|\frac{(-3)(-11)}{-2}\right| = \frac{33}{2}$

Ans: A parallelogram of area $\frac{33}{2}$

---

19. Find the image of $(-5, -3)$ in the line $3x + 4y - 3 = 0$. [2]

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2} = \frac{60}{25}$

$(h, k) = (\frac{11}{5}, \frac{33}{5})$

Ans: $(\frac{11}{5}, \frac{33}{5})$

---

20. A line through $(1, 25)$ meets the positive x- and y-axes at A and B. Find the least value of $OA + OB$. [3]

$(\sqrt{x_1} + \sqrt{y_1})^2 = (\sqrt{1} + \sqrt{25})^2 = 36$

Ans: 36
