EXPRESSWALA ACADEMY

Maths 1B: Straight Lines

Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the slope of the line through $(0, -4)$ and $(-5, 6)$. [1 mark]

2. Find a if the distance between $(a, -7)$ and $(-1, -4)$ is 5. [2 marks]

3. Find the point dividing the join of $A(-4, 4)$ and $B(7, 0)$ internally in the ratio $2 : 1$. [2 marks]

4. The midpoint of AB is $(-1, 2)$ and $A = (3, -6)$. Find B. [1 mark]

5. Show that the points $A(-3, 3)$, $B(-6, 0)$, $C(3, -9)$, $D(6, -6)$ taken in order form a rectangle, and find its area. [5 marks]

6. Find the area of the quadrilateral whose vertices, taken in order, are $(-1, -4)$, $(-2, -2)$, $(1, 0)$, $(1, -4)$. [3 marks]

7. Find the locus of a point P such that $PA^2 + PB^2 = 32$, where $A = (3, 4)$ and $B = (-3, -4)$. [3 marks]

8. Find the equation of the line through $(2, 6)$ parallel to $3x - 2y - 3 = 0$. [2 marks]

9. Find the equation of the line with x-intercept 4 and y-intercept -4. [1 mark]

10. Write $\sqrt{3}\,x - y + 4 = 0$ in the normal form and find p and α. [2 marks]

11. Find the points on the line through $(-6, 6)$ with $\tan\theta = -\frac{5}{12}$, at a distance 26 from $(-6, 6)$. [3 marks]

12. Find the perpendicular distance from $(3, 1)$ to $8x - 15y + 15 = 0$. [1 mark]

13. In what ratio does $x + 6y - 4 = 0$ divide the join of $(-3, 6)$ and $(1, -4)$? [1 mark]

14. Find the point of intersection of $2x + y = 0$ and $y + 4 = 0$. [2 marks]

15. Find the orthocentre of the triangle with vertices $A(1, 4)$, $B(-1, 0)$, $C(-1, -1)$. [5 marks]

16. Find a if $3x + ay + 4 = 0$ is parallel to $4x - 4y - 5 = 0$. [1 mark]

17. Find the bisectors of the angles between $12x - 5y + 3 = 0$ and $3x + 4y + 8 = 0$, and say which one bisects the acute angle. [5 marks]

18. Show that the lines $x - 3y + 1 = 0$, $x - 3y + 8 = 0$, $x + 4y - 1 = 0$, $x + 4y + 3 = 0$ form a parallelogram and find its area. [5 marks]

19. Find the image of $(-5, -4)$ in the line $x + 3y + 2 = 0$. [2 marks]

20. A line through $(1, 9)$ meets the positive x- and y-axes at A and B. Find the least value of $OA + OB$. [3 marks]

Key sheet

Q	Answer
1	-2
2	$a = 3$ or $a = -5$
3	$(\frac{10}{3}, \frac{4}{3})$
4	$B = (-5, 10)$
5	A rectangle of area 54
6	8 square units
7	$x^2 + y^2 + 9 = 0$
8	$3x - 2y + 6 = 0$
9	$x - y - 4 = 0$
10	$x\cos 150^\circ + y\sin 150^\circ = 2,\ p = 2,\ \alpha = 150^\circ$
11	$(18, -4),\ (-30, 16)$
12	$\frac{24}{17}$
13	$29 : 27$ internally
14	$(2, -4)$
15	$(-11, 4)$
16	$a = -3$
17	$21x - 77y - 89 = 0,\ 99x + 27y + 119 = 0$; the acute angle bisector is $99x + 27y + 119 = 0$
18	A parallelogram of area 4
19	$(-2, 5)$
20	16

Solutions

1. Find the slope of the line through $(0, -4)$ and $(-5, 6)$. [1]

$m = \frac{6-(-4)}{-5-0} = -2$

Ans: -2

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2. Find a if the distance between $(a, -7)$ and $(-1, -4)$ is 5. [2]

$(a-(-1))^2 + (-7-(-4))^2 = 5^2$

$(a-(-1))^2 = 25 - 9 = 16$

$a-(-1) = \pm 4$

$a = 3 \text{ or } -5$

Ans: $a = 3$ or $a = -5$

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3. Find the point dividing the join of $A(-4, 4)$ and $B(7, 0)$ internally in the ratio $2:1$. [2]

$P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$= \left(\frac{2(7)+1(-4)}{3}, \frac{2(0)+1(4)}{3}\right)$

$= \left(\frac{10}{3}, \frac{4}{3}\right)$

Ans: $\left(\frac{10}{3}, \frac{4}{3}\right)$

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4. The midpoint of AB is $(-1, 2)$ and $A = (3, -6)$. Find B. [1]

$B = 2M - A = (-2-3, 4-(-6)) = (-5, 10)$

Ans: $B = (-5, 10)$

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5. Show that the points $A(-3, 3)$, $B(-6, 0)$, $C(3, -9)$, $D(6, -6)$ taken in order form a rectangle, and find its area. [5]

Midpoint of $AC = (0, -3)$ = midpoint of BD, so the diagonals bisect each other: $ABCD$ is a parallelogram.

$AB^2 = 18,\ BC^2 = 162,\ AC^2 = 180,\ BD^2 = 180$

$AB \neq BC$ but $AC = BD$: equal diagonals, so it is a rectangle.

$\text{Area} = \frac{1}{2}\left|(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)\right| = 54$

Ans: A rectangle of area 54

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6. Find the area of the quadrilateral whose vertices, taken in order, are $(-1, -4)$, $(-2, -2)$, $(1, 0)$, $(1, -4)$. [3]

Cross-multiply around the boundary:

$\frac{1}{2}\left|(-6) + 2 + (-4) + (-8)\right| = 8$

Ans: 8 **square units**

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7. Find the locus of a point P such that $PA^2 + PB^2 = 32$, where $A = (3, 4)$ and $B = (-3, -4)$. [3]

Let $P(x, y)$ be on the locus: $PA^2 + PB^2 = 32$

$x^2 + y^2 + 9 = 0$

This is a circle.

Ans: $x^2 + y^2 + 9 = 0$

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8. Find the equation of the line through $(2, 6)$ parallel to $3x - 2y - 3 = 0$. [2]

A parallel line is $3x - 2y = k$

$k = -6$

$3x - 2y + 6 = 0$

Ans: $3x - 2y + 6 = 0$

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9. Find the equation of the line with x-intercept 4 and y-intercept -4. [1]

$\frac{x}{4} + \frac{y}{-4} = 1$

$x - y - 4 = 0$

Ans: $x - y - 4 = 0$

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10. Write $\sqrt{3}\,x - y + 4 = 0$ in the normal form and find p and α. [2]

Move the constant to the right so that it is positive, then divide by $\sqrt{a^2 + b^2} = 2$

$x\cos 150^\circ + y\sin 150^\circ = 2$

Ans: $x\cos 150^\circ + y\sin 150^\circ = 2,\ p = 2,\ \alpha = 150^\circ$

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11. Find the points on the line through $(-6, 6)$ with $\tan\theta = -\frac{5}{12}$, at a distance 26 from $(-6, 6)$. [3]

$\cos\theta = \frac{12}{13},\ \sin\theta = -\frac{5}{13}$

$(x_1, y_1) \pm r(\cos\theta, \sin\theta) = (-6, 6) \pm 26\left(\frac{12}{13}, -\frac{5}{13}\right)$

$(18, -4)$ and $(-30, 16)$

Ans: $(18, -4),\ (-30, 16)$

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12. Find the perpendicular distance from $(3, 1)$ to $8x - 15y + 15 = 0$. [1]

$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|24|}{\sqrt{289}} = \frac{24}{17}$

Ans: $\frac{24}{17}$

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13. In what ratio does $x + 6y - 4 = 0$ divide the join of $(-3, 6)$ and $(1, -4)$? [1]

$L_{11} = 29,\ L_{22} = -27$

Ratio $-L_{11} : L_{22} = -29 : -27$, that is $29 : 27$ internally

Ans: $29 : 27$ **internally**

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14. Find the point of intersection of $2x + y = 0$ and $y + 4 = 0$. [2]

$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$x = \frac{4}{2},\ y = \frac{-8}{2}$

$(2, -4)$

Ans: $(2, -4)$

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15. Find the orthocentre of the triangle with vertices $A(1, 4)$, $B(-1, 0)$, $C(-1, -1)$. [5]

The altitude through A is perpendicular to BC: $y - 4 = 0$

The altitude through B is perpendicular to CA: $2x + 5y + 2 = 0$

Solving the two altitudes: the orthocentre is $(-11, 4)$

Ans: $(-11, 4)$

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16. Find a if $3x + ay + 4 = 0$ is parallel to $4x - 4y - 5 = 0$. [1]

$\frac{a_1}{a_2} = \frac{b_1}{b_2} : \frac{3}{4} = \frac{a}{-4}$

$a = -3$

Ans: $a = -3$

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17. Find the bisectors of the angles between $12x - 5y + 3 = 0$ and $3x + 4y + 8 = 0$, and say which one bisects the acute angle. [5]

$\frac{12x-5y+3}{13} = \pm\frac{3x+4y+8}{5}$

$+$ sign: $21x - 77y - 89 = 0$

$-$ sign: $99x + 27y + 119 = 0$

Both constants are positive and $a_1a_2 + b_1b_2 = 16 > 0$, so the $-$ sign gives the acute angle bisector.

Acute angle bisector: $99x + 27y + 119 = 0$

Ans: $21x - 77y - 89 = 0,\ 99x + 27y + 119 = 0$**; the acute angle bisector is** $99x + 27y + 119 = 0$

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18. Show that the lines $x - 3y + 1 = 0$, $x - 3y + 8 = 0$, $x + 4y - 1 = 0$, $x + 4y + 3 = 0$ form a parallelogram and find its area. [5]

The first two lines are parallel, and so are the last two.

$d_1 = \frac{|-7|}{\sqrt{10}} = \frac{7\sqrt{10}}{10},\ d_2 = \frac{|-4|}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$

$aa_1 + bb_1 = -11$: the pairs are not perpendicular, and the widths are unequal, so it is a parallelogram.

$\text{Area} = \left|\frac{(c_1-c_2)(d_1-d_2)}{a_1b_2-a_2b_1}\right| = \left|\frac{(-7)(-4)}{7}\right| = 4$

Ans: A parallelogram of area 4

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19. Find the image of $(-5, -4)$ in the line $x + 3y + 2 = 0$. [2]

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2} = \frac{30}{10}$

$(h, k) = (-2, 5)$

Ans: $(-2, 5)$

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20. A line through $(1, 9)$ meets the positive x- and y-axes at A and B. Find the least value of $OA + OB$. [3]

$(\sqrt{x_1} + \sqrt{y_1})^2 = (\sqrt{1} + \sqrt{9})^2 = 16$

Ans: 16
