EXPRESSWALA ACADEMY

Maths 1B: Straight Lines

Question Paper, Set 07

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the inclination of a line with slope $\frac{\sqrt{3}}{3}$. [1 mark]

2. Find a if the distance between $(a, 30)$ and $(-4, 6)$ is 25. [2 marks]

3. Find the point dividing the join of $A(2, 1)$ and $B(3, -5)$ internally in the ratio $1 : 4$. [2 marks]

4. The midpoint of AB is $(2, 4)$ and $A = (1, 2)$. Find B. [1 mark]

5. Show that the points $A(1, -3)$, $B(4, 1)$, $C(8, 4)$, $D(5, 0)$ taken in order form a rhombus, and find its area. [5 marks]

6. Find a if the points $(1, 4)$, $(-5, 7)$ and $(5, a)$ are collinear. [3 marks]

7. Find the locus of a point P such that $PA^2 + PB^2 = 74$, where $A = (5, -2)$ and $B = (-3, -1)$. [3 marks]

8. Find the equation of the line through $(-3, 3)$ perpendicular to $5x + y - 9 = 0$. [2 marks]

9. Find the intercepts made by $2x + y - 8 = 0$ on the axes. [1 mark]

10. Write $\sqrt{3}\,x + y + 2 = 0$ in the normal form and find p and α. [2 marks]

11. Find the points on the line through $(-2, 2)$ with inclination 30°, at a distance 6 from $(-2, 2)$. [3 marks]

12. Find the perpendicular distance from $(6, 3)$ to $x - 3y + 4 = 0$. [1 mark]

13. In what ratio does $x + 5y - 2 = 0$ divide the join of $(1, 5)$ and $(3, -2)$? [1 mark]

14. Find the equation of the line through the point of intersection of $x = 0$ and $2x + y - 5 = 0$ and the point $(-6, 6)$. [2 marks]

15. Find the circumcentre of the triangle with vertices $A(1, 2)$, $B(-1, 6)$, $C(-4, 2)$. [5 marks]

16. Find a if $4x + ay - 1 = 0$ is perpendicular to $7x - 5y - 5 = 0$. [1 mark]

17. Find the bisectors of the angles between $3x + 4y + 3 = 0$ and $8x + 15y + 7 = 0$, and say which one bisects the acute angle. [5 marks]

18. Show that the lines $3y + 3 = 0$, $3y - 6 = 0$, $x - 3y + 5 = 0$, $x - 3y - 8 = 0$ form a parallelogram and find its area. [5 marks]

19. Find the image of $(-1, 7)$ in the line $x + 2y + 3 = 0$. [2 marks]

20. P is a point on the x-axis, $A = (0, 3)$ and $B = (-1, 2)$. Find the least value of $PA + PB$. [3 marks]

Key sheet

Q	Answer
1	30°
2	$a = 3$ or $a = -11$
3	$(\frac{11}{5}, -\frac{1}{5})$
4	$B = (3, 6)$
5	A rhombus of area 7
6	$a = 2$
7	$2x^2 + 2y^2 - 4x + 6y - 35 = 0$
8	$x - 5y + 18 = 0$
9	$4,\ 8$
10	$x\cos 210^\circ + y\sin 210^\circ = 1,\ p = 1,\ \alpha = 210^\circ$
11	$(-2 + 3\sqrt{3}, 5),\ (-3\sqrt{3} - 2, -1)$
12	$\frac{\sqrt{10}}{10}$
13	$8 : 3$ internally
14	$x + 6y - 30 = 0$
15	$(-\frac{3}{2}, \frac{13}{4})$
16	$a = \frac{28}{5}$
17	$11x - 7y + 16 = 0,\ 91x + 143y + 86 = 0$; the acute angle bisector is $91x + 143y + 86 = 0$
18	A parallelogram of area 39
19	$(-\frac{37}{5}, -\frac{29}{5})$
20	$\sqrt{26}$

Solutions

1. Find the inclination of a line with slope $\frac{\sqrt{3}}{3}$. [1]

$\tan\theta = \frac{\sqrt{3}}{3},\ 0 \le \theta < 180^\circ$

$\theta = 30^\circ$

Ans: 30°

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2. Find a if the distance between $(a, 30)$ and $(-4, 6)$ is 25. [2]

$(a-(-4))^2 + (30-6)^2 = 25^2$

$(a-(-4))^2 = 625 - 576 = 49$

$a-(-4) = \pm 7$

$a = 3 \text{ or } -11$

Ans: $a = 3$ or $a = -11$

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3. Find the point dividing the join of $A(2, 1)$ and $B(3, -5)$ internally in the ratio $1 : 4$. [2]

$P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$= \left(\frac{1(3)+4(2)}{5}, \frac{1(-5)+4(1)}{5}\right)$

$= \left(\frac{11}{5}, -\frac{1}{5}\right)$

Ans: $\left(\frac{11}{5}, -\frac{1}{5}\right)$

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4. The midpoint of AB is $(2, 4)$ and $A = (1, 2)$. Find B. [1]

$B = 2M - A = (4-1, 8-2) = (3, 6)$

Ans: $B = (3, 6)$

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5. Show that the points $A(1, -3)$, $B(4, 1)$, $C(8, 4)$, $D(5, 0)$ taken in order form a rhombus, and find its area. [5]

Midpoint of $AC = \left(\frac{9}{2}, \frac{1}{2}\right)$ = midpoint of BD, so the diagonals bisect each other: $ABCD$ is a parallelogram.

$AB^2 = 25,\ BC^2 = 25,\ AC^2 = 98,\ BD^2 = 2$

$AB = BC$ but $AC \neq BD$: equal sides, unequal diagonals, so it is a rhombus.

$\text{Area} = \frac{1}{2}\left|(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)\right| = 7$

Ans: A rhombus of area 7

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6. Find a if the points $(1, 4)$, $(-5, 7)$ and $(5, a)$ are collinear. [3]

Area $= 0$: $1(7-a) + (-5)(a-4) + 5(4-7) = 0$

$a = 2$

Ans: $a = 2$

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7. Find the locus of a point P such that $PA^2 + PB^2 = 74$, where $A = (5, -2)$ and $B = (-3, -1)$. [3]

Let $P(x, y)$ be on the locus: $PA^2 + PB^2 = 74$

$2x^2 + 2y^2 - 4x + 6y - 35 = 0$

This is a circle.

Ans: $2x^2 + 2y^2 - 4x + 6y - 35 = 0$

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8. Find the equation of the line through $(-3, 3)$ perpendicular to $5x + y - 9 = 0$. [2]

A perpendicular line is $x - 5y = k$

$k = -18$

$x - 5y + 18 = 0$

Ans: $x - 5y + 18 = 0$

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9. Find the intercepts made by $2x + y - 8 = 0$ on the axes. [1]

x-intercept $= -\frac{c}{a} = 4,\ y$-intercept $= -\frac{c}{b} = 8$

Ans: $4,\ 8$

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10. Write $\sqrt{3}\,x + y + 2 = 0$ in the normal form and find p and α. [2]

Move the constant to the right so that it is positive, then divide by $\sqrt{a^2 + b^2} = 2$

$x\cos 210^\circ + y\sin 210^\circ = 1$

Ans: $x\cos 210^\circ + y\sin 210^\circ = 1,\ p = 1,\ \alpha = 210^\circ$

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11. Find the points on the line through $(-2, 2)$ with inclination 30°, at a distance 6 from $(-2, 2)$. [3]

$\cos\theta = \frac{\sqrt{3}}{2},\ \sin\theta = \frac{1}{2}$

$(x_1, y_1) \pm r(\cos\theta, \sin\theta) = (-2, 2) \pm 6\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$(-2 + 3\sqrt{3}, 5)$ and $(-3\sqrt{3} - 2, -1)$

Ans: $(-2 + 3\sqrt{3}, 5),\ (-3\sqrt{3} - 2, -1)$

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12. Find the perpendicular distance from $(6, 3)$ to $x - 3y + 4 = 0$. [1]

$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|1|}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

Ans: $\frac{\sqrt{10}}{10}$

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13. In what ratio does $x + 5y - 2 = 0$ divide the join of $(1, 5)$ and $(3, -2)$? [1]

$L_{11} = 24,\ L_{22} = -9$

Ratio $-L_{11} : L_{22} = -24 : -9$, that is $8 : 3$ internally

Ans: $8 : 3$ **internally**

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14. Find the equation of the line through the point of intersection of $x = 0$ and $2x + y - 5 = 0$ and the point $(-6, 6)$. [2]

The line is $(x) + \lambda(2x + y - 5) = 0$

At $(-6, 6)$: $-6 + \lambda(-11) = 0 \Rightarrow \lambda = -\frac{6}{11}$

$x + 6y - 30 = 0$

Ans: $x + 6y - 30 = 0$

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15. Find the circumcentre of the triangle with vertices $A(1, 2)$, $B(-1, 6)$, $C(-4, 2)$. [5]

The perpendicular bisector of AB (through $(0, 4)$): $x - 2y + 8 = 0$

The perpendicular bisector of BC (through $(-\frac{5}{2}, 4)$): $6x + 8y - 17 = 0$

Solving: the circumcentre is $(-\frac{3}{2}, \frac{13}{4})$

Ans: $(-\frac{3}{2}, \frac{13}{4})$

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16. Find a if $4x + ay - 1 = 0$ is perpendicular to $7x - 5y - 5 = 0$. [1]

$a_1a_2 + b_1b_2 = 0 : \ 4(7) + a(-5) = 0$

$a = \frac{28}{5}$

Ans: $a = \frac{28}{5}$

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17. Find the bisectors of the angles between $3x + 4y + 3 = 0$ and $8x + 15y + 7 = 0$, and say which one bisects the acute angle. [5]

$\frac{3x+4y+3}{5} = \pm\frac{8x+15y+7}{17}$

$+$ sign: $11x - 7y + 16 = 0$

$-$ sign: $91x + 143y + 86 = 0$

Both constants are positive and $a_1a_2 + b_1b_2 = 84 > 0$, so the $-$ sign gives the acute angle bisector.

Acute angle bisector: $91x + 143y + 86 = 0$

Ans: $11x - 7y + 16 = 0,\ 91x + 143y + 86 = 0$**; the acute angle bisector is** $91x + 143y + 86 = 0$

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18. Show that the lines $3y + 3 = 0$, $3y - 6 = 0$, $x - 3y + 5 = 0$, $x - 3y - 8 = 0$ form a parallelogram and find its area. [5]

The first two lines are parallel, and so are the last two.

$d_1 = \frac{|9|}{\sqrt{9}} = 3,\ d_2 = \frac{|13|}{\sqrt{10}} = \frac{13\sqrt{10}}{10}$

$aa_1 + bb_1 = -9$: the pairs are not perpendicular, and the widths are unequal, so it is a parallelogram.

$\text{Area} = \left|\frac{(c_1-c_2)(d_1-d_2)}{a_1b_2-a_2b_1}\right| = \left|\frac{(9)(13)}{-3}\right| = 39$

Ans: A parallelogram of area 39

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19. Find the image of $(-1, 7)$ in the line $x + 2y + 3 = 0$. [2]

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2} = \frac{-32}{5}$

$(h, k) = (-\frac{37}{5}, -\frac{29}{5})$

Ans: $(-\frac{37}{5}, -\frac{29}{5})$

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20. P is a point on the x-axis, $A = (0, 3)$ and $B = (-1, 2)$. Find the least value of $PA + PB$. [3]

A and B are on the same side of the x-axis; reflect B to $B'(-1, -2)$.

$PA + PB \geq AB' = \sqrt{26} = \sqrt{26}$

Equality when $P = (-\frac{3}{5}, 0)$.

Ans: $\sqrt{26}$
