EXPRESSWALA ACADEMY

Maths 1B: Straight Lines

Question Paper, Set 06

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the slope of a line perpendicular to the line through $(4, 2)$ and $(-3, -6)$. [1 mark]

2. Find a if the distance between $(a, -24)$ and $(4, 0)$ is 25. [2 marks]

3. Find the point dividing the join of $A(5, -2)$ and $B(6, 4)$ internally in the ratio $3 : 4$. [2 marks]

4. The midpoint of AB is $(5, -5)$ and $A = (-6, -6)$. Find B. [1 mark]

5. Show that the points $A(3, 2)$, $B(6, 6)$, $C(11, 6)$, $D(8, 2)$ taken in order form a rhombus, and find its area. [5 marks]

6. Find the area of the triangle with vertices $(0, -4)$, $(7, 5)$, $(-4, -3)$. [3 marks]

7. Find the locus of a point equidistant from $A(3, 1)$ and $B(-3, 4)$. [3 marks]

8. Find the equation of the line through $(3, 4)$ with slope $-\frac{2}{3}$. [2 marks]

9. Find the intercepts made by $3x + 2y - 15 = 0$ on the axes. [1 mark]

10. Write $x + y + 2 = 0$ in the normal form and find p and α. [2 marks]

11. Find the points on the line through $(0, 3)$ with inclination 135°, at a distance $6\sqrt{2}$ from $(0, 3)$. [3 marks]

12. Find the perpendicular distance from $(-2, 5)$ to $8x - 15y + 10 = 0$. [1 mark]

13. In what ratio does $2x + y - 3 = 0$ divide the join of $(-2, -1)$ and $(5, -1)$? [1 mark]

14. Find the point of intersection of $x - 5y - 24 = 0$ and $x + 2y + 4 = 0$. [2 marks]

15. Find the orthocentre of the triangle with vertices $A(2, 0)$, $B(6, -4)$, $C(-3, 2)$. [5 marks]

16. Find a if $3x + ay + 4 = 0$ is perpendicular to $x + 5y + 6 = 0$. [1 mark]

17. Find the bisectors of the angles between $15x + 8y + 5 = 0$ and $4x + 3y + 4 = 0$, and say which one bisects the acute angle. [5 marks]

18. Show that the lines $3x + 3y + 6 = 0$, $3x + 3y - 1 = 0$, $3x + 1 = 0$, $3x - 5 = 0$ form a parallelogram and find its area. [5 marks]

19. Find the foot of the perpendicular from $(-3, 1)$ to the line $3x + 4y = 0$. [2 marks]

20. A line through $(4, 5)$ cuts off a triangle from the positive axes. Find the least area of the triangle. [3 marks]

Key sheet

Q	Answer
1	$-\frac{7}{8}$
2	$a = 11$ or $a = -3$
3	$(\frac{38}{7}, \frac{4}{7})$
4	$B = (16, -4)$
5	A rhombus of area 20
6	$\frac{43}{2}$ square units
7	$4x - 2y + 5 = 0$
8	$2x + 3y - 18 = 0$
9	$5,\ \frac{15}{2}$
10	$x\cos 225^\circ + y\sin 225^\circ = \sqrt{2},\ p = \sqrt{2},\ \alpha = 225^\circ$
11	$(-6, 9),\ (6, -3)$
12	$\frac{81}{17}$
13	$4:3$ internally
14	$(4, -4)$
15	$(16, 21)$
16	$a = -\frac{3}{5}$
17	$7x - 11y - 43 = 0,\ 143x + 91y + 93 = 0$; the acute angle bisector is $143x + 91y + 93 = 0$
18	A parallelogram of area $\frac{14}{3}$
19	$(-\frac{12}{5}, \frac{9}{5})$
20	40

Solutions

1. Find the slope of a line perpendicular to the line through $(4, 2)$ and $(-3, -6)$. [1]

$m = \frac{-6-2}{-3-4} = \frac{8}{7}$

A perpendicular line has slope $-\frac{1}{m} = -\frac{7}{8}$

Ans: $-\frac{7}{8}$

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2. Find a if the distance between $(a, -24)$ and $(4, 0)$ is 25. [2]

$(a-4)^2 + (-24-0)^2 = 25^2$

$(a-4)^2 = 625 - 576 = 49$

$a - 4 = \pm 7$

$a = 11 \text{ or } -3$

Ans: $a = 11$ or $a = -3$

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3. Find the point dividing the join of $A(5, -2)$ and $B(6, 4)$ internally in the ratio $3 : 4$. [2]

$P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$= \left(\frac{3(6)+4(5)}{7}, \frac{3(4)+4(-2)}{7}\right)$

$= \left(\frac{38}{7}, \frac{4}{7}\right)$

Ans: $\left(\frac{38}{7}, \frac{4}{7}\right)$

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4. The midpoint of AB is $(5, -5)$ and $A = (-6, -6)$. Find B. [1]

$B = 2M - A = (10 - (-6), -10 - (-6)) = (16, -4)$

Ans: $B = (16, -4)$

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5. Show that the points $A(3, 2)$, $B(6, 6)$, $C(11, 6)$, $D(8, 2)$ taken in order form a rhombus, and find its area. [5]

Midpoint of $AC = (7, 4)$ = midpoint of BD, so the diagonals bisect each other: $ABCD$ is a parallelogram.

$AB^2 = 25,\ BC^2 = 25,\ AC^2 = 80,\ BD^2 = 20$

$AB = BC$ but $AC \neq BD$: equal sides, unequal diagonals, so it is a rhombus.

$\text{Area} = \frac{1}{2}\left|(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)\right| = 20$

Ans: A rhombus of area 20

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6. Find the area of the triangle with vertices $(0, -4)$, $(7, 5)$, $(-4, -3)$. [3]

$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$= \frac{1}{2}\left|0(8) + 7(1) + (-4)(-9)\right|$

$= \frac{43}{2}$

Ans: $\frac{43}{2}$ **square units**

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7. Find the locus of a point equidistant from $A(3,1)$ and $B(-3,4)$. [3]

Let $P(x,y)$ be on the locus: $PA^2 = PB^2$

$4x - 2y + 5 = 0$

This is the perpendicular bisector of AB.

Ans: $4x - 2y + 5 = 0$

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8. Find the equation of the line through $(3,4)$ with slope $-\frac{2}{3}$. [2]

$y - y_1 = m(x - x_1)$

$2x + 3y - 18 = 0$

Ans: $2x + 3y - 18 = 0$

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9. Find the intercepts made by $3x + 2y - 15 = 0$ on the axes. [1]

$x\text{-intercept} = -\frac{c}{a} = 5,\ y\text{-intercept} = -\frac{c}{b} = \frac{15}{2}$

Ans: $5,\ \frac{15}{2}$

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10. Write $x + y + 2 = 0$ in the normal form and find p and α. [2]

Move the constant to the right so that it is positive, then divide by $\sqrt{a^2+b^2} = \sqrt{2}$

$x\cos 225^\circ + y\sin 225^\circ = \sqrt{2}$

Ans: $x\cos 225^\circ + y\sin 225^\circ = \sqrt{2},\ p = \sqrt{2},\ \alpha = 225^\circ$

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11. Find the points on the line through $(0,3)$ with inclination 135°, at a distance $6\sqrt{2}$ from $(0,3)$. [3]

$\cos\theta = -\frac{\sqrt{2}}{2},\ \sin\theta = \frac{\sqrt{2}}{2}$

$(x_1, y_1) \pm r(\cos\theta, \sin\theta) = (0,3) \pm 6\sqrt{2}\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$(-6, 9)$ and $(6, -3)$

Ans: $(-6,9),\ (6,-3)$

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12. Find the perpendicular distance from $(-2,5)$ to $8x - 15y + 10 = 0$. [1]

$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \frac{|-81|}{\sqrt{289}} = \frac{81}{17}$

Ans: $\frac{81}{17}$

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13. In what ratio does $2x + y - 3 = 0$ divide the join of $(-2,-1)$ and $(5,-1)$? [1]

$L_{11} = -8,\ L_{22} = 6$

Ratio $-L_{11} : L_{22} = 8 : 6$, that is $4 : 3$ internally

Ans: $4 : 3$ **internally**

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14. Find the point of intersection of $x - 5y - 24 = 0$ and $x + 2y + 4 = 0$. [2]

$\frac{x}{b_1c_2-b_2c_1} = \frac{y}{c_1a_2-c_2a_1} = \frac{1}{a_1b_2-a_2b_1}$

$x = \frac{28}{7},\ y = \frac{-28}{7}$

$(4, -4)$

Ans: $(4,-4)$

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15. Find the orthocentre of the triangle with vertices $A(2,0)$, $B(6,-4)$, $C(-3,2)$. [5]

The altitude through A is perpendicular to BC: $3x-2y-6=0$

The altitude through B is perpendicular to CA: $5x-2y-38=0$

Solving the two altitudes: the orthocentre is $(16,21)$

Ans: $(16,21)$

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16. Find a if $3x+ay+4=0$ is perpendicular to $x+5y+6=0$. [1]

$a_1a_2+b_1b_2=0:\ 3(1)+a(5)=0$

$a=-\frac{3}{5}$

Ans: $a=-\frac{3}{5}$

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17. Find the bisectors of the angles between $15x+8y+5=0$ and $4x+3y+4=0$, and say which one bisects the acute angle. [5]

$\frac{15x+8y+5}{17}=\pm\frac{4x+3y+4}{5}$

$+$ sign: $7x-11y-43=0$

$-$ sign: $143x+91y+93=0$

Both constants are positive and $a_1a_2+b_1b_2=84>0$, so the $-$ sign gives the acute angle bisector.

Acute angle bisector: $143x+91y+93=0$

Ans: $7x-11y-43=0,\ 143x+91y+93=0$**; the acute angle bisector is** $143x+91y+93=0$

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18. Show that the lines $3x+3y+6=0$, $3x+3y-1=0$, $3x+1=0$, $3x-5=0$ form a parallelogram and find its area. [5]

The first two lines are parallel, and so are the last two.

$d_1=\frac{|7|}{\sqrt{18}}=\frac{7\sqrt{2}}{6},\ d_2=\frac{|6|}{\sqrt{9}}=2$

$aa_1+bb_1=9$: the pairs are not perpendicular, and the widths are unequal, so it is a parallelogram.

$\text{Area}=\left|\frac{(c_1-c_2)(d_1-d_2)}{a_1b_2-a_2b_1}\right|=\left|\frac{(7)(6)}{-9}\right|=\frac{14}{3}$

Ans: A parallelogram of area $\frac{14}{3}$

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19. Find the foot of the perpendicular from $(-3,1)$ to the line $3x+4y=0$. [2]

$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}=\frac{5}{25}$

$(h,k)=(-\frac{12}{5},\frac{9}{5})$

Ans: $(-\frac{12}{5},\frac{9}{5})$

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20. A line through $(4,5)$ cuts off a triangle from the positive axes. Find the least area of the triangle. [3]

The area is least when $(4,5)$ is the midpoint of the intercepted segment: intercepts 8 and 10.

$\text{Least area}=2|x_1y_1|=40$

Ans: 40
