EXPRESSWALA ACADEMY

Maths 1B: Straight Lines

Question Paper, Set 05

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. A line makes an angle of 150° with the positive x-axis (anticlockwise). Find its slope. [1 mark]

2. Find a if the distance between $(a, 7)$ and $(-4, 4)$ is 5. [2 marks]

3. Find the point dividing the join of $A(6, -1)$ and $B(-5, 6)$ externally in the ratio $3 : 4$. [2 marks]

4. The midpoint of AB is $(-5, 1)$ and $A = (-5, -1)$. Find B. [1 mark]

5. Show that the points $A(5, 4)$, $B(4, 6)$, $C(4, 4)$, $D(5, 2)$ taken in order form a parallelogram, and find its area. [5 marks]

6. Find the area of the quadrilateral whose vertices, taken in order, are $(6, 0)$, $(2, 5)$, $(1, 6)$, $(3, 0)$. [3 marks]

7. Find the locus of a point P at which the segment joining $A(2, -2)$ and $B(2, -1)$ subtends a right angle. [3 marks]

8. Find the equation of the line through $(-6, 4)$ with slope $\frac{4}{3}$. [2 marks]

9. Find the area of the triangle formed by $2x + y - 5 = 0$ with the coordinate axes. [1 mark]

10. Write $\sqrt{3}\,x - y + 10 = 0$ in the normal form and find p and α. [2 marks]

11. Find the points on the line through $(-1, 2)$ with inclination 135°, at a distance $6\sqrt{2}$ from $(-1, 2)$. [3 marks]

12. Find the perpendicular distance from $(-4, -6)$ to $x + y + 5 = 0$. [1 mark]

13. Is $(-6, 3)$ on the same side of $x - 5y + 8 = 0$ as the origin? [1 mark]

14. Find a if the lines $2x + y + 3 = 0$, $x + 1 = 0$ and $5x + ay - 9 = 0$ are concurrent. [2 marks]

15. Find the orthocentre of the triangle with vertices $A(-2, 4)$, $B(6, 5)$, $C(2, -2)$. [5 marks]

16. Find a if $5x + ay - 2 = 0$ is parallel to $7x - 5y + 8 = 0$. [1 mark]

17. Find the bisectors of the angles between $12x + 5y + 1 = 0$ and $4x - 3y + 9 = 0$, and say which one bisects the acute angle. [5 marks]

18. Show that the lines $2x + y + 3 = 0$, $2x + y - 3 = 0$, $x + 2y + 3 = 0$, $x + 2y + 5 = 0$ form a parallelogram and find its area. [5 marks]

19. Find the image of $(2, 1)$ in the line $x - y + 2 = 0$. [2 marks]

20. A line through $(7, 1)$ cuts off a triangle from the positive axes. Find the least area of the triangle. [3 marks]

Key sheet

Q	Answer
1	$-\frac{\sqrt{3}}{3}$
2	$a = 0$ or $a = -8$
3	$(39, -22)$
4	$B = (-5, 3)$
5	A parallelogram of area 2
6	$\frac{19}{2}$ square units
7	$x^2 + y^2 - 4x + 3y + 6 = 0$
8	$4x - 3y + 36 = 0$
9	$\frac{25}{4}$
10	$x\cos 150^\circ + y\sin 150^\circ = 5,\ p = 5,\ \alpha = 150^\circ$
11	$(-7, 8),\ (5, -4)$
12	$\frac{5\sqrt{2}}{2}$
13	No, the opposite side
14	$a = -14$
15	$(\frac{3}{2}, 2)$
16	$a = -\frac{25}{7}$
17	$x + 8y - 14 = 0,\ 56x - 7y + 61 = 0$; the acute angle bisector is $56x - 7y + 61 = 0$
18	A parallelogram of area 4
19	$(-1, 4)$
20	14

Solutions

1. A line makes an angle of 150° with the positive x-axis (anticlockwise). Find its slope. [1]

$m = \tan 150^\circ = -\frac{\sqrt{3}}{3}$

Ans: $-\frac{\sqrt{3}}{3}$

---

2. Find a if the distance between $(a, 7)$ and $(-4, 4)$ is 5. [2]

$(a-(-4))^2 + (7-4)^2 = 5^2$

$(a-(-4))^2 = 25 - 9 = 16$

$a-(-4) = \pm 4$

$a = 0 \text{ or } -8$

Ans: $a = 0$ or $a = -8$

---

3. Find the point dividing the join of $A(6, -1)$ and $B(-5, 6)$ externally in the ratio $3:4$. [2]

$P = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

$= \left(\frac{3(-5)-4(6)}{-1}, \frac{3(6)-4(-1)}{-1}\right)$

$= (39, -22)$

Ans: $(39, -22)$

---

4. The midpoint of AB is $(-5, 1)$ and $A = (-5, -1)$. Find B. [1]

$B = 2M - A = (-10-(-5), 2-(-1)) = (-5, 3)$

Ans: $B = (-5, 3)$

---

5. Show that the points $A(5, 4)$, $B(4, 6)$, $C(4, 4)$, $D(5, 2)$ taken in order form a parallelogram, and find its area. [5]

Midpoint of $AC = (\frac{9}{2}, 4)$ = midpoint of BD, so the diagonals bisect each other: $ABCD$ is a parallelogram.

$AB^2 = 5,\ BC^2 = 4,\ AC^2 = 1,\ BD^2 = 17$

$AB \neq BC$ and $AC \neq BD$, so it is a parallelogram (not a rectangle or rhombus).

$\text{Area} = \frac{1}{2}\left|(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)\right| = 2$

Ans: A parallelogram of area 2

---

6. Find the area of the quadrilateral whose vertices, taken in order, are $(6, 0)$, $(2, 5)$, $(1, 6)$, $(3, 0)$. [3]

Cross-multiply around the boundary:

$\frac{1}{2}\left|30 + 7 + (-18) + 0\right| = \frac{19}{2}$

Ans: $\frac{19}{2}$ **square units**

---

7. Find the locus of a point P at which the segment joining $A(2, -2)$ and $B(2, -1)$ subtends a right angle. [3]

Let $P(x, y)$ be on the locus: $PA \perp PB:\ (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$x^2 + y^2 - 4x + 3y + 6 = 0$

This is a circle.

Ans: $x^2 + y^2 - 4x + 3y + 6 = 0$

---

8. Find the equation of the line through $(-6, 4)$ with slope $\frac{4}{3}$. [2]

$y - y_1 = m(x - x_1)$

$4x - 3y + 36 = 0$

Ans: $4x - 3y + 36 = 0$

---

9. Find the area of the triangle formed by $2x + y - 5 = 0$ with the coordinate axes. [1]

Intercepts $\frac{5}{2}$ and 5.

$\text{Area} = \frac{c^2}{2|ab|} = \frac{25}{4} = \frac{25}{4}$

Ans: $\frac{25}{4}$

---

10. Write $\sqrt{3}\,x - y + 10 = 0$ in the normal form and find p and α. [2]

Move the constant to the right so that it is positive, then divide by $\sqrt{a^2 + b^2} = 2$

$x\cos 150^\circ + y\sin 150^\circ = 5$

Ans: $x\cos 150^\circ + y\sin 150^\circ = 5,\ p = 5,\ \alpha = 150^\circ$

---

11. Find the points on the line through $(-1, 2)$ with inclination 135°, at a distance $6\sqrt{2}$ from $(-1, 2)$. [3]

$\cos\theta = -\frac{\sqrt{2}}{2},\ \sin\theta = \frac{\sqrt{2}}{2}$

$(x_1, y_1) \pm r(\cos\theta, \sin\theta) = (-1, 2) \pm 6\sqrt{2}\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$(-7, 8)$ and $(5, -4)$

Ans: $(-7, 8),\ (5, -4)$

---

12. Find the perpendicular distance from $(-4, -6)$ to $x + y + 5 = 0$. [1]

$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|-5|}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

Ans: $\frac{5\sqrt{2}}{2}$

---

13. Is $(-6, 3)$ on the same side of $x - 5y + 8 = 0$ as the origin? [1]

$c = 8,\ L_{11} = -13$

$cL_{11} < 0$: opposite side to the origin.

Ans: No, the opposite side

---

14. Find a if the lines $2x + y + 3 = 0$, $x + 1 = 0$ and $5x + ay - 9 = 0$ are concurrent. [2]

The first two lines meet at $(-1, -1)$.

It lies on the third line: $-5 + (-1)a - 9 = 0$

$a = -14$

Ans: $a = -14$

---

15. Find the orthocentre of the triangle with vertices $A(-2, 4)$, $B(6, 5)$, $C(2, -2)$. [5]

The altitude through A is perpendicular to BC: $4x + 7y - 20 = 0$

The altitude through B is perpendicular to CA: $2x - 3y + 3 = 0$

Solving the two altitudes: the orthocentre is $(\frac{3}{2}, 2)$

Ans: $(\frac{3}{2}, 2)$

---

16. Find a if $5x + ay - 2 = 0$ is parallel to $7x - 5y + 8 = 0$. [1]

$\frac{a_1}{a_2} = \frac{b_1}{b_2} : \frac{5}{7} = \frac{a}{-5}$

$a = -\frac{25}{7}$

Ans: $a = -\frac{25}{7}$

---

17. Find the bisectors of the angles between $12x + 5y + 1 = 0$ and $4x - 3y + 9 = 0$, and say which one bisects the acute angle. [5]

$\frac{12x+5y+1}{13} = \pm\frac{4x-3y+9}{5}$

$+$ sign: $x + 8y - 14 = 0$

$-$ sign: $56x - 7y + 61 = 0$

Both constants are positive and $a_1a_2 + b_1b_2 = 33 > 0$, so the $-$ sign gives the acute angle bisector.

Acute angle bisector: $56x - 7y + 61 = 0$

Ans: $x + 8y - 14 = 0,\ 56x - 7y + 61 = 0$; **the acute angle bisector is** $56x - 7y + 61 = 0$

---

18. Show that the lines $2x + y + 3 = 0$, $2x + y - 3 = 0$, $x + 2y + 3 = 0$, $x + 2y + 5 = 0$ form a parallelogram and find its area. [5]

The first two lines are parallel, and so are the last two.

$d_1 = \frac{|6|}{\sqrt{5}} = \frac{6\sqrt{5}}{5},\ d_2 = \frac{|-2|}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$aa_1 + bb_1 = 4$: the pairs are not perpendicular, and the widths are unequal, so it is a parallelogram.

$\text{Area} = \left|\frac{(c_1-c_2)(d_1-d_2)}{a_1b_2-a_2b_1}\right| = \left|\frac{(6)(-2)}{3}\right| = 4$

Ans: A parallelogram of area 4

---

19. Find the image of $(2, 1)$ in the line $x - y + 2 = 0$. [2]

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2} = \frac{-6}{2}$

$(h, k) = (-1, 4)$

Ans: $(-1, 4)$

---

20. A line through $(7, 1)$ cuts off a triangle from the positive axes. Find the least area of the triangle. [3]

The area is least when $(7, 1)$ is the midpoint of the intercepted segment: intercepts 14 and 2.

$\text{Least area} = 2|x_1y_1| = 14$

Ans: 14
