EXPRESSWALA ACADEMY

Maths 1B: Straight Lines

Question Paper, Set 10

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the slope of a line perpendicular to the line through $(-6, -1)$ and $(0, 2)$. [1 mark]

2. Find a if the distance between $(a, -11)$ and $(4, 1)$ is 13. [2 marks]

3. Find the point dividing the join of $A(-5, 3)$ and $B(5, 5)$ externally in the ratio $3 : 2$. [2 marks]

4. The midpoint of AB is $(0, 4)$ and $A = (-4, -4)$. Find B. [1 mark]

5. Show that the points $A(3, 1)$, $B(7, 4)$, $C(7, 9)$, $D(3, 6)$ taken in order form a rhombus, and find its area. [5 marks]

6. Find a if the points $(5, -3)$, $(3, 0)$ and $(-5, a)$ are collinear. [3 marks]

7. Find the locus of a point P such that $PA = 3\,PB$, where $A = (0, 1)$ and $B = (-4, -4)$. [3 marks]

8. Find the equation of the line through $(-4, -3)$ and $(7, 1)$. [2 marks]

9. Find the intercepts made by $6x - 2y - 15 = 0$ on the axes. [1 mark]

10. Write $x + y - 6 = 0$ in the normal form and find p and α. [2 marks]

11. Find the points on the line through $(-4, 5)$ with inclination 30°, at a distance 4 from $(-4, 5)$. [3 marks]

12. Find the perpendicular distance from $(2, 1)$ to $2x - y + 8 = 0$. [1 mark]

13. In what ratio does $2x + 4y - 1 = 0$ divide the join of $(-1, 6)$ and $(3, -6)$? [1 mark]

14. Find the point of intersection of $5x + 4y - 1 = 0$ and $3x + 4y - 7 = 0$. [2 marks]

15. Find the circumcentre of the triangle with vertices $A(0, 4)$, $B(6, 4)$, $C(0, -2)$. [5 marks]

16. Find a if $5x + ay - 2 = 0$ is parallel to $7x - y + 1 = 0$. [1 mark]

17. Find the bisectors of the angles between $4x - 3y + 7 = 0$ and $12x - 5y + 4 = 0$, and say which one bisects the acute angle. [5 marks]

18. Show that the lines $2x - 3y + 4 = 0$, $2x - 3y - 1 = 0$, $2y + 2 = 0$, $2y - 6 = 0$ form a parallelogram and find its area. [5 marks]

19. Find the foot of the perpendicular from $(7, -5)$ to the line $x + y + 6 = 0$. [2 marks]

20. A line through $(4, 36)$ meets the positive x- and y-axes at A and B. Find the least value of $OA + OB$. [3 marks]

Key sheet

Q	Answer
1	-2
2	$a = 9$ or $a = -1$
3	$(25, 9)$
4	$B = (4, 12)$
5	A rhombus of area 20
6	$a = 12$
7	$8x^2 + 8y^2 + 72x + 74y + 287 = 0$
8	$4x - 11y - 17 = 0$
9	$\frac{5}{2},\ -\frac{15}{2}$
10	$x\cos 45^\circ + y\sin 45^\circ = 3\sqrt{2},\ p = 3\sqrt{2},\ \alpha = 45^\circ$
11	$(-4 + 2\sqrt{3}, 7),\ (-4 - 2\sqrt{3}, 3)$
12	$\frac{11\sqrt{5}}{5}$
13	$21 : 19$ internally
14	$(-3, 4)$
15	$(3, 1)$
16	$a = -\frac{5}{7}$
17	$8x + 14y - 71 = 0,\ 112x - 64y + 111 = 0$; the acute angle bisector is $112x - 64y + 111 = 0$
18	A parallelogram of area 10
19	$(3, -9)$
20	64

Solutions

1. Find the slope of a line perpendicular to the line through $(-6, -1)$ and $(0, 2)$. [1]

$m = \frac{2-(-1)}{0-(-6)} = \frac{1}{2}$

A perpendicular line has slope $-\frac{1}{m} = -2$

Ans: -2

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2. Find a if the distance between $(a, -11)$ and $(4, 1)$ is 13. [2]

$(a-4)^2 + (-11-1)^2 = 13^2$

$(a-4)^2 = 169 - 144 = 25$

$a - 4 = \pm 5$

$a = 9 \text{ or } -1$

Ans: $a = 9$ or $a = -1$

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3. Find the point dividing the join of $A(-5, 3)$ and $B(5, 5)$ externally in the ratio $3 : 2$. [2]

$P = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

$= \left(\frac{3(5)-2(-5)}{1}, \frac{3(5)-2(3)}{1}\right)$

$= (25, 9)$

Ans: $(25, 9)$

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4. The midpoint of AB is $(0, 4)$ and $A = (-4, -4)$. Find B. [1]

$B = 2M - A = (0-(-4), 8-(-4)) = (4, 12)$

Ans: $B = (4, 12)$

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5. Show that the points $A(3, 1)$, $B(7, 4)$, $C(7, 9)$, $D(3, 6)$ taken in order form a rhombus, and find its area. [5]

Midpoint of $AC = (5, 5)$ = midpoint of BD, so the diagonals bisect each other: $ABCD$ is a parallelogram.

$AB^2 = 25,\ BC^2 = 25,\ AC^2 = 80,\ BD^2 = 20$

$AB = BC$ but $AC \neq BD$: equal sides, unequal diagonals, so it is a rhombus.

$\text{Area} = \frac{1}{2}\left|(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)\right| = 20$

Ans: A rhombus of area 20

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6. Find a if the points $(5, -3)$, $(3, 0)$ and $(-5, a)$ are collinear. [3]

Area $= 0$: $5(0 - a) + 3(a - (-3)) + (-5)(-3 - 0) = 0$

$a = 12$

Ans: $a = 12$

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7. Find the locus of a point P such that $PA = 3\,PB$, where $A = (0, 1)$ and $B = (-4, -4)$. [3]

Let $P(x, y)$ be on the locus: $PA^2 = 9\,PB^2$

$8x^2 + 8y^2 + 72x + 74y + 287 = 0$

This is a circle.

Ans: $8x^2 + 8y^2 + 72x + 74y + 287 = 0$

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8. Find the equation of the line through $(-4, -3)$ and $(7, 1)$. [2]

$m = \frac{4}{11}$

$y + (3) = \frac{4}{11}(x + (4))$

$4x - 11y - 17 = 0$

Ans: $4x - 11y - 17 = 0$

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9. Find the intercepts made by $6x - 2y - 15 = 0$ on the axes. [1]

$x\text{-intercept} = -\frac{c}{a} = \frac{5}{2},\ y\text{-intercept} = -\frac{c}{b} = -\frac{15}{2}$

Ans: $\frac{5}{2},\ -\frac{15}{2}$

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10. Write $x + y - 6 = 0$ in the normal form and find p and α. [2]

Move the constant to the right so that it is positive, then divide by $\sqrt{a^2 + b^2} = \sqrt{2}$

$x\cos 45^\circ + y\sin 45^\circ = 3\sqrt{2}$

Ans: $x\cos 45^\circ + y\sin 45^\circ = 3\sqrt{2},\ p = 3\sqrt{2},\ \alpha = 45^\circ$

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11. Find the points on the line through $(-4, 5)$ with inclination 30°, at a distance 4 from $(-4, 5)$. [3]

$\cos\theta = \frac{\sqrt{3}}{2},\ \sin\theta = \frac{1}{2}$

$(x_1, y_1) \pm r(\cos\theta, \sin\theta) = (-4, 5) \pm 4\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$(-4 + 2\sqrt{3}, 7)$ and $(-4 - 2\sqrt{3}, 3)$

Ans: $(-4 + 2\sqrt{3}, 7),\ (-4 - 2\sqrt{3}, 3)$

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12. Find the perpendicular distance from $(2, 1)$ to $2x - y + 8 = 0$. [1]

$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|11|}{\sqrt{5}} = \frac{11\sqrt{5}}{5}$

Ans: $\frac{11\sqrt{5}}{5}$

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13. In what ratio does $2x + 4y - 1 = 0$ divide the join of $(-1, 6)$ and $(3, -6)$? [1]

$L_{11} = 21,\ L_{22} = -19$

Ratio $-L_{11} : L_{22} = -21 : -19$, that is $21 : 19$ internally

Ans: $21 : 19$ **internally**

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14. Find the point of intersection of $5x + 4y - 1 = 0$ and $3x + 4y - 7 = 0$. [2]

$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$x = \frac{-24}{8},\ y = \frac{32}{8}$

$(-3, 4)$

Ans: $(-3, 4)$

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15. Find the circumcentre of the triangle with vertices $A(0, 4)$, $B(6, 4)$, $C(0, -2)$. [5]

The perpendicular bisector of AB (through $(3, 4)$): $x - 3 = 0$

The perpendicular bisector of BC (through $(3, 1)$): $x + y - 4 = 0$

Solving: the circumcentre is $(3, 1)$

Ans: $(3, 1)$

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16. Find a if $5x + ay - 2 = 0$ is parallel to $7x - y + 1 = 0$. [1]

$\frac{a_1}{a_2} = \frac{b_1}{b_2} : \frac{5}{7} = \frac{a}{-1}$

$a = -\frac{5}{7}$

Ans: $a = -\frac{5}{7}$

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17. Find the bisectors of the angles between $4x - 3y + 7 = 0$ and $12x - 5y + 4 = 0$, and say which one bisects the acute angle. [5]

$\frac{4x-3y+7}{5} = \pm\frac{12x-5y+4}{13}$

$+$ sign: $8x + 14y - 71 = 0$

$-$ sign: $112x - 64y + 111 = 0$

Both constants are positive and $a_1a_2 + b_1b_2 = 63 > 0$, so the $-$ sign gives the acute angle bisector.

Acute angle bisector: $112x - 64y + 111 = 0$

Ans: $8x + 14y - 71 = 0,\ 112x - 64y + 111 = 0$**; the acute angle bisector is** $112x - 64y + 111 = 0$

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18. Show that the lines $2x - 3y + 4 = 0$, $2x - 3y - 1 = 0$, $2y + 2 = 0$, $2y - 6 = 0$ form a parallelogram and find its area. [5]

The first two lines are parallel, and so are the last two.

$d_1 = \frac{|5|}{\sqrt{13}} = \frac{5\sqrt{13}}{13},\ d_2 = \frac{|8|}{\sqrt{4}} = 4$

$aa_1 + bb_1 = -6$: the pairs are not perpendicular, and the widths are unequal, so it is a parallelogram.

$\text{Area} = \left|\frac{(c_1-c_2)(d_1-d_2)}{a_1b_2-a_2b_1}\right| = \left|\frac{(5)(8)}{4}\right| = 10$

Ans: A parallelogram of area 10

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19. Find the foot of the perpendicular from $(7, -5)$ to the line $x + y + 6 = 0$. [2]

$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2} = \frac{-8}{2}$

$(h, k) = (3, -9)$

Ans: $(3, -9)$

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20. A line through $(4, 36)$ meets the positive x- and y-axes at A and B. Find the least value of $OA + OB$. [3]

$(\sqrt{x_1} + \sqrt{y_1})^2 = (\sqrt{4} + \sqrt{36})^2 = 64$

Ans: 64
