EXPRESSWALA ACADEMY

# Maths 1A: Properties of Triangles

## Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

**1.** In triangle $ABC$, $A = 90^\circ$, $B = 30^\circ$ and $a = 12\sqrt{2}$. Find $b$. [1 mark]

**2.** In triangle $ABC$, $b = 4$, $c = 7$ and $A = 120^\circ$. Find $a$. [2 marks]

**3.** In triangle $ABC$, $a = 9$, $c = 8$ and $\cos B = \frac{17}{18}$. Find $b\cos C$. [1 mark]

**4.** Use Napier's rule to find $\tan\frac{B-C}{2}$ when $b = 5$, $c = 8$, $A = 30^\circ$. [2 marks]

**5.** In triangle $ABC$, $a = 25$, $b = 15$, $c = 20$. Find $\sin\frac{A}{2}$. [2 marks]

**6.** Find the area of triangle $ABC$ with $a = 10$, $b = 9$ and $C = 45^\circ$. [1 mark]

**7.** For the triangle with $a = 28$, $b = 39$, $c = 17$, find the area and $\sin B$. [3 marks]

**8.** Using the area, find $\cot C$ for the triangle with $a = 30$, $b = 25$, $c = 25$. [2 marks]

**9.** If $a : b : c = 4 : 9 : 7$, find $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2}$. [1 mark]

**10.** Find the length of the altitude from $C$ to the opposite side in the triangle with $a = 5$, $b = 13$, $c = 12$. [2 marks]

**11.** In triangle $ABC$, $b = 10$, $c = 12$ and $A = 120^\circ$. The bisector of angle $A$ meets $BC$ at $D$. Find $AD$, $a$ and $BD$. [3 marks]

**12.** The sides of triangle $ABC$ are $a = 3\sqrt{2}$, $b = 2\sqrt{6}$, $c = \sqrt{6}$. Find its angles. [5 marks]

**13.** How many triangles have $a = 9$, $b = 6$ and $A = 45^\circ$? [1 mark]

**14.** Show that two triangles have $a = 9$, $b = 16$, $A = 30^\circ$, and find the two values of $c$. [3 marks]

**15.** For the triangle with $a = 20$, $b = 20$, $c = 24$, find the inradius, the three exradii and the circumradius. [5 marks]

**16.** If $r_1 = 6$, $r_2 = 24$ and $r_3 = 6$, find $r$, $\Delta$, $s$ and the sides of the triangle. [5 marks]

**17.** The triangle $ABC$ has $a = 40$, $b = 29$, $c = 29$. Find $R$, the sides of its pedal triangle and the distance $AO$ of $A$ from the orthocentre $O$. [5 marks]

**18.** Each interior angle of a regular polygon is $156^\circ$. How many sides has it? [1 mark]

**19.** Find the circumradius, the inradius and the area of a square of side $8$. [3 marks]

**20.** In triangle $ABC$, $BC = 26$, $AC = 26$, $AB = 20$. $D$ is on $AB$ with $BD = 4$. Find $CD$ using Stewart's theorem. [2 marks]

## Key sheet

| Q | Answer |
|---|---|
| 1 | $b = 6\sqrt{2}$ |
| 2 | $a = \sqrt{93}$ |
| 3 | $\frac{13}{9}$ |
| 4 | $-\frac{6}{13} - \frac{3\sqrt{3}}{13}$ |
| 5 | $\frac{\sqrt{2}}{2}$ |
| 6 | $\Delta = \frac{45\sqrt{2}}{2}$ |
| 7 | $\Delta = 210$, $\sin B = \frac{15}{17}$ |
| 8 | $\frac{3}{4}$ |
| 9 | $6 : 1 : 3$ |
| 10 | $5$ |
| 11 | $AD = \frac{60}{11}$, $a = 2\sqrt{91}$, $BD = \frac{12\sqrt{91}}{11}$ |
| 12 | $A = 60^\circ$, $B = 90^\circ$, $C = 30^\circ$ |
| 13 | One triangle |
| 14 | $c = \sqrt{17} + 8\sqrt{3}$ or $c = 8\sqrt{3} - \sqrt{17}$ |
| 15 | $r = 6$, $r_1 = 16$, $r_2 = 16$, $r_3 = 24$, $R = \frac{25}{2}$ |
| 16 | $r = \frac{8}{3}$, $\Delta = 48$, $s = 18$; $a = 10$, $b = 16$, $c = 10$ |
| 17 | $R = \frac{841}{42}$; pedal sides $\frac{1640}{841}, 20, 20$; $AO = \frac{41}{21}$ |
| 18 | $n = 15$ |
| 19 | $R = 4\sqrt{2}$, $r = 4$, area $= 64$ |
| 20 | $CD = 6\sqrt{17}$ |

## Solutions

**1.** In triangle $ABC$, $A = 90^\circ$, $B = 30^\circ$ and $a = 12\sqrt{2}$. Find $b$. [1]

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$b = \frac{12\sqrt{2}\sin 30^\circ}{\sin 90^\circ} = 6\sqrt{2}$

**Ans:** $b = 6\sqrt{2}$

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**2.** In triangle $ABC$, $b = 4$, $c = 7$ and $A = 120^\circ$. Find $a$. [2]

$a^2 = b^2 + c^2 - 2bc\cos A = 16 + 49 - 56\cos 120^\circ$

$a^2 = 93$

$a = \sqrt{93}$

**Ans:** $a = \sqrt{93}$

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**3.** In triangle $ABC$, $a = 9$, $c = 8$ and $\cos B = \frac{17}{18}$. Find $b\cos C$. [1]

$a = b\cos C + c\cos B$

$b\cos C = 9 - 8\cdot\frac{17}{18} = \frac{13}{9}$

**Ans:** $\frac{13}{9}$

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**4.** Use Napier's rule to find $\tan\frac{B-C}{2}$ when $b = 5$, $c = 8$, $A = 30^\circ$. [2]

$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

$= \frac{-3}{13}\cot 15^\circ = -\frac{3}{13}\cdot\sqrt{3} + 2 = -\frac{6}{13} - \frac{3\sqrt{3}}{13}$

**Ans:** $-\frac{6}{13} - \frac{3\sqrt{3}}{13}$

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**5.** In triangle $ABC$, $a = 25$, $b = 15$, $c = 20$. Find $\sin\frac{A}{2}$. [2]

$s = 30$

$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} = \sqrt{\frac{15\cdot 10}{15\cdot 20}} = \frac{\sqrt{2}}{2}$

**Ans:** $\frac{\sqrt{2}}{2}$

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**6.** Find the area of triangle $ABC$ with $a = 10$, $b = 9$ and $C = 45^\circ$. [1]

$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}\cdot 10\cdot 9\cdot\frac{\sqrt{2}}{2}$

$= \frac{45\sqrt{2}}{2}$

**Ans:** $\Delta = \frac{45\sqrt{2}}{2}$

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**7.** For the triangle with $a = 28$, $b = 39$, $c = 17$, find the area and $\sin B$. [3]

$s = \frac{28+39+17}{2} = 42$

$\Delta = \sqrt{42\cdot 14\cdot 3\cdot 25} = 210$

$\sin B = \frac{2\Delta}{ac} = \frac{420}{476} = \frac{15}{17}$

**Ans:** $\Delta = 210$, $\sin B = \frac{15}{17}$

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**8.** Using the area, find $\cot C$ for the triangle with $a = 30$, $b = 25$, $c = 25$. [2]

$s = \frac{30+25+25}{2} = 40$

$\Delta = \sqrt{40 \cdot 10 \cdot 15 \cdot 15} = 300$

$\cot C = \frac{a^2+b^2-c^2}{4\Delta} = \frac{900}{1200} = \frac{3}{4}$

**Ans:** $\frac{3}{4}$

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**9.** If $a : b : c = 4 : 9 : 7$, find $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2}$. [1]

$\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2} = (s-a) : (s-b) : (s-c) = (y+z-x) : (z+x-y) : (x+y-z)$

$= 12 : 2 : 6 = 6 : 1 : 3$

**Ans:** $6 : 1 : 3$

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**10.** Find the length of the altitude from $C$ to the opposite side in the triangle with $a = 5$, $b = 13$, $c = 12$. [2]

$s = \frac{5+13+12}{2} = 15$

$\Delta = \sqrt{15 \cdot 10 \cdot 2 \cdot 3} = 30$

$\text{altitude} = \frac{2\Delta}{c} = \frac{60}{12} = 5$

**Ans:** $5$

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**11.** In triangle $ABC$, $b = 10$, $c = 12$ and $A = 120^\circ$. The bisector of angle $A$ meets $BC$ at $D$. Find $AD$, $a$ and $BD$. [3]

$AD = \frac{2bc}{b+c}\cos\frac{A}{2} = \frac{240}{22}\cos 60^\circ = \frac{60}{11}$

$a^2 = 100 + 144 - 240\cos 120^\circ = 364,\ a = 2\sqrt{91}$

$BD : DC = c : b = 12 : 10 \Rightarrow BD = \frac{12}{22} \cdot 2\sqrt{91} = \frac{12\sqrt{91}}{11}$

**Ans:** $AD = \frac{60}{11}$, $a = 2\sqrt{91}$, $BD = \frac{12\sqrt{91}}{11}$

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**12.** The sides of triangle $ABC$ are $a = 3\sqrt{2}$, $b = 2\sqrt{6}$, $c = \sqrt{6}$. Find its angles. [5]

$a^2 = 18,\ b^2 = 24,\ c^2 = 6$

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{1}{2} \Rightarrow A = 60^\circ$

$\cos B = \frac{c^2+a^2-b^2}{2ca} = 0 \Rightarrow B = 90^\circ$

$C = 180^\circ - 60^\circ - 90^\circ = 30^\circ$

**Ans:** $A = 60^\circ$, $B = 90^\circ$, $C = 30^\circ$

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**13.** How many triangles have $a = 9$, $b = 6$ and $A = 45^\circ$? [1]

$b\sin A = 6 \cdot \frac{\sqrt{2}}{2} = 3\sqrt{2}$

$a \geq b$: exactly one triangle.

**Ans: One triangle**

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**14.** Show that two triangles have $a = 9$, $b = 16$, $A = 30^\circ$, and find the two values of $c$. [3]

$b\sin A = 8$, and $8 < 9 < 16$: two triangles.

$c = b\cos A \pm \sqrt{a^2 - b^2\sin^2 A} = 8\sqrt{3} \pm \sqrt{81 - 64}$

$c = \sqrt{17} + 8\sqrt{3}$ or $8\sqrt{3} - \sqrt{17}$

**Ans:** $c = \sqrt{17} + 8\sqrt{3}$ **or** $c = 8\sqrt{3} - \sqrt{17}$

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**15.** For the triangle with $a = 20$, $b = 20$, $c = 24$, find the inradius, the three exradii and the circumradius. [5]

$s = \frac{20+20+24}{2} = 32$
$\Delta = \sqrt{32 \cdot 12 \cdot 12 \cdot 8} = 192$
$r = \frac{\Delta}{s} = 6, \quad r_1 = \frac{\Delta}{s-a} = 16$
$r_2 = \frac{\Delta}{s-b} = 16, \quad r_3 = \frac{\Delta}{s-c} = 24$
$4R = r_1 + r_2 + r_3 - r = 50 \Rightarrow R = \frac{25}{2}$
**Ans:** $r = 6$, $r_1 = 16$, $r_2 = 16$, $r_3 = 24$, $R = \frac{25}{2}$

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**16.** If $r_1 = 6$, $r_2 = 24$ and $r_3 = 6$, find $r$, $\Delta$, $s$ and the sides of the triangle. [5]

$\frac{1}{r} = \frac{1}{6} + \frac{1}{24} + \frac{1}{6} = \frac{3}{8} \Rightarrow r = \frac{8}{3}$
$\Delta^2 = r r_1 r_2 r_3 = 2304 \Rightarrow \Delta = 48$
$s = \frac{\Delta}{r} = 18$
$s - a = \frac{\Delta}{r_1} = 8,\ s - b = \frac{\Delta}{r_2} = 2,\ s - c = \frac{\Delta}{r_3} = 8$
$a = 10,\ b = 16,\ c = 10$
**Ans:** $r = \frac{8}{3}$, $\Delta = 48$, $s = 18$; $a = 10$, $b = 16$, $c = 10$

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**17.** The triangle $ABC$ has $a = 40$, $b = 29$, $c = 29$. Find $R$, the sides of its pedal triangle and the distance $AO$ of $A$ from the orthocentre $O$. [5]

$s = \frac{40+29+29}{2} = 49$
$\Delta = \sqrt{49 \cdot 9 \cdot 20 \cdot 20} = 420$
$R = \frac{abc}{4\Delta} = \frac{841}{42}$
$\cos A = \frac{41}{841},\ \cos B = \frac{20}{29},\ \cos C = \frac{20}{29}$ (all positive: acute)
Pedal sides: $a\cos A = \frac{1640}{841},\ b\cos B = 20,\ c\cos C = 20$
$AO = 2R\cos A = \frac{41}{21}$
**Ans:** $R = \frac{841}{42}$; **pedal sides** $\frac{1640}{841}, 20, 20$; $AO = \frac{41}{21}$

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**18.** Each interior angle of a regular polygon is $156^\circ$. How many sides has it? [1]

$\frac{(n-2)180^\circ}{n} = 156^\circ$
$\Rightarrow n = 15$
**Ans:** $n = 15$

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**19.** Find the circumradius, the inradius and the area of a square of side $8$. [3]

$\frac{\pi}{n} = 45^\circ$
$R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n} = 4\sqrt{2}$
$r = \frac{a}{2}\cot\frac{\pi}{n} = 4$
$\text{Area} = \frac{1}{4}na^2\cot\frac{\pi}{n} = 64$
**Ans:** $R = 4\sqrt{2}$, $r = 4$, **area** $= 64$

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**20.** In triangle $ABC$, $BC = 26$, $AC = 26$, $AB = 20$. $D$ is on $AB$ with $BD = 4$. Find $CD$ using Stewart's theorem. [2]

$DA = 20 - 4 = 16$

$a^2n + b^2m = c(d^2 + mn)$

$676 \cdot 16 + 676 \cdot 4 = 20(d^2 + 64)$

$d^2 = 612,\ CD = 6\sqrt{17}$

**Ans:** $CD = 6\sqrt{17}$

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