EXPRESSWALA ACADEMY

Maths 1A: Properties of Triangles

Question Paper, Set 07

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. In triangle ABC, $a = 2\sqrt{2}$ and $A = 45^\circ$. Find the circumradius R. [1 mark]

2. In triangle ABC, $a = 56$, $b = 24$, $c = 40$. Find the angle A. [2 marks]

3. In triangle ABC, $a = 18$, $c = 15$ and $\cos B = \frac{3}{5}$. Find $b\cos C$. [1 mark]

4. In triangle ABC, $b = 12$, $c = 11$ and $A = 45^\circ$. Find $\tan\frac{B-C}{2}$. [2 marks]

5. In triangle ABC, $a = 39$, $b = 25$, $c = 16$. Find $\cot\frac{C}{2}$. [2 marks]

6. Find the area of triangle ABC with $b = 15$, $c = 4$ and $A = 150^\circ$. [1 mark]

7. Find the area and the circumradius of the triangle with $a = 21$, $b = 28$, $c = 35$. [3 marks]

8. For the triangle with $a = 30$, $b = 11$, $c = 25$, find $\cot A + \cot B + \cot C$. [2 marks]

9. In triangle ABC, $a + c = \frac{5}{4}b$. Find $\tan\frac{A}{2}\tan\frac{C}{2}$. [1 mark]

10. Find the length of the median from B in the triangle with $a = 4$, $b = 6$, $c = 7$. [2 marks]

11. In triangle ABC, $b = 12$, $c = 5$ and $A = 120^\circ$. The bisector of angle A meets BC at D. Find AD, a and BD. [3 marks]

12. Solve the triangle ABC in which $A = 45^\circ$, $B = 75^\circ$ and $c = 2\sqrt{6}$ (find C, a, b and R). [5 marks]

13. How many triangles have $a = 4\sqrt{3}$, $b = 8$ and $A = 60^\circ$? [1 mark]

14. Show that two triangles have $a = 6$, $b = 8$, $A = 30^\circ$, and find the two values of c. [3 marks]

15. Find r, r_1, r_2, r_3 and R for the triangle with $a = 20$, $b = 20$, $c = 24$. [5 marks]

16. Find the sides a, b, c of the triangle in which $r_1 = 8$, $r_2 = 72$, $r_3 = 9$. [5 marks]

17. For the triangle with $a = 35$, $b = 34$, $c = 15$, find R, r, the distance IS between the incentre and the circumcentre, and the distance of S from BC. [5 marks]

18. Find the sum of the interior angles of a polygon with 13 sides, in right angles and in degrees. [1 mark]

19. Find the circumradius, the inradius and the area of a regular dodecagon (12 sides) of side 5. [3 marks]

20. Find the area of a cyclic quadrilateral with sides 10, 9, 8, 3. [2 marks]

Key sheet

Q	Answer
1	$R = 2$
2	$A = 120^\circ$
3	9
4	$\frac{1}{23} + \frac{\sqrt{2}}{23}$
5	8
6	$\Delta = 15$
7	$\Delta = 294$, $R = \frac{35}{2}$
8	$\frac{823}{264}$
9	$\frac{1}{9}$
10	$\frac{\sqrt{94}}{2}$
11	$AD = \frac{60}{17}$, $a = \sqrt{229}$, $BD = \frac{5\sqrt{229}}{17}$
12	$C = 60^\circ$, $a = 4$, $b = 2 + 2\sqrt{3}$, $R = 2\sqrt{2}$
13	One triangle
14	$c = 2\sqrt{5} + 4\sqrt{3}$ or $c = 4\sqrt{3} - 2\sqrt{5}$
15	$r = 6$, $r_1 = 16$, $r_2 = 16$, $r_3 = 24$, $R = \frac{25}{2}$
16	$a = 18$, $b = 34$, $c = 20$
17	$R = \frac{425}{24}$, $r = 6$, $IS = \frac{5\sqrt{2329}}{24}$, distance $= \frac{65}{24}$
18	22 right angles $= 1980^\circ$
19	$R = \frac{5\sqrt{2}+5\sqrt{6}}{2}$, $r = \frac{5\sqrt{3}}{2} + 5$, area $= 75\sqrt{3} + 150$
20	$6\sqrt{70}$

Solutions

1. In triangle ABC, $a = 2\sqrt{2}$ and $A = 45^\circ$. Find the circumradius R. [1]

$\frac{a}{\sin A} = 2R$

$2R = \frac{2\sqrt{2}}{\sin 45^\circ} = \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = 4$

$R = 2$

Ans: $R = 2$

---

2. In triangle ABC, $a = 56$, $b = 24$, $c = 40$. Find the angle A. [2]

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{576+1600-3136}{1920} = -\frac{1}{2}$

$A = 120^\circ$

Ans: $A = 120^\circ$

---

3. In triangle ABC, $a = 18$, $c = 15$ and $\cos B = \frac{3}{5}$. Find $b\cos C$. [1]

$a = b\cos C + c\cos B$

$b\cos C = 18 - 15 \cdot \frac{3}{5} = 9$

Ans: 9

---

4. In triangle ABC, $b = 12$, $c = 11$ and $A = 45^\circ$. Find $\tan\frac{B-C}{2}$. [2]

$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

$= \frac{1}{23}\cot\frac{45^\circ}{2} = \frac{1}{23}\cdot 1+\sqrt{2} = \frac{1}{23}+\frac{\sqrt{2}}{23}$

Ans: $\frac{1}{23}+\frac{\sqrt{2}}{23}$

---

5. In triangle ABC, $a = 39$, $b = 25$, $c = 16$. Find $\cot\frac{C}{2}$. [2]

$s = \frac{39+25+16}{2} = 40$

$\Delta = \sqrt{40\cdot 1\cdot 15\cdot 24} = 120$

$\cot\frac{C}{2} = \frac{s(s-c)}{\Delta} = \frac{40\cdot 24}{120} = 8$

Ans: 8

---

6. Find the area of triangle ABC with $b = 15$, $c = 4$ and $A = 150^\circ$. [1]

$\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}\cdot 15\cdot 4\cdot\frac{1}{2}$

$= 15$

Ans: $\Delta = 15$

---

7. Find the area and the circumradius of the triangle with $a = 21$, $b = 28$, $c = 35$. [3]

$s = \frac{21+28+35}{2} = 42$

$\Delta = \sqrt{42\cdot 21\cdot 14\cdot 7} = 294$

$R = \frac{abc}{4\Delta} = \frac{20580}{1176} = \frac{35}{2}$

Ans: $\Delta = 294$, $R = \frac{35}{2}$

---

8. For the triangle with $a = 30$, $b = 11$, $c = 25$, find $\cot A + \cot B + \cot C$. [2]

$s = \frac{30+11+25}{2} = 33$

$\Delta = \sqrt{33 \cdot 3 \cdot 22 \cdot 8} = 132$

$\cot A + \cot B + \cot C = \frac{a^2+b^2+c^2}{4\Delta} = \frac{1646}{528} = \frac{823}{264}$

Ans: $\frac{823}{264}$

---

9. In triangle ABC, $a + c = \frac{5}{4}b$. Find $\tan\frac{A}{2}\tan\frac{C}{2}$. [1]

$\tan\frac{A}{2}\tan\frac{C}{2} = \frac{n-1}{n+1}$

$= \frac{\frac{5}{4}-1}{\frac{5}{4}+1} = \frac{1}{9}$

Ans: $\frac{1}{9}$

---

10. Find the length of the median from B in the triangle with $a = 4$, $b = 6$, $c = 7$. [2]

$m = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2} = \frac{1}{2}\sqrt{32 + 98 - 36}$

$= \frac{1}{2}\sqrt{94} = \frac{\sqrt{94}}{2}$

Ans: $\frac{\sqrt{94}}{2}$

---

11. In triangle ABC, $b = 12$, $c = 5$ and $A = 120^\circ$. The bisector of angle A meets BC at D. Find AD, a and BD. [3]

$AD = \frac{2bc}{b+c}\cos\frac{A}{2} = \frac{120}{17}\cos 60^\circ = \frac{60}{17}$

$a^2 = 144 + 25 - 120\cos 120^\circ = 229,\ a = \sqrt{229}$

$BD : DC = c : b = 5 : 12 \Rightarrow BD = \frac{5}{17} \cdot \sqrt{229} = \frac{5\sqrt{229}}{17}$

Ans: $AD = \frac{60}{17}$, $a = \sqrt{229}$, $BD = \frac{5\sqrt{229}}{17}$

---

12. Solve the triangle ABC in which $A = 45^\circ$, $B = 75^\circ$ and $c = 2\sqrt{6}$ (find C, a, b and R). [5]

$C = 180^\circ - (45^\circ + 75^\circ) = 60^\circ$

$2R = \frac{c}{\sin C} = \frac{2\sqrt{6}}{\frac{\sqrt{3}}{2}} = 4\sqrt{2}$

$a = 2R\sin A = 4\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 4$

$b = 2R\sin B = 4\sqrt{2} \cdot \frac{\sqrt{2}+\sqrt{6}}{4} = 2 + 2\sqrt{3}$

$R = 2\sqrt{2}$

Ans: $C = 60^\circ$, $a = 4$, $b = 2 + 2\sqrt{3}$, $R = 2\sqrt{2}$

---

13. How many triangles have $a = 4\sqrt{3}$, $b = 8$ and $A = 60^\circ$? [1]

$b\sin A = 8 \cdot \frac{\sqrt{3}}{2} = 4\sqrt{3}$

$a = b\sin A$: exactly one triangle, right-angled at B.

Ans: One triangle

---

14. Show that two triangles have $a = 6$, $b = 8$, $A = 30^\circ$, and find the two values of c. [3]

$b\sin A = 4$, and $4 < 6 < 8$: two triangles.

$c = b\cos A \pm \sqrt{a^2 - b^2\sin^2 A} = 4\sqrt{3} \pm \sqrt{36 - 16}$

$c = 2\sqrt{5} + 4\sqrt{3}$ or $4\sqrt{3} - 2\sqrt{5}$

Ans: $c = 2\sqrt{5} + 4\sqrt{3}$ **or** $c = 4\sqrt{3} - 2\sqrt{5}$

---

15. Find r, r_1, r_2, r_3 and R for the triangle with $a = 20$, $b = 20$, $c = 24$. [5]

$s = \frac{20+20+24}{2} = 32$

$\Delta = \sqrt{32 \cdot 12 \cdot 12 \cdot 8} = 192$

$r = \frac{\Delta}{s} = 6, \quad r_1 = \frac{\Delta}{s-a} = 16$

$r_2 = \frac{\Delta}{s-b} = 16, \quad r_3 = \frac{\Delta}{s-c} = 24$

$4R = r_1 + r_2 + r_3 - r = 50 \Rightarrow R = \frac{25}{2}$

Ans: $r = 6$, $r_1 = 16$, $r_2 = 16$, $r_3 = 24$, $R = \frac{25}{2}$

---

16. Find the sides a, b, c of the triangle in which $r_1 = 8$, $r_2 = 72$, $r_3 = 9$. [5]

$\frac{1}{r} = \frac{1}{8} + \frac{1}{72} + \frac{1}{9} = \frac{1}{4} \Rightarrow r = 4$

$\Delta^2 = r r_1 r_2 r_3 = 20736 \Rightarrow \Delta = 144$

$s = \frac{\Delta}{r} = 36$

$s - a = \frac{\Delta}{r_1} = 18, \; s - b = \frac{\Delta}{r_2} = 2, \; s - c = \frac{\Delta}{r_3} = 16$

$a = 18, \; b = 34, \; c = 20$

Ans: $a = 18$, $b = 34$, $c = 20$

---

17. For the triangle with $a = 35$, $b = 34$, $c = 15$, find R, r, the distance IS between the incentre and the circumcentre, and the distance of S from BC. [5]

$s = \frac{35+34+15}{2} = 42$

$\Delta = \sqrt{42 \cdot 7 \cdot 8 \cdot 27} = 252$

$R = \frac{abc}{4\Delta} = \frac{425}{24}, \quad r = \frac{\Delta}{s} = 6$

$IS = \sqrt{R^2 - 2rR} = \sqrt{\frac{180625}{576} - \frac{425}{2}} = \frac{5\sqrt{2329}}{24}$

$\cos A = \frac{13}{85}, \;$ distance of S from $BC = R\cos A = \frac{65}{24}$

Ans: $R = \frac{425}{24}$, $r = 6$, $IS = \frac{5\sqrt{2329}}{24}$, **distance** $= \frac{65}{24}$

---

18. Find the sum of the interior angles of a polygon with 13 sides, in right angles and in degrees. [1]

$(2n - 4)$ right angles $= 22$ right angles

$= 22 \times 90^\circ = 1980^\circ$

Ans: 22 **right angles** $= 1980^\circ$

---

19. Find the circumradius, the inradius and the area of a regular dodecagon (12 sides) of side 5. [3]

$\frac{\pi}{n} = 15^\circ$

$R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n} = \frac{5\sqrt{2}+5\sqrt{6}}{2}$

$r = \frac{a}{2}\cot\frac{\pi}{n} = \frac{5\sqrt{3}}{2} + 5$

$\text{Area} = \frac{1}{4}na^2\cot\frac{\pi}{n} = 75\sqrt{3} + 150$

Ans: $R = \frac{5\sqrt{2}+5\sqrt{6}}{2}$, $r = \frac{5\sqrt{3}}{2} + 5$, **area** $= 75\sqrt{3} + 150$

---

20. Find the area of a cyclic quadrilateral with sides 10, 9, 8, 3. [2]

$s = \frac{10+9+8+3}{2} = 15$

$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)} = \sqrt{5 \cdot 6 \cdot 7 \cdot 12} = 6\sqrt{70}$

Ans: $6\sqrt{70}$
