EXPRESSWALA ACADEMY

Maths 1A: Properties of Triangles

Question Paper, Set 05

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. In triangle ABC, $a = 12$ and $A = 150^\circ$. Find the circumradius R. [1 mark]

2. In triangle ABC, $a = 24$, $b = 35$, $c = 31$. Find the angle C. [2 marks]

3. In triangle ABC, $a = 24$, $b = 20$ and $\cos C = \frac{3}{5}$. Find $c\cos B$. [1 mark]

4. In triangle ABC, $b = 2$, $c = 13$ and $A = 120^\circ$. Find $\tan\frac{B-C}{2}$. [2 marks]

5. In triangle ABC, $a = 30$, $b = 29$, $c = 5$. Find $\tan\frac{A}{2}$. [2 marks]

6. Find the area of triangle ABC with $a = 13$, $b = 4$ and $C = 30^\circ$. [1 mark]

7. Find the area and the circumradius of the triangle with $a = 24$, $b = 13$, $c = 13$. [3 marks]

8. For the triangle with $a = 16$, $b = 25$, $c = 39$, find $\cot A + \cot B + \cot C$. [2 marks]

9. In triangle ABC, $a + c = \frac{7}{2}b$. Find $\tan\frac{A}{2}\tan\frac{C}{2}$. [1 mark]

10. Find the length of the median from C in the triangle with $a = 8$, $b = 12$, $c = 6$. [2 marks]

11. In triangle ABC, $b = 9$, $c = 2$ and $A = 90^\circ$. The bisector of angle A meets BC at D. Find AD, a and BD. [3 marks]

12. Solve the triangle ABC in which $A = 60^\circ$, $B = 75^\circ$ and $c = 2\sqrt{3}$ (find C, a, b and R). [5 marks]

13. How many triangles have $a = 10$, $b = 14$ and $A = 30^\circ$? [1 mark]

14. Show that two triangles have $a = 12$, $b = 18$, $A = 30^\circ$, and find the two values of c. [3 marks]

15. Find r, r_1, r_2, r_3 and R for the triangle with $a = 26$, $b = 37$, $c = 15$. [5 marks]

16. Find the sides a, b, c of the triangle in which $r_1 = 12$, $r_2 = 20$, $r_3 = 5$. [5 marks]

17. For the triangle with $a = 40$, $b = 29$, $c = 29$, find R, r, the distance IS between the incentre and the circumcentre, and the distance of S from BC. [5 marks]

18. Find the sum of the interior angles of a polygon with 22 sides, in right angles and in degrees. [1 mark]

19. Find the circumradius, the inradius and the area of a regular dodecagon (12 sides) of side 3. [3 marks]

20. In triangle ABC, $BC = 6$, $AC = 5$, $AB = 5$. D is on AB with $BD = 2$. Find CD using Stewart's theorem. [2 marks]

Key sheet

Q	Answer
1	$R = 12$
2	$C = 60^\circ$
3	12
4	$-\frac{11\sqrt{3}}{45}$
5	$\frac{9}{8}$
6	$\Delta = 13$
7	$\Delta = 60$, $R = \frac{169}{10}$
8	$\frac{1201}{240}$
9	$\frac{5}{9}$
10	$\sqrt{95}$
11	$AD = \frac{18\sqrt{2}}{11}$, $a = \sqrt{85}$, $BD = \frac{2\sqrt{85}}{11}$
12	$C = 45^\circ$, $a = 3\sqrt{2}$, $b = \sqrt{3} + 3$, $R = \sqrt{6}$
13	Two triangles
14	$c = 3\sqrt{7} + 9\sqrt{3}$ or $c = 9\sqrt{3} - 3\sqrt{7}$
15	$r = 4$, $r_1 = 12$, $r_2 = 78$, $r_3 = \frac{13}{2}$, $R = \frac{185}{8}$
16	$a = 15$, $b = 17$, $c = 8$
17	$R = \frac{841}{42}$, $r = \frac{60}{7}$, $IS = \frac{319}{42}$, distance $= \frac{41}{42}$
18	40 right angles $= 3600^\circ$
19	$R = \frac{3\sqrt{2}+3\sqrt{6}}{2}$, $r = \frac{3\sqrt{3}}{2} + 3$, area $= 27\sqrt{3} + 54$
20	$CD = \frac{8\sqrt{10}}{5}$

Solutions

1. In triangle ABC, $a = 12$ and $A = 150°$. Find the circumradius R. [1]

$\frac{a}{\sin A} = 2R$

$2R = \frac{12}{\sin 150°} = \frac{12}{\frac{1}{2}} = 24$

$R = 12$

Ans: $R = 12$

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2. In triangle ABC, $a = 24$, $b = 35$, $c = 31$. Find the angle C. [2]

$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{576+1225-961}{1680} = \frac{1}{2}$

$C = 60°$

Ans: $C = 60°$

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3. In triangle ABC, $a = 24$, $b = 20$ and $\cos C = \frac{3}{5}$. Find $c\cos B$. [1]

$a = b\cos C + c\cos B$

$c\cos B = 24 - 20 \cdot \frac{3}{5} = 12$

Ans: 12

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4. In triangle ABC, $b = 2$, $c = 13$ and $A = 120°$. Find $\tan\frac{B-C}{2}$. [2]

$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

$= \frac{-11}{15}\cot 60° = -\frac{11}{15} \cdot \frac{\sqrt{3}}{3} = -\frac{11\sqrt{3}}{45}$

Ans: $-\frac{11\sqrt{3}}{45}$

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5. In triangle ABC, $a = 30$, $b = 29$, $c = 5$. Find $\tan\frac{A}{2}$. [2]

$s = \frac{30+29+5}{2} = 32$

$\Delta = \sqrt{32 \cdot 2 \cdot 3 \cdot 27} = 72$

$\tan\frac{A}{2} = \frac{(s-b)(s-c)}{\Delta} = \frac{3 \cdot 27}{72} = \frac{9}{8}$

Ans: $\frac{9}{8}$

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6. Find the area of triangle ABC with $a = 13$, $b = 4$ and $C = 30°$. [1]

$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2} \cdot 13 \cdot 4 \cdot \frac{1}{2}$

$= 13$

Ans: $\Delta = 13$

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7. Find the area and the circumradius of the triangle with $a = 24$, $b = 13$, $c = 13$. [3]

$s = \frac{24+13+13}{2} = 25$

$\Delta = \sqrt{25 \cdot 1 \cdot 12 \cdot 12} = 60$

$R = \frac{abc}{4\Delta} = \frac{4056}{240} = \frac{169}{10}$

Ans: $\Delta = 60$, $R = \frac{169}{10}$

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8. For the triangle with $a = 16$, $b = 25$, $c = 39$, find $\cot A + \cot B + \cot C$. [2]

$s = \frac{16+25+39}{2} = 40$

$\Delta = \sqrt{40 \cdot 24 \cdot 15 \cdot 1} = 120$

$\cot A + \cot B + \cot C = \frac{a^2+b^2+c^2}{4\Delta} = \frac{2402}{480} = \frac{1201}{240}$

Ans: $\frac{1201}{240}$

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9. In triangle ABC, $a + c = \frac{7}{2}b$. Find $\tan\frac{A}{2}\tan\frac{C}{2}$. [1]

$\tan\frac{A}{2}\tan\frac{C}{2} = \frac{n-1}{n+1}$

$= \frac{\frac{7}{2}-1}{\frac{7}{2}+1} = \frac{5}{9}$

Ans: $\frac{5}{9}$

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10. Find the length of the median from C in the triangle with $a = 8$, $b = 12$, $c = 6$. [2]

$m = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2} = \frac{1}{2}\sqrt{128 + 288 - 36}$

$= \frac{1}{2}\sqrt{380} = \sqrt{95}$

Ans: $\sqrt{95}$

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11. In triangle ABC, $b = 9$, $c = 2$ and $A = 90^\circ$. The bisector of angle A meets BC at D. Find AD, a and BD. [3]

$AD = \frac{2bc}{b+c}\cos\frac{A}{2} = \frac{36}{11}\cos 45^\circ = \frac{18\sqrt{2}}{11}$

$a^2 = 81 + 4 - 36\cos 90^\circ = 85,\ a = \sqrt{85}$

$BD : DC = c : b = 2 : 9 \Rightarrow BD = \frac{2}{11}\cdot\sqrt{85} = \frac{2\sqrt{85}}{11}$

Ans: $AD = \frac{18\sqrt{2}}{11}$, $a = \sqrt{85}$, $BD = \frac{2\sqrt{85}}{11}$

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12. Solve the triangle ABC in which $A = 60^\circ$, $B = 75^\circ$ and $c = 2\sqrt{3}$ (find C, a, b and R). [5]

$C = 180^\circ - (60^\circ + 75^\circ) = 45^\circ$

$2R = \frac{c}{\sin C} = \frac{2\sqrt{3}}{\frac{\sqrt{2}}{2}} = 2\sqrt{6}$

$a = 2R\sin A = 2\sqrt{6}\cdot\frac{\sqrt{3}}{2} = 3\sqrt{2}$

$b = 2R\sin B = 2\sqrt{6}\cdot\frac{\sqrt{2}+\sqrt{6}}{4} = \sqrt{3}+3$

$R = \sqrt{6}$

Ans: $C = 45^\circ$, $a = 3\sqrt{2}$, $b = \sqrt{3}+3$, $R = \sqrt{6}$

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13. How many triangles have $a = 10$, $b = 14$ and $A = 30^\circ$? [1]

$b\sin A = 14\cdot\frac{1}{2} = 7$

$b\sin A < a < b$: two triangles.

Ans: Two triangles

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14. Show that two triangles have $a = 12$, $b = 18$, $A = 30^\circ$, and find the two values of c. [3]

$b\sin A = 9$, and $9 < 12 < 18$: two triangles.

$c = b\cos A \pm \sqrt{a^2 - b^2\sin^2 A} = 9\sqrt{3} \pm \sqrt{144 - 81}$

$c = 3\sqrt{7} + 9\sqrt{3}$ or $9\sqrt{3} - 3\sqrt{7}$

Ans: $c = 3\sqrt{7} + 9\sqrt{3}$ **or** $c = 9\sqrt{3} - 3\sqrt{7}$

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15. Find r, r_1, r_2, r_3 and R for the triangle with $a = 26$, $b = 37$, $c = 15$. [5]

$s = \frac{26+37+15}{2} = 39$

$\Delta = \sqrt{39 \cdot 13 \cdot 2 \cdot 24} = 156$

$r = \frac{\Delta}{s} = 4, \quad r_1 = \frac{\Delta}{s-a} = 12$

$r_2 = \frac{\Delta}{s-b} = 78, \quad r_3 = \frac{\Delta}{s-c} = \frac{13}{2}$

$4R = r_1 + r_2 + r_3 - r = \frac{185}{2} \Rightarrow R = \frac{185}{8}$

Ans: $r = 4$, $r_1 = 12$, $r_2 = 78$, $r_3 = \frac{13}{2}$, $R = \frac{185}{8}$

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16. Find the sides a, b, c of the triangle in which $r_1 = 12$, $r_2 = 20$, $r_3 = 5$. [5]

$\frac{1}{r} = \frac{1}{12} + \frac{1}{20} + \frac{1}{5} = \frac{1}{3} \Rightarrow r = 3$

$\Delta^2 = r r_1 r_2 r_3 = 3600 \Rightarrow \Delta = 60$

$s = \frac{\Delta}{r} = 20$

$s - a = \frac{\Delta}{r_1} = 5, \; s - b = \frac{\Delta}{r_2} = 3, \; s - c = \frac{\Delta}{r_3} = 12$

$a = 15, \; b = 17, \; c = 8$

Ans: $a = 15$, $b = 17$, $c = 8$

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17. For the triangle with $a = 40$, $b = 29$, $c = 29$, find R, r, the distance IS between the incentre and the circumcentre, and the distance of S from BC. [5]

$s = \frac{40+29+29}{2} = 49$

$\Delta = \sqrt{49 \cdot 9 \cdot 20 \cdot 20} = 420$

$R = \frac{abc}{4\Delta} = \frac{841}{42}, \quad r = \frac{\Delta}{s} = \frac{60}{7}$

$IS = \sqrt{R^2 - 2rR} = \sqrt{\frac{707281}{1764} - \frac{16820}{49}} = \frac{319}{42}$

$\cos A = \frac{41}{841}, \;$ distance of S from $BC = R \cos A = \frac{41}{42}$

Ans: $R = \frac{841}{42}$, $r = \frac{60}{7}$, $IS = \frac{319}{42}$, **distance** $= \frac{41}{42}$

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18. Find the sum of the interior angles of a polygon with 22 sides, in right angles and in degrees. [1]

$(2n - 4)$ right angles $= 40$ right angles

$= 40 \times 90^\circ = 3600^\circ$

Ans: 40 **right angles** $= 3600^\circ$

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19. Find the circumradius, the inradius and the area of a regular dodecagon (12 sides) of side 3. [3]

$\frac{\pi}{n} = 15^\circ$

$R = \frac{a}{2} \operatorname{cosec} \frac{\pi}{n} = \frac{3\sqrt{2}+3\sqrt{6}}{2}$

$r = \frac{a}{2} \cot \frac{\pi}{n} = \frac{3\sqrt{3}}{2} + 3$

$\text{Area} = \frac{1}{4} n a^2 \cot \frac{\pi}{n} = 27\sqrt{3} + 54$

Ans: $R = \frac{3\sqrt{2}+3\sqrt{6}}{2}$, $r = \frac{3\sqrt{3}}{2} + 3$, **area** $= 27\sqrt{3} + 54$

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20. In triangle ABC, $BC = 6$, $AC = 5$, $AB = 5$. D is on AB with $BD = 2$. Find CD using Stewart's theorem. [2]

$DA = 5 - 2 = 3$

$a^2n + b^2m = c(d^2 + mn)$

$36 \cdot 3 + 25 \cdot 2 = 5(d^2 + 6)$

$d^2 = \frac{128}{5}$, $CD = \frac{8\sqrt{10}}{5}$

Ans: $CD = \frac{8\sqrt{10}}{5}$
