EXPRESSWALA ACADEMY

Maths 1A: Properties of Triangles

Question Paper, Set 04

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. In triangle ABC, $A = 120^\circ$, $B = 30^\circ$ and $a = 21$. Find b. [1 mark]

2. In triangle ABC, $b = 11$, $c = 12$ and $A = 135^\circ$. Find a. [2 marks]

3. In triangle ABC, $a = 13$, $b = 12$ and $\cos C = \frac{11}{39}$. Find $c\cos B$. [1 mark]

4. Use Napier's rule to find $\tan\frac{B-C}{2}$ when $b = 11$, $c = 2$, $A = 45^\circ$. [2 marks]

5. In triangle ABC, $a = 5$, $b = 30$, $c = 29$. Find $\sin\frac{C}{2}$. [2 marks]

6. Find the area of triangle ABC with $b = 12$, $c = 7$ and $A = 150^\circ$. [1 mark]

7. For the triangle with $a = 16$, $b = 10$, $c = 10$, find the area and $\sin C$. [3 marks]

8. Using the area, find $\cot A$ for the triangle with $a = 18$, $b = 15$, $c = 15$. [2 marks]

9. If $a : b : c = 6 : 11 : 9$, find $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2}$. [1 mark]

10. Find the length of the altitude from B to the opposite side in the triangle with $a = 29$, $b = 40$, $c = 29$. [2 marks]

11. In triangle ABC, $b = 12$, $c = 2$ and $A = 60^\circ$. The bisector of angle A meets BC at D. Find AD, a and BD. [3 marks]

12. The sides of triangle ABC are $a = 1$, $b = \sqrt{3}$, $c = 2$. Find its angles. [5 marks]

13. How many triangles have $a = 19$, $b = 20\sqrt{2}$ and $A = 45^\circ$? [1 mark]

14. Show that two triangles have $a = 16$, $b = 20$, $A = 45^\circ$, and find the two values of c. [3 marks]

15. For the triangle with $a = 12$, $b = 37$, $c = 35$, find the inradius, the three exradii and the circumradius. [5 marks]

16. If $r_1 = 6$, $r_2 = 12$ and $r_3 = 28$, find r, Δ, s and the sides of the triangle. [5 marks]

17. The triangle ABC has $a = 39$, $b = 25$, $c = 40$. Find R, the sides of its pedal triangle and the distance AO of A from the orthocentre O. [5 marks]

18. Each interior angle of a regular polygon is $\frac{1980}{13}^\circ$. How many sides has it? [1 mark]

19. Find the circumradius, the inradius and the area of an equilateral triangle of side 4. [3 marks]

20. Find the area of a cyclic quadrilateral with sides 9, 12, 5, 7. [2 marks]

Key sheet

Q	Answer
1	$b = 7\sqrt{3}$
2	$a = \sqrt{132\sqrt{2} + 265}$
3	$\frac{125}{13}$
4	$\frac{9}{13} + \frac{9\sqrt{2}}{13}$
5	$\frac{3}{5}$
6	$\Delta = 21$
7	$\Delta = 48$, $\sin C = \frac{3}{5}$
8	$\frac{7}{24}$
9	$7 : 2 : 4$
10	21
11	$AD = \frac{12\sqrt{3}}{7}$, $a = 2\sqrt{31}$, $BD = \frac{2\sqrt{31}}{7}$
12	$A = 30^\circ$, $B = 60^\circ$, $C = 90^\circ$
13	No triangle
14	$c = 2\sqrt{14} + 10\sqrt{2}$ or $c = 10\sqrt{2} - 2\sqrt{14}$
15	$r = 5$, $r_1 = 7$, $r_2 = 42$, $r_3 = 30$, $R = \frac{37}{2}$
16	$r = \frac{7}{2}$, $\Delta = 84$, $s = 24$; $a = 10$, $b = 17$, $c = 21$
17	$R = \frac{125}{6}$; pedal sides $\frac{1716}{125}, 20, \frac{56}{5}$; $AO = \frac{44}{3}$
18	$n = 13$
19	$R = \frac{4\sqrt{3}}{3}$, $r = \frac{2\sqrt{3}}{3}$, area $= 4\sqrt{3}$
20	$\frac{3\sqrt{6555}}{4}$

Solutions

1. In triangle ABC, $A = 120°$, $B = 30°$ and $a = 21$. Find b. [1]

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$b = \frac{21 \sin 30°}{\sin 120°} = 7\sqrt{3}$

Ans: $b = 7\sqrt{3}$

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2. In triangle ABC, $b = 11$, $c = 12$ and $A = 135°$. Find a. [2]

$a^2 = b^2 + c^2 - 2bc\cos A = 121 + 144 - 264\cos 135°$

$a^2 = 132\sqrt{2} + 265$

$a = \sqrt{132\sqrt{2} + 265}$

Ans: $a = \sqrt{132\sqrt{2} + 265}$

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3. In triangle ABC, $a = 13$, $b = 12$ and $\cos C = \frac{11}{39}$. Find $c\cos B$. [1]

$a = b\cos C + c\cos B$

$c\cos B = 13 - 12 \cdot \frac{11}{39} = \frac{125}{13}$

Ans: $\frac{125}{13}$

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4. Use Napier's rule to find $\tan\frac{B-C}{2}$ when $b = 11$, $c = 2$, $A = 45°$. [2]

$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

$= \frac{9}{13}\cot\frac{45}{2}° = \frac{9}{13} \cdot 1 + \sqrt{2} = \frac{9}{13} + \frac{9\sqrt{2}}{13}$

Ans: $\frac{9}{13} + \frac{9\sqrt{2}}{13}$

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5. In triangle ABC, $a = 5$, $b = 30$, $c = 29$. Find $\sin\frac{C}{2}$. [2]

$s = 32$

$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}} = \sqrt{\frac{27 \cdot 2}{5 \cdot 30}} = \frac{3}{5}$

Ans: $\frac{3}{5}$

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6. Find the area of triangle ABC with $b = 12$, $c = 7$ and $A = 150°$. [1]

$\Delta = \frac{1}{2}bc\sin A = \frac{1}{2} \cdot 12 \cdot 7 \cdot \frac{1}{2}$

$= 21$

Ans: $\Delta = 21$

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7. For the triangle with $a = 16$, $b = 10$, $c = 10$, find the area and $\sin C$. [3]

$s = \frac{16+10+10}{2} = 18$

$\Delta = \sqrt{18 \cdot 2 \cdot 8 \cdot 8} = 48$

$\sin C = \frac{2\Delta}{ab} = \frac{96}{160} = \frac{3}{5}$

Ans: $\Delta = 48$, $\sin C = \frac{3}{5}$

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8. Using the area, find $\cot A$ for the triangle with $a = 18$, $b = 15$, $c = 15$. [2]

$s = \frac{18+15+15}{2} = 24$

$\Delta = \sqrt{24 \cdot 6 \cdot 9 \cdot 9} = 108$

$\cot A = \frac{b^2+c^2-a^2}{4\Delta} = \frac{126}{432} = \frac{7}{24}$

Ans: $\frac{7}{24}$

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9. If $a : b : c = 6 : 11 : 9$, find $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2}$. [1]

$\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2} = (s-a) : (s-b) : (s-c) = (y+z-x) : (z+x-y) : (x+y-z)$

$= 14 : 4 : 8 = 7 : 2 : 4$

Ans: $7 : 2 : 4$

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10. Find the length of the altitude from B to the opposite side in the triangle with $a = 29$, $b = 40$, $c = 29$. [2]

$s = \frac{29+40+29}{2} = 49$

$\Delta = \sqrt{49 \cdot 20 \cdot 9 \cdot 20} = 420$

$\text{altitude} = \frac{2\Delta}{b} = \frac{840}{40} = 21$

Ans: 21

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11. In triangle ABC, $b = 12$, $c = 2$ and $A = 60^\circ$. The bisector of angle A meets BC at D. Find AD, a and BD. [3]

$AD = \frac{2bc}{b+c}\cos\frac{A}{2} = \frac{48}{14}\cos 30^\circ = \frac{12\sqrt{3}}{7}$

$a^2 = 144 + 4 - 48\cos 60^\circ = 124,\ a = 2\sqrt{31}$

$BD : DC = c : b = 2 : 12 \Rightarrow BD = \frac{2}{14} \cdot 2\sqrt{31} = \frac{2\sqrt{31}}{7}$

Ans: $AD = \frac{12\sqrt{3}}{7}$, $a = 2\sqrt{31}$, $BD = \frac{2\sqrt{31}}{7}$

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12. The sides of triangle ABC are $a = 1$, $b = \sqrt{3}$, $c = 2$. Find its angles. [5]

$a^2 = 1,\ b^2 = 3,\ c^2 = 4$

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{\sqrt{3}}{2} \Rightarrow A = 30^\circ$

$\cos B = \frac{c^2+a^2-b^2}{2ca} = \frac{1}{2} \Rightarrow B = 60^\circ$

$C = 180^\circ - 30^\circ - 60^\circ = 90^\circ$

Ans: $A = 30^\circ$, $B = 60^\circ$, $C = 90^\circ$

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13. How many triangles have $a = 19$, $b = 20\sqrt{2}$ and $A = 45^\circ$? [1]

$b\sin A = 20\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 20$

$a < b\sin A$: no triangle.

Ans: No triangle

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14. Show that two triangles have $a = 16$, $b = 20$, $A = 45^\circ$, and find the two values of c. [3]

$b\sin A = 10\sqrt{2}$, and $10\sqrt{2} < 16 < 20$: two triangles.

$c = b\cos A \pm \sqrt{a^2 - b^2\sin^2 A} = 10\sqrt{2} \pm \sqrt{256 - 200}$

$c = 2\sqrt{14} + 10\sqrt{2}$ or $10\sqrt{2} - 2\sqrt{14}$

Ans: $c = 2\sqrt{14} + 10\sqrt{2}$ **or** $c = 10\sqrt{2} - 2\sqrt{14}$

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15. For the triangle with $a = 12$, $b = 37$, $c = 35$, find the inradius, the three exradii and the circumradius. [5]

$s = \frac{12+37+35}{2} = 42$
$\Delta = \sqrt{42 \cdot 30 \cdot 5 \cdot 7} = 210$
$r = \frac{\Delta}{s} = 5, \quad r_1 = \frac{\Delta}{s-a} = 7$
$r_2 = \frac{\Delta}{s-b} = 42, \quad r_3 = \frac{\Delta}{s-c} = 30$
$4R = r_1 + r_2 + r_3 - r = 74 \Rightarrow R = \frac{37}{2}$
Ans: $r = 5$, $r_1 = 7$, $r_2 = 42$, $r_3 = 30$, $R = \frac{37}{2}$

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16. If $r_1 = 6$, $r_2 = 12$ and $r_3 = 28$, find r, Δ, s and the sides of the triangle. [5]

$\frac{1}{r} = \frac{1}{6} + \frac{1}{12} + \frac{1}{28} = \frac{2}{7} \Rightarrow r = \frac{7}{2}$
$\Delta^2 = r r_1 r_2 r_3 = 7056 \Rightarrow \Delta = 84$
$s = \frac{\Delta}{r} = 24$
$s - a = \frac{\Delta}{r_1} = 14,\ s - b = \frac{\Delta}{r_2} = 7,\ s - c = \frac{\Delta}{r_3} = 3$
$a = 10,\ b = 17,\ c = 21$
Ans: $r = \frac{7}{2}$, $\Delta = 84$, $s = 24$; $a = 10$, $b = 17$, $c = 21$

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17. The triangle ABC has $a = 39$, $b = 25$, $c = 40$. Find R, the sides of its pedal triangle and the distance AO of A from the orthocentre O. [5]

$s = \frac{39+25+40}{2} = 52$
$\Delta = \sqrt{52 \cdot 13 \cdot 27 \cdot 12} = 468$
$R = \frac{abc}{4\Delta} = \frac{125}{6}$
$\cos A = \frac{44}{125},\ \cos B = \frac{4}{5},\ \cos C = \frac{7}{25}$ (all positive: acute)
Pedal sides: $a\cos A = \frac{1716}{125},\ b\cos B = 20,\ c\cos C = \frac{56}{5}$
$AO = 2R\cos A = \frac{44}{3}$
Ans: $R = \frac{125}{6}$; **pedal sides** $\frac{1716}{125}, 20, \frac{56}{5}$; $AO = \frac{44}{3}$

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18. Each interior angle of a regular polygon is $\frac{1980}{13}^\circ$. How many sides has it? [1]

$\frac{(n-2)180^\circ}{n} = \frac{1980}{13}^\circ$
$\Rightarrow n = 13$
Ans: $n = 13$

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19. Find the circumradius, the inradius and the area of an equilateral triangle of side 4. [3]

$\frac{\pi}{n} = 60^\circ$
$R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n} = \frac{4\sqrt{3}}{3}$
$r = \frac{a}{2}\cot\frac{\pi}{n} = \frac{2\sqrt{3}}{3}$
$\text{Area} = \frac{1}{4}na^2\cot\frac{\pi}{n} = 4\sqrt{3}$
Ans: $R = \frac{4\sqrt{3}}{3}$, $r = \frac{2\sqrt{3}}{3}$, **area** $= 4\sqrt{3}$

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20. Find the area of a cyclic quadrilateral with sides 9, 12, 5, 7. [2]

$s = \frac{9+12+5+7}{2} = \frac{33}{2}$
$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)} = \sqrt{\frac{15}{2}\cdot\frac{9}{2}\cdot\frac{23}{2}\cdot\frac{19}{2}} = \frac{3\sqrt{6555}}{4}$
Ans: $\frac{3\sqrt{6555}}{4}$
