EXPRESSWALA ACADEMY

Maths 1A: Properties of Triangles

Question Paper, Set 02

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. In triangle ABC, $A = 30°$, $B = 90°$ and $a = 8\sqrt{3}$. Find b. [1 mark]

2. In triangle ABC, $b = 2$, $c = 3$ and $A = 30°$. Find a. [2 marks]

3. In triangle ABC, $a = 7$, $b = 12$ and $\cos C = -\frac{4}{21}$. Find $c\cos B$. [1 mark]

4. Use Napier's rule to find $\tan\frac{B-C}{2}$ when $b = 8$, $c = 14$, $A = 120°$. [2 marks]

5. In triangle ABC, $a = 32$, $b = 34$, $c = 34$. Find $\cos\frac{A}{2}$. [2 marks]

6. Find the area of triangle ABC with $c = 5$, $a = 13$ and $B = 90°$. [1 mark]

7. For the triangle with $a = 25$, $b = 36$, $c = 29$, find the area and $\sin A$. [3 marks]

8. Using the area, find $\cot A$ for the triangle with $a = 15$, $b = 20$, $c = 25$. [2 marks]

9. If $a : b : c = 6 : 10 : 8$, find $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2}$. [1 mark]

10. Find the length of the altitude from C to the opposite side in the triangle with $a = 12$, $b = 10$, $c = 10$. [2 marks]

11. In triangle ABC, $b = 10$, $c = 2$ and $A = 120°$. The bisector of angle A meets BC at D. Find AD, a and BD. [3 marks]

12. The sides of triangle ABC are $a = \sqrt{3}$, $b = 2$, $c = 1$. Find its angles. [5 marks]

13. How many triangles have $a = 18$, $b = 14$ and $A = 30°$? [1 mark]

14. Show that two triangles have $a = 19$, $b = 20$, $A = 45°$, and find the two values of c. [3 marks]

15. For the triangle with $a = 35$, $b = 29$, $c = 8$, find the inradius, the three exradii and the circumradius. [5 marks]

16. If $r_1 = 5$, $r_2 = 60$ and $r_3 = 5$, find r, Δ, s and the sides of the triangle. [5 marks]

17. The triangle ABC has $a = 13$, $b = 13$, $c = 10$. Find R, the sides of its pedal triangle and the distance AO of A from the orthocentre O. [5 marks]

18. Each interior angle of a regular polygon is $162°$. How many sides has it? [1 mark]

19. Find the circumradius, the inradius and the area of a regular dodecagon (12 sides) of side 10. [3 marks]

20. In triangle ABC, $BC = 18$, $AC = 20$, $AB = 34$. D is on AB with $BD = \frac{136}{5}$. Find CD using Stewart's theorem. [2 marks]

Key sheet

Q	Answer
1	$b = 16\sqrt{3}$
2	$a = \sqrt{13 - 6\sqrt{3}}$
3	$\frac{65}{7}$
4	$-\frac{\sqrt{3}}{11}$
5	$\frac{15}{17}$
6	$\Delta = \frac{65}{2}$
7	$\Delta = 360$, $\sin A = \frac{20}{29}$
8	$\frac{4}{3}$
9	$3 : 1 : 2$
10	$\frac{48}{5}$
11	$AD = \frac{5}{3}$, $a = 2\sqrt{31}$, $BD = \frac{\sqrt{31}}{3}$
12	$A = 60^\circ$, $B = 90^\circ$, $C = 30^\circ$
13	One triangle
14	$c = \sqrt{161} + 10\sqrt{2}$ or $c = 10\sqrt{2} - \sqrt{161}$
15	$r = \frac{7}{3}$, $r_1 = 84$, $r_2 = 12$, $r_3 = 3$, $R = \frac{145}{6}$
16	$r = \frac{12}{5}$, $\Delta = 60$, $s = 25$; $a = 13$, $b = 24$, $c = 13$
17	$R = \frac{169}{24}$; pedal sides $5, 5, \frac{1190}{169}$; $AO = \frac{65}{12}$
18	$n = 20$
19	$R = 5\sqrt{2} + 5\sqrt{6}$, $r = 5\sqrt{3} + 10$, area $= 300\sqrt{3} + 600$
20	$CD = \frac{2\sqrt{1249}}{5}$

Solutions

1. In triangle ABC, $A = 30°$, $B = 90°$ and $a = 8\sqrt{3}$. Find b. [1]

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$b = \frac{8\sqrt{3}\sin 90°}{\sin 30°} = 16\sqrt{3}$

Ans: $b = 16\sqrt{3}$

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2. In triangle ABC, $b = 2$, $c = 3$ and $A = 30°$. Find a. [2]

$a^2 = b^2 + c^2 - 2bc\cos A = 4 + 9 - 12\cos 30°$

$a^2 = 13 - 6\sqrt{3}$

$a = \sqrt{13 - 6\sqrt{3}}$

Ans: $a = \sqrt{13 - 6\sqrt{3}}$

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3. In triangle ABC, $a = 7$, $b = 12$ and $\cos C = -\frac{4}{21}$. Find $c\cos B$. [1]

$a = b\cos C + c\cos B$

$c\cos B = 7 - 12 \cdot -\frac{4}{21} = \frac{65}{7}$

Ans: $\frac{65}{7}$

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4. Use Napier's rule to find $\tan\frac{B-C}{2}$ when $b = 8$, $c = 14$, $A = 120°$. [2]

$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

$= \frac{-6}{22}\cot 60° = -\frac{3}{11} \cdot \frac{\sqrt{3}}{3} = -\frac{\sqrt{3}}{11}$

Ans: $-\frac{\sqrt{3}}{11}$

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5. In triangle ABC, $a = 32$, $b = 34$, $c = 34$. Find $\cos\frac{A}{2}$. [2]

$s = 50$

$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}} = \sqrt{\frac{50 \cdot 18}{34 \cdot 34}} = \frac{15}{17}$

Ans: $\frac{15}{17}$

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6. Find the area of triangle ABC with $c = 5$, $a = 13$ and $B = 90°$. [1]

$\Delta = \frac{1}{2}ca\sin B = \frac{1}{2} \cdot 5 \cdot 13 \cdot 1$

$= \frac{65}{2}$

Ans: $\Delta = \frac{65}{2}$

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7. For the triangle with $a = 25$, $b = 36$, $c = 29$, find the area and $\sin A$. [3]

$s = \frac{25+36+29}{2} = 45$

$\Delta = \sqrt{45 \cdot 20 \cdot 9 \cdot 16} = 360$

$\sin A = \frac{2\Delta}{bc} = \frac{720}{1044} = \frac{20}{29}$

Ans: $\Delta = 360$, $\sin A = \frac{20}{29}$

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8. Using the area, find $\cot A$ for the triangle with $a = 15$, $b = 20$, $c = 25$. [2]

$s = \frac{15+20+25}{2} = 30$

$\Delta = \sqrt{30 \cdot 15 \cdot 10 \cdot 5} = 150$

$\cot A = \frac{b^2+c^2-a^2}{4\Delta} = \frac{800}{600} = \frac{4}{3}$

Ans: $\frac{4}{3}$

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9. If $a : b : c = 6 : 10 : 8$, find $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2}$. [1]

$\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2} = (s-a) : (s-b) : (s-c) = (y+z-x) : (z+x-y) : (x+y-z)$

$= 12 : 4 : 8 = 3 : 1 : 2$

Ans: $3 : 1 : 2$

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10. Find the length of the altitude from C to the opposite side in the triangle with $a = 12$, $b = 10$, $c = 10$. [2]

$s = \frac{12+10+10}{2} = 16$

$\Delta = \sqrt{16 \cdot 4 \cdot 6 \cdot 6} = 48$

$\text{altitude} = \frac{2\Delta}{c} = \frac{96}{10} = \frac{48}{5}$

Ans: $\frac{48}{5}$

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11. In triangle ABC, $b = 10$, $c = 2$ and $A = 120^\circ$. The bisector of angle A meets BC at D. Find AD, a and BD. [3]

$AD = \frac{2bc}{b+c}\cos\frac{A}{2} = \frac{40}{12}\cos 60^\circ = \frac{5}{3}$

$a^2 = 100 + 4 - 40\cos 120^\circ = 124,\ a = 2\sqrt{31}$

$BD : DC = c : b = 2 : 10 \Rightarrow BD = \frac{2}{12} \cdot 2\sqrt{31} = \frac{\sqrt{31}}{3}$

Ans: $AD = \frac{5}{3}$, $a = 2\sqrt{31}$, $BD = \frac{\sqrt{31}}{3}$

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12. The sides of triangle ABC are $a = \sqrt{3}$, $b = 2$, $c = 1$. Find its angles. [5]

$a^2 = 3,\ b^2 = 4,\ c^2 = 1$

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{1}{2} \Rightarrow A = 60^\circ$

$\cos B = \frac{c^2+a^2-b^2}{2ca} = 0 \Rightarrow B = 90^\circ$

$C = 180^\circ - 60^\circ - 90^\circ = 30^\circ$

Ans: $A = 60^\circ$, $B = 90^\circ$, $C = 30^\circ$

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13. How many triangles have $a = 18$, $b = 14$ and $A = 30^\circ$? [1]

$b\sin A = 14 \cdot \frac{1}{2} = 7$

$a \geq b$: exactly one triangle.

Ans: One triangle

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14. Show that two triangles have $a = 19$, $b = 20$, $A = 45^\circ$, and find the two values of c. [3]

$b\sin A = 10\sqrt{2}$, and $10\sqrt{2} < 19 < 20$: two triangles.

$c = b\cos A \pm \sqrt{a^2 - b^2\sin^2 A} = 10\sqrt{2} \pm \sqrt{361 - 200}$

$c = \sqrt{161} + 10\sqrt{2}$ or $10\sqrt{2} - \sqrt{161}$

Ans: $c = \sqrt{161} + 10\sqrt{2}$ **or** $c = 10\sqrt{2} - \sqrt{161}$

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15. For the triangle with $a = 35$, $b = 29$, $c = 8$, find the inradius, the three exradii and the circumradius. [5]

$s = \frac{35+29+8}{2} = 36$
$\Delta = \sqrt{36 \cdot 1 \cdot 7 \cdot 28} = 84$
$r = \frac{\Delta}{s} = \frac{7}{3}, \quad r_1 = \frac{\Delta}{s-a} = 84$
$r_2 = \frac{\Delta}{s-b} = 12, \quad r_3 = \frac{\Delta}{s-c} = 3$
$4R = r_1 + r_2 + r_3 - r = \frac{290}{3} \Rightarrow R = \frac{145}{6}$
Ans: $r = \frac{7}{3}$, $r_1 = 84$, $r_2 = 12$, $r_3 = 3$, $R = \frac{145}{6}$

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16. If $r_1 = 5$, $r_2 = 60$ and $r_3 = 5$, find r, Δ, s and the sides of the triangle. [5]

$\frac{1}{r} = \frac{1}{5} + \frac{1}{60} + \frac{1}{5} = \frac{5}{12} \Rightarrow r = \frac{12}{5}$
$\Delta^2 = r r_1 r_2 r_3 = 3600 \Rightarrow \Delta = 60$
$s = \frac{\Delta}{r} = 25$
$s - a = \frac{\Delta}{r_1} = 12,\ s - b = \frac{\Delta}{r_2} = 1,\ s - c = \frac{\Delta}{r_3} = 12$
$a = 13,\ b = 24,\ c = 13$
Ans: $r = \frac{12}{5}$, $\Delta = 60$, $s = 25$; $a = 13$, $b = 24$, $c = 13$

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17. The triangle ABC has $a = 13$, $b = 13$, $c = 10$. Find R, the sides of its pedal triangle and the distance AO of A from the orthocentre O. [5]

$s = \frac{13+13+10}{2} = 18$
$\Delta = \sqrt{18 \cdot 5 \cdot 5 \cdot 8} = 60$
$R = \frac{abc}{4\Delta} = \frac{169}{24}$
$\cos A = \frac{5}{13},\ \cos B = \frac{5}{13},\ \cos C = \frac{119}{169}$ (all positive: acute)
Pedal sides: $a\cos A = 5,\ b\cos B = 5,\ c\cos C = \frac{1190}{169}$
$AO = 2R\cos A = \frac{65}{12}$
Ans: $R = \frac{169}{24}$; **pedal sides** $5, 5, \frac{1190}{169}$; $AO = \frac{65}{12}$

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18. Each interior angle of a regular polygon is 162°. How many sides has it? [1]

$\frac{(n-2)180^\circ}{n} = 162^\circ$
$\Rightarrow n = 20$
Ans: $n = 20$

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19. Find the circumradius, the inradius and the area of a regular dodecagon (12 sides) of side 10. [3]

$\frac{\pi}{n} = 15^\circ$
$R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n} = 5\sqrt{2} + 5\sqrt{6}$
$r = \frac{a}{2}\cot\frac{\pi}{n} = 5\sqrt{3} + 10$
$\text{Area} = \frac{1}{4}na^2\cot\frac{\pi}{n} = 300\sqrt{3} + 600$
Ans: $R = 5\sqrt{2} + 5\sqrt{6}$, $r = 5\sqrt{3} + 10$, **area** $= 300\sqrt{3} + 600$

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20. In triangle ABC, $BC = 18$, $AC = 20$, $AB = 34$. D is on AB with $BD = \frac{136}{5}$. Find CD using Stewart's theorem. [2]

$DA = 34 - \frac{136}{5} = \frac{34}{5}$

$a^2n + b^2m = c(d^2 + mn)$

$324 \cdot \frac{34}{5} + 400 \cdot \frac{136}{5} = 34(d^2 + \frac{4624}{25})$

$d^2 = \frac{4996}{25}$, $CD = \frac{2\sqrt{1249}}{5}$

Ans: $CD = \frac{2\sqrt{1249}}{5}$