EXPRESSWALA ACADEMY

Maths 1A: Properties of Triangles

Question Paper, Set 01

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. In triangle ABC, $a = 4$ and $A = 135^\circ$. Find the circumradius R. [1 mark]

2. In triangle ABC, $a = 26$, $b = 16$, $c = 30$. Find the angle A. [2 marks]

3. In triangle ABC, $a = 5$, $b = 12$ and $\cos C = 0$. Find $c \cos B$. [1 mark]

4. In triangle ABC, $b = 13$, $c = 3$ and $A = 90^\circ$. Find $\tan \frac{B-C}{2}$. [2 marks]

5. In triangle ABC, $a = 12$, $b = 20$, $c = 16$. Find $\tan \frac{C}{2}$. [2 marks]

6. Find the area of triangle ABC with $c = 8$, $a = 10$ and $B = 45^\circ$. [1 mark]

7. Find the area and the circumradius of the triangle with $a = 15$, $b = 26$, $c = 37$. [3 marks]

8. For the triangle with $a = 20$, $b = 20$, $c = 32$, find $\cot A + \cot B + \cot C$. [2 marks]

9. In triangle ABC, $a + c = 15b$. Find $\tan \frac{A}{2} \tan \frac{C}{2}$. [1 mark]

10. Find the length of the median from C in the triangle with $a = 6$, $b = 8$, $c = 9$. [2 marks]

11. In triangle ABC, $b = 5$, $c = 12$ and $A = 60^\circ$. The bisector of angle A meets BC at D. Find AD, a and BD. [3 marks]

12. Solve the triangle ABC in which $A = 30^\circ$, $B = 90^\circ$ and $c = 4\sqrt{3}$ (find C, a, b and R). [5 marks]

13. How many triangles have $a = 23$, $b = 14\sqrt{2}$ and $A = 45^\circ$? [1 mark]

14. Show that two triangles have $a = 17$, $b = 20$, $A = 45^\circ$, and find the two values of c. [3 marks]

15. Find r, r_1, r_2, r_3 and R for the triangle with $a = 10$, $b = 35$, $c = 39$. [5 marks]

16. Find the sides a, b, c of the triangle in which $r_1 = 36$, $r_2 = 18$, $r_3 = 12$. [5 marks]

17. For the triangle with $a = 17$, $b = 25$, $c = 26$, find R, r, the distance IS between the incentre and the circumcentre, and the distance of S from BC. [5 marks]

18. Find the sum of the interior angles of a polygon with 18 sides, in right angles and in degrees. [1 mark]

19. Find the circumradius, the inradius and the area of a regular hexagon of side 3. [3 marks]

20. Find the area of a cyclic quadrilateral with sides 3, 13, 12, 2. [2 marks]

Key sheet

Q	Answer
1	$R = 2\sqrt{2}$
2	$A = 60^\circ$
3	5
4	$\frac{5}{8}$
5	$\frac{1}{2}$
6	$\Delta = 20\sqrt{2}$
7	$\Delta = 156$, $R = \frac{185}{8}$
8	$\frac{19}{8}$
9	$\frac{7}{8}$
10	$\frac{\sqrt{119}}{2}$
11	$AD = \frac{60\sqrt{3}}{17}$, $a = \sqrt{109}$, $BD = \frac{12\sqrt{109}}{17}$
12	$C = 60^\circ$, $a = 4$, $b = 8$, $R = 4$
13	One triangle
14	$c = \sqrt{89} + 10\sqrt{2}$ or $c = 10\sqrt{2} - \sqrt{89}$
15	$r = 4$, $r_1 = \frac{21}{4}$, $r_2 = 24$, $r_3 = 56$, $R = \frac{325}{16}$
16	$a = 30$, $b = 24$, $c = 18$
17	$R = \frac{325}{24}$, $r = 6$, $IS = \frac{5\sqrt{481}}{24}$, distance $= \frac{253}{24}$
18	32 right angles $= 2880^\circ$
19	$R = 3$, $r = \frac{3\sqrt{3}}{2}$, area $= \frac{27\sqrt{3}}{2}$
20	$6\sqrt{26}$

Solutions

1. In triangle ABC, $a = 4$ and $A = 135^\circ$. Find the circumradius R. [1]

$\frac{a}{\sin A} = 2R$

$2R = \frac{4}{\sin 135^\circ} = \frac{4}{\frac{\sqrt{2}}{2}} = 4\sqrt{2}$

$R = 2\sqrt{2}$

Ans: $R = 2\sqrt{2}$

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2. In triangle ABC, $a = 26$, $b = 16$, $c = 30$. Find the angle A. [2]

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{256+900-676}{960} = \frac{1}{2}$

$A = 60^\circ$

Ans: $A = 60^\circ$

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3. In triangle ABC, $a = 5$, $b = 12$ and $\cos C = 0$. Find $c\cos B$. [1]

$a = b\cos C + c\cos B$

$c\cos B = 5 - 12 \cdot 0 = 5$

Ans: 5

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4. In triangle ABC, $b = 13$, $c = 3$ and $A = 90^\circ$. Find $\tan\frac{B-C}{2}$. [2]

$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

$= \frac{10}{16}\cot 45^\circ = \frac{5}{8} \cdot 1 = \frac{5}{8}$

Ans: $\frac{5}{8}$

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5. In triangle ABC, $a = 12$, $b = 20$, $c = 16$. Find $\tan\frac{C}{2}$. [2]

$s = \frac{12+20+16}{2} = 24$

$\Delta = \sqrt{24 \cdot 12 \cdot 4 \cdot 8} = 96$

$\tan\frac{C}{2} = \frac{(s-a)(s-b)}{\Delta} = \frac{12 \cdot 4}{96} = \frac{1}{2}$

Ans: $\frac{1}{2}$

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6. Find the area of triangle ABC with $c = 8$, $a = 10$ and $B = 45^\circ$. [1]

$\Delta = \frac{1}{2}ca\sin B = \frac{1}{2} \cdot 8 \cdot 10 \cdot \frac{\sqrt{2}}{2}$

$= 20\sqrt{2}$

Ans: $\Delta = 20\sqrt{2}$

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7. Find the area and the circumradius of the triangle with $a = 15$, $b = 26$, $c = 37$. [3]

$s = \frac{15+26+37}{2} = 39$

$\Delta = \sqrt{39 \cdot 24 \cdot 13 \cdot 2} = 156$

$R = \frac{abc}{4\Delta} = \frac{14430}{624} = \frac{185}{8}$

Ans: $\Delta = 156$, $R = \frac{185}{8}$

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8. For the triangle with $a = 20$, $b = 20$, $c = 32$, find $\cot A + \cot B + \cot C$. [2]

$s = \frac{20+20+32}{2} = 36$

$\Delta = \sqrt{36 \cdot 16 \cdot 16 \cdot 4} = 192$

$\cot A + \cot B + \cot C = \frac{a^2+b^2+c^2}{4\Delta} = \frac{1824}{768} = \frac{19}{8}$

Ans: $\frac{19}{8}$

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9. In triangle ABC, $a + c = 15b$. Find $\tan\frac{A}{2}\tan\frac{C}{2}$. [1]

$\tan\frac{A}{2}\tan\frac{C}{2} = \frac{n-1}{n+1}$

$= \frac{15-1}{15+1} = \frac{7}{8}$

Ans: $\frac{7}{8}$

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10. Find the length of the median from C in the triangle with $a = 6$, $b = 8$, $c = 9$. [2]

$m = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2} = \frac{1}{2}\sqrt{72 + 128 - 81}$

$= \frac{1}{2}\sqrt{119} = \frac{\sqrt{119}}{2}$

Ans: $\frac{\sqrt{119}}{2}$

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11. In triangle ABC, $b = 5$, $c = 12$ and $A = 60°$. The bisector of angle A meets BC at D. Find AD, a and BD. [3]

$AD = \frac{2bc}{b+c}\cos\frac{A}{2} = \frac{120}{17}\cos 30° = \frac{60\sqrt{3}}{17}$

$a^2 = 25 + 144 - 120\cos 60° = 109,\ a = \sqrt{109}$

$BD : DC = c : b = 12 : 5 \Rightarrow BD = \frac{12}{17} \cdot \sqrt{109} = \frac{12\sqrt{109}}{17}$

Ans: $AD = \frac{60\sqrt{3}}{17}$, $a = \sqrt{109}$, $BD = \frac{12\sqrt{109}}{17}$

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12. Solve the triangle ABC in which $A = 30°$, $B = 90°$ and $c = 4\sqrt{3}$ (find C, a, b and R). [5]

$C = 180° - (30° + 90°) = 60°$

$2R = \frac{c}{\sin C} = \frac{4\sqrt{3}}{\frac{\sqrt{3}}{2}} = 8$

$a = 2R\sin A = 8 \cdot \frac{1}{2} = 4$

$b = 2R\sin B = 8 \cdot 1 = 8$

$R = 4$

Ans: $C = 60°$, $a = 4$, $b = 8$, $R = 4$

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13. How many triangles have $a = 23$, $b = 14\sqrt{2}$ and $A = 45°$? [1]

$b\sin A = 14\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 14$

$a \geq b$: exactly one triangle.

Ans: One triangle

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14. Show that two triangles have $a = 17$, $b = 20$, $A = 45°$, and find the two values of c. [3]

$b\sin A = 10\sqrt{2}$, and $10\sqrt{2} < 17 < 20$: two triangles.

$c = b\cos A \pm \sqrt{a^2 - b^2\sin^2 A} = 10\sqrt{2} \pm \sqrt{289 - 200}$

$c = \sqrt{89} + 10\sqrt{2}$ or $10\sqrt{2} - \sqrt{89}$

Ans: $c = \sqrt{89} + 10\sqrt{2}$ **or** $c = 10\sqrt{2} - \sqrt{89}$

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15. Find r, r_1, r_2, r_3 and R for the triangle with $a = 10$, $b = 35$, $c = 39$. [5]

$s = \frac{10+35+39}{2} = 42$

$\Delta = \sqrt{42 \cdot 32 \cdot 7 \cdot 3} = 168$

$r = \frac{\Delta}{s} = 4, \quad r_1 = \frac{\Delta}{s-a} = \frac{21}{4}$

$r_2 = \frac{\Delta}{s-b} = 24, \quad r_3 = \frac{\Delta}{s-c} = 56$

$4R = r_1 + r_2 + r_3 - r = \frac{325}{4} \Rightarrow R = \frac{325}{16}$

Ans: $r = 4$, $r_1 = \frac{21}{4}$, $r_2 = 24$, $r_3 = 56$, $R = \frac{325}{16}$

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16. Find the sides a, b, c of the triangle in which $r_1 = 36$, $r_2 = 18$, $r_3 = 12$. [5]

$\frac{1}{r} = \frac{1}{36} + \frac{1}{18} + \frac{1}{12} = \frac{1}{6} \Rightarrow r = 6$

$\Delta^2 = r r_1 r_2 r_3 = 46656 \Rightarrow \Delta = 216$

$s = \frac{\Delta}{r} = 36$

$s - a = \frac{\Delta}{r_1} = 6, \; s - b = \frac{\Delta}{r_2} = 12, \; s - c = \frac{\Delta}{r_3} = 18$

$a = 30, \; b = 24, \; c = 18$

Ans: $a = 30$, $b = 24$, $c = 18$

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17. For the triangle with $a = 17$, $b = 25$, $c = 26$, find R, r, the distance IS between the incentre and the circumcentre, and the distance of S from BC. [5]

$s = \frac{17+25+26}{2} = 34$

$\Delta = \sqrt{34 \cdot 17 \cdot 9 \cdot 8} = 204$

$R = \frac{abc}{4\Delta} = \frac{325}{24}, \quad r = \frac{\Delta}{s} = 6$

$IS = \sqrt{R^2 - 2rR} = \sqrt{\frac{105625}{576} - \frac{325}{2}} = \frac{5\sqrt{481}}{24}$

$\cos A = \frac{253}{325}, \;$ distance of S from $BC = R\cos A = \frac{253}{24}$

Ans: $R = \frac{325}{24}$, $r = 6$, $IS = \frac{5\sqrt{481}}{24}$, **distance** $= \frac{253}{24}$

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18. Find the sum of the interior angles of a polygon with 18 sides, in right angles and in degrees. [1]

$(2n - 4)$ right angles $= 32$ right angles

$= 32 \times 90^\circ = 2880^\circ$

Ans: 32 **right angles** $= 2880^\circ$

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19. Find the circumradius, the inradius and the area of a regular hexagon of side 3. [3]

$\frac{\pi}{n} = 30^\circ$

$R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n} = 3$

$r = \frac{a}{2}\cot\frac{\pi}{n} = \frac{3\sqrt{3}}{2}$

$\text{Area} = \frac{1}{4}na^2\cot\frac{\pi}{n} = \frac{27\sqrt{3}}{2}$

Ans: $R = 3$, $r = \frac{3\sqrt{3}}{2}$, **area** $= \frac{27\sqrt{3}}{2}$

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20. Find the area of a cyclic quadrilateral with sides 3, 13, 12, 2. [2]

$s = \frac{3+13+12+2}{2} = 15$

$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)} = \sqrt{12 \cdot 2 \cdot 3 \cdot 13} = 6\sqrt{26}$

Ans: $6\sqrt{26}$
